Ginzburg Landau Approach to Superconductivity in Cuprates

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- A superconductor is characterized by a nonzero quantity $\psi(\mathbf{r})$

(Ginzburg and Landau, 1952)

(amplitude for electron **pairs**)

$$\Psi = \Delta \exp(i\varphi) \ (\Delta \text{ and } \varphi \text{ real})$$

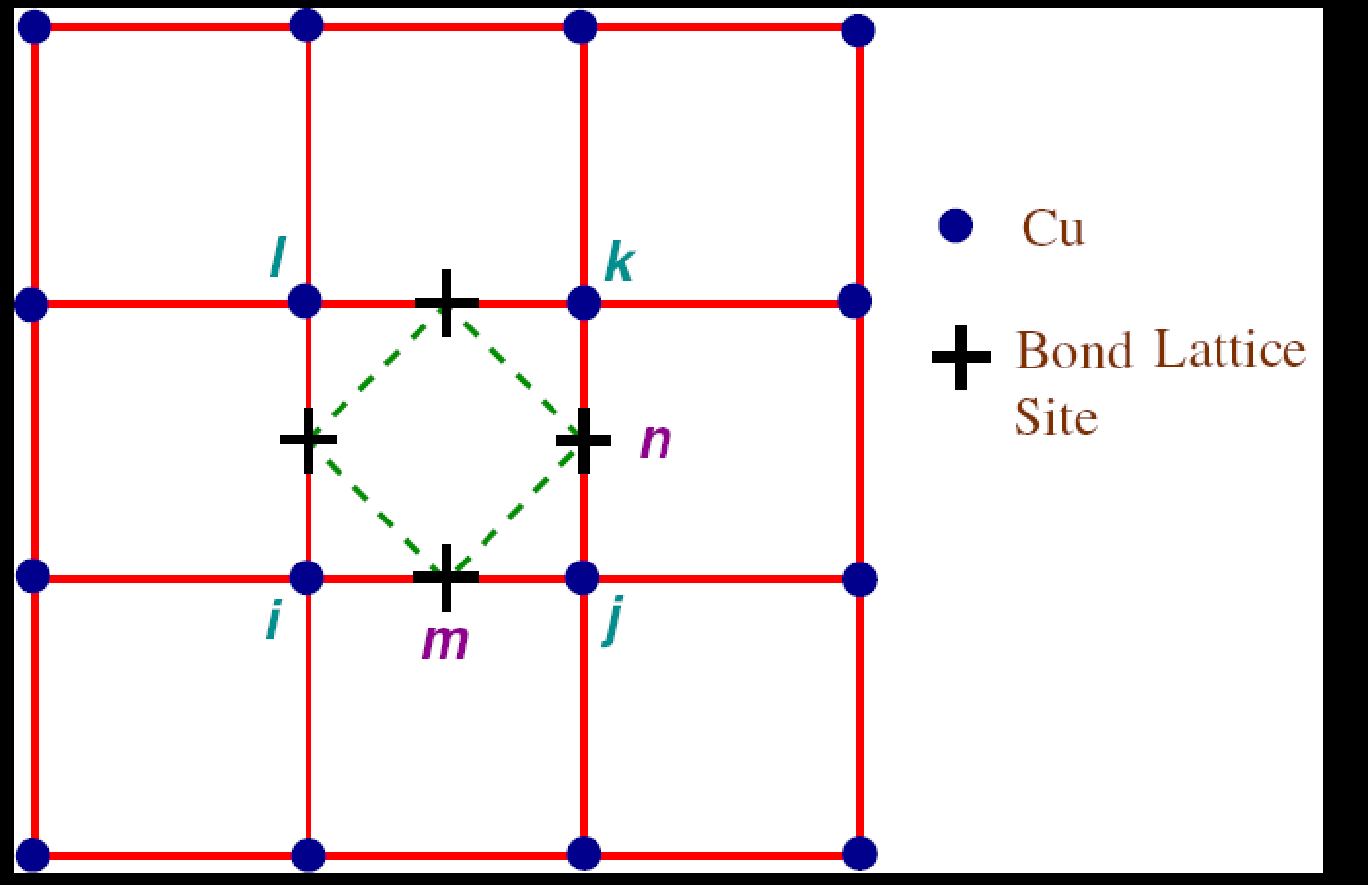
The free energy F is a functional of $\psi(\mathbf{r})$

A common form is

$$F= \int \{\alpha \,|\psi|^2 + (\beta/2)\,|\psi|^4 + \gamma|\partial\psi\,|^2\}$$

Ginzburg and Landau obtained a ‘Schroedinger’ like equation for $\psi(\mathbf{r})$

Deep,versatile description of superconductivity



F is expressed as a functional of $\psi_{ij} = \psi_m = \Delta_m \exp(i\varphi_m)$
The form is similar to the original GL form.
The coefficients are inspired by experiment.

- $F = F_o + F_1$ where

$$F_o = \sum_m \{ \alpha\Delta_m{}^2 + (\beta/2)\Delta_m{}^4 \}$$

and $$F_1 = \gamma \sum_{mn} \Delta_m \Delta_n \cos(\varphi_m - \varphi_n)$$

$$Z = \int \exp\{- F/k_B T\}$$

- Model of classical planar spins, length Δ_m and direction φ_m with nearest neighbour coupling

  (d-wave superconductivity → AF or Neel order)

- α, β and γ inspired by experiment;

  eg $\alpha \sim T_o (1 – (x/x_c))$ ('pseudogap' line)

  $\gamma \sim x$

- Calculate for example T_c (for γ positive ; onset of d-wave phase coherence)
- Should calculate the coefficients from microscopic electron theory; eg a la Gorkov for BCS
- We do not do this; however, for example, it is easy to see that

  $\gamma \sim xt'$ for strong correlation.

One approach in principle: start with a microscopic (eg strong local Hubbard repulsion U) lattice Hamiltonian with J (nearest neighbour superexchange for which there is strong experimental evidence at zero doping, with J ~ 0.15 eV)

≡ nn singlet pair self interaction

Hubbard Stratonovich : electrons in time dependent nn singlet (bond) pair potential.

An interacting two fluid (bosonic pair amplitude, and fermionic electrons) picture results

Ignore quantum fluctuations in Bose field and integrate out electrons, eg on the nn scale

F is the resulting 'bosonic' effective Hamiltonian

There are other terms of order x, eg 'nematic' ; these as well as no-single site terms in F_o are ignored.

- Inputs : $\alpha = 4\{T - T_o(1 - x/0.3)\} \exp(T/T_o)$

  $\beta = 1.6\ T_o$ and

  $\gamma = 4xT_o$
- Output :

  Onset of nonzero phase stiffness ('Neel' order) or T_c

  $\sqrt{\langle \Delta_m^2 \rangle}$

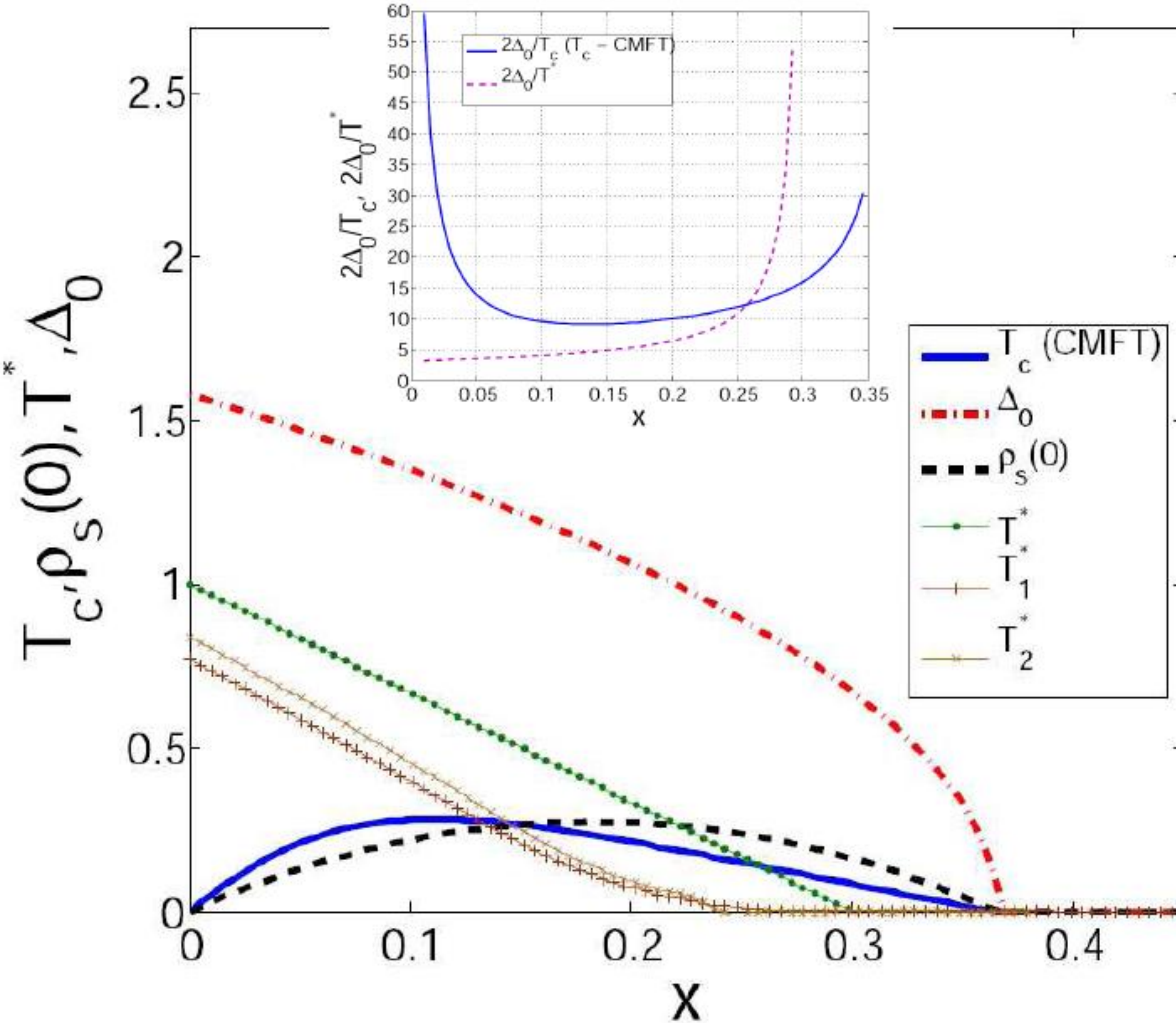
  ρ_s

  C_v

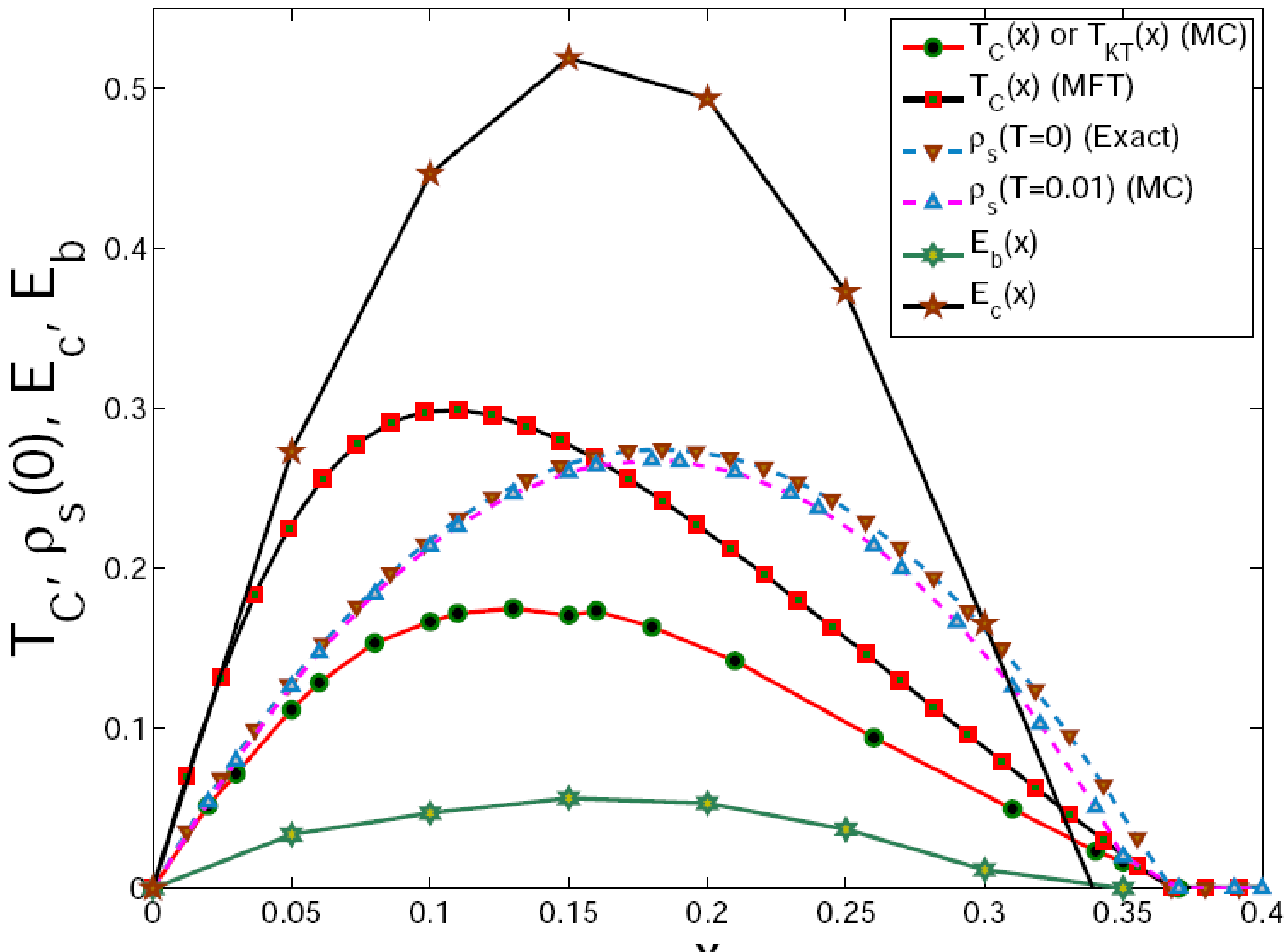
  Vortex structure and energetics

  Electron Green's function using $\langle \psi_m^* \psi_n \rangle$

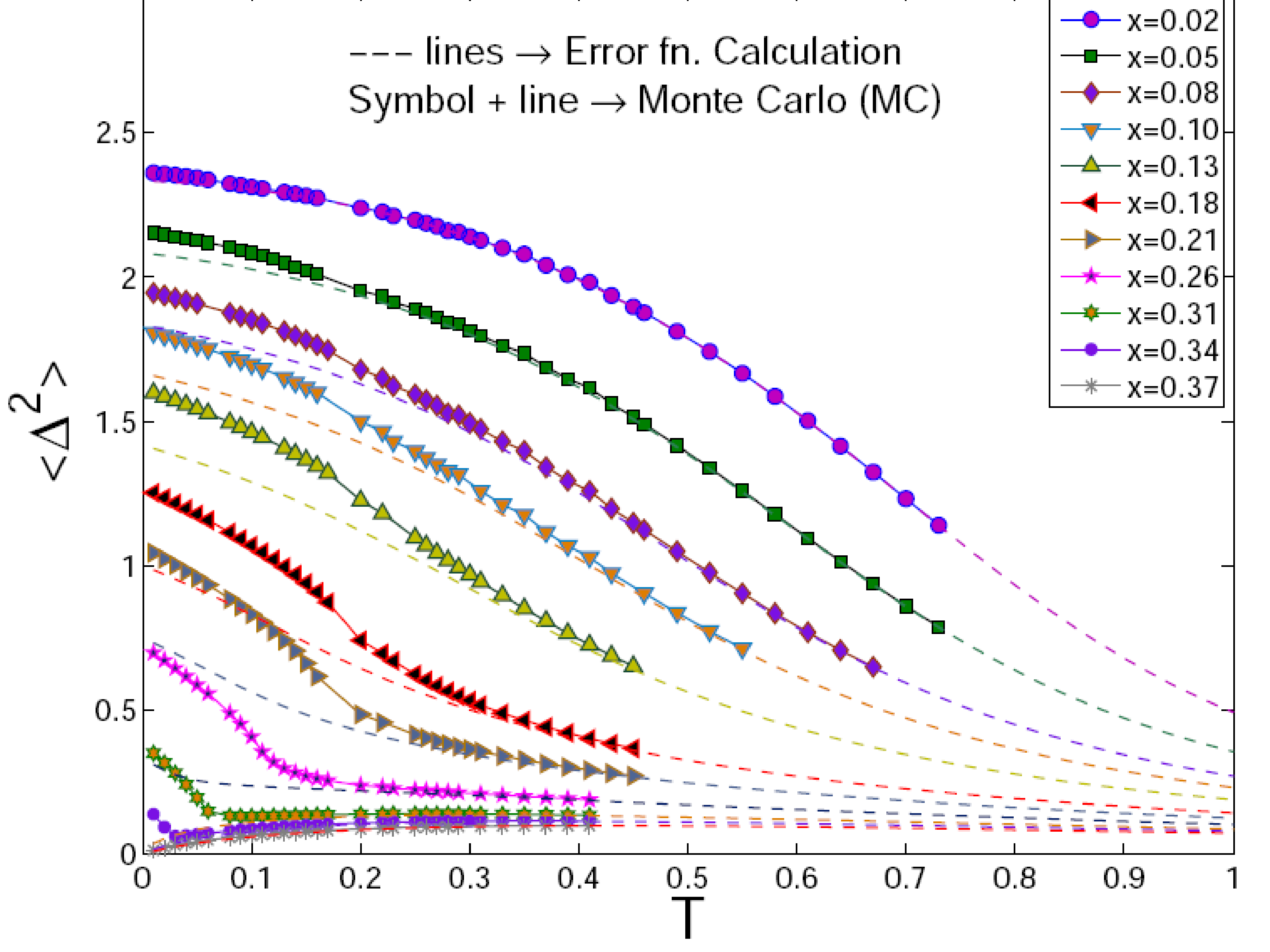
  coupled to electrons (especially useful for large $|m-n|$)

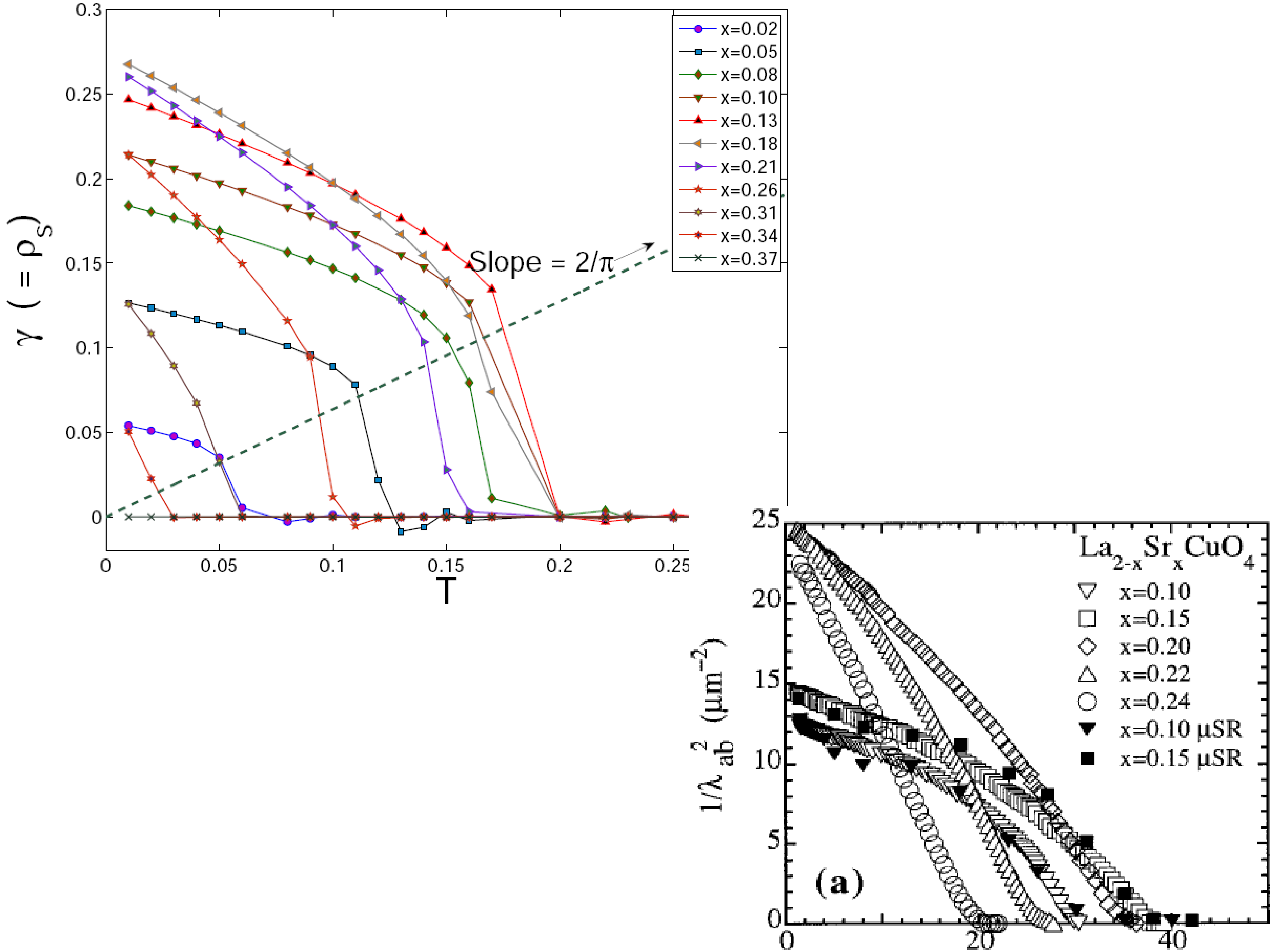


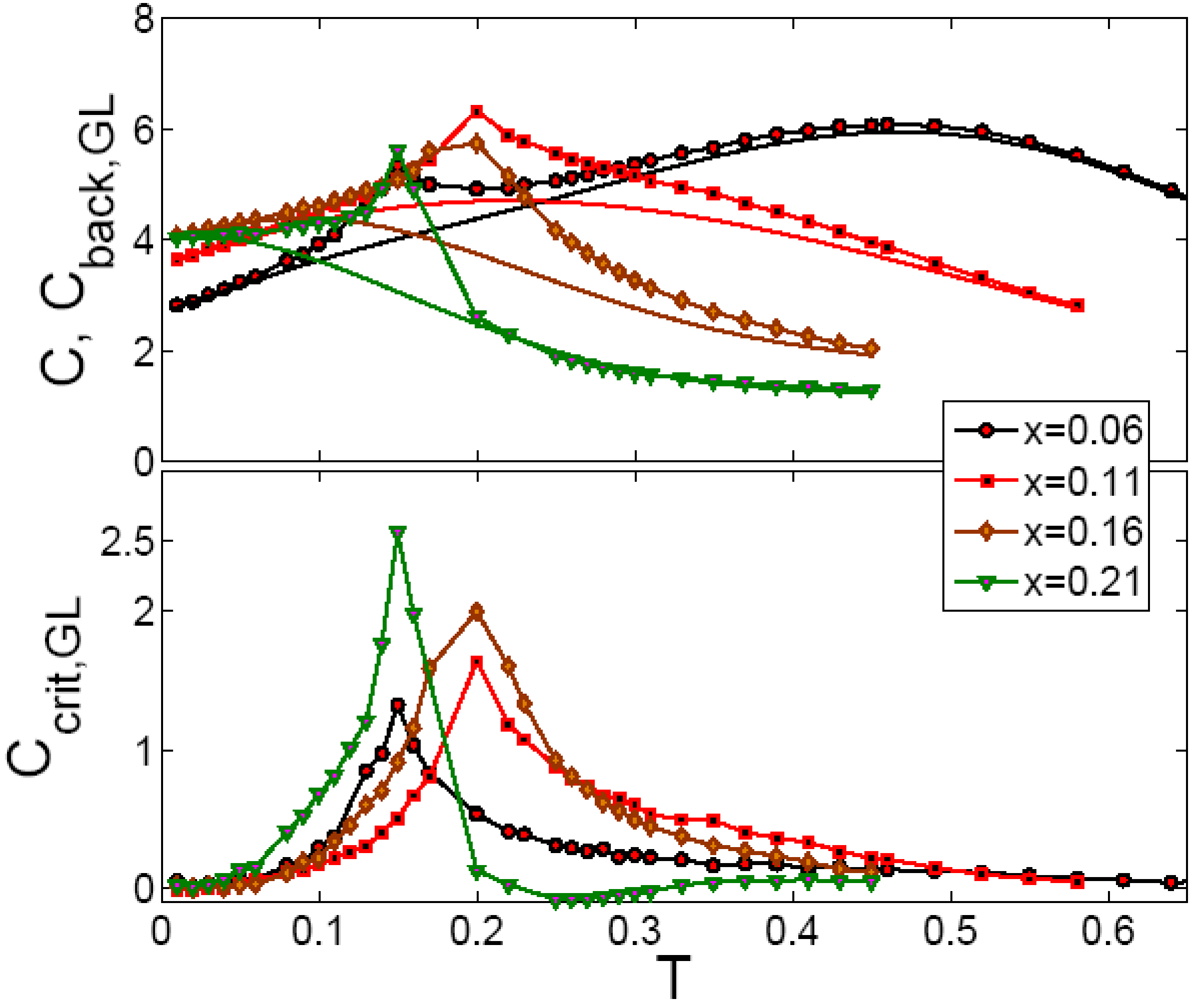
2Δ0/Tc (Tc - CMFT)
2Δ0/T*
2Δ0/Tc, 2Δ0/T*
x
Tc (CMFT)
Δ0
ρs(0)
T*
T*1
T*2
Tc, ρs(0), T*, Δ0
x



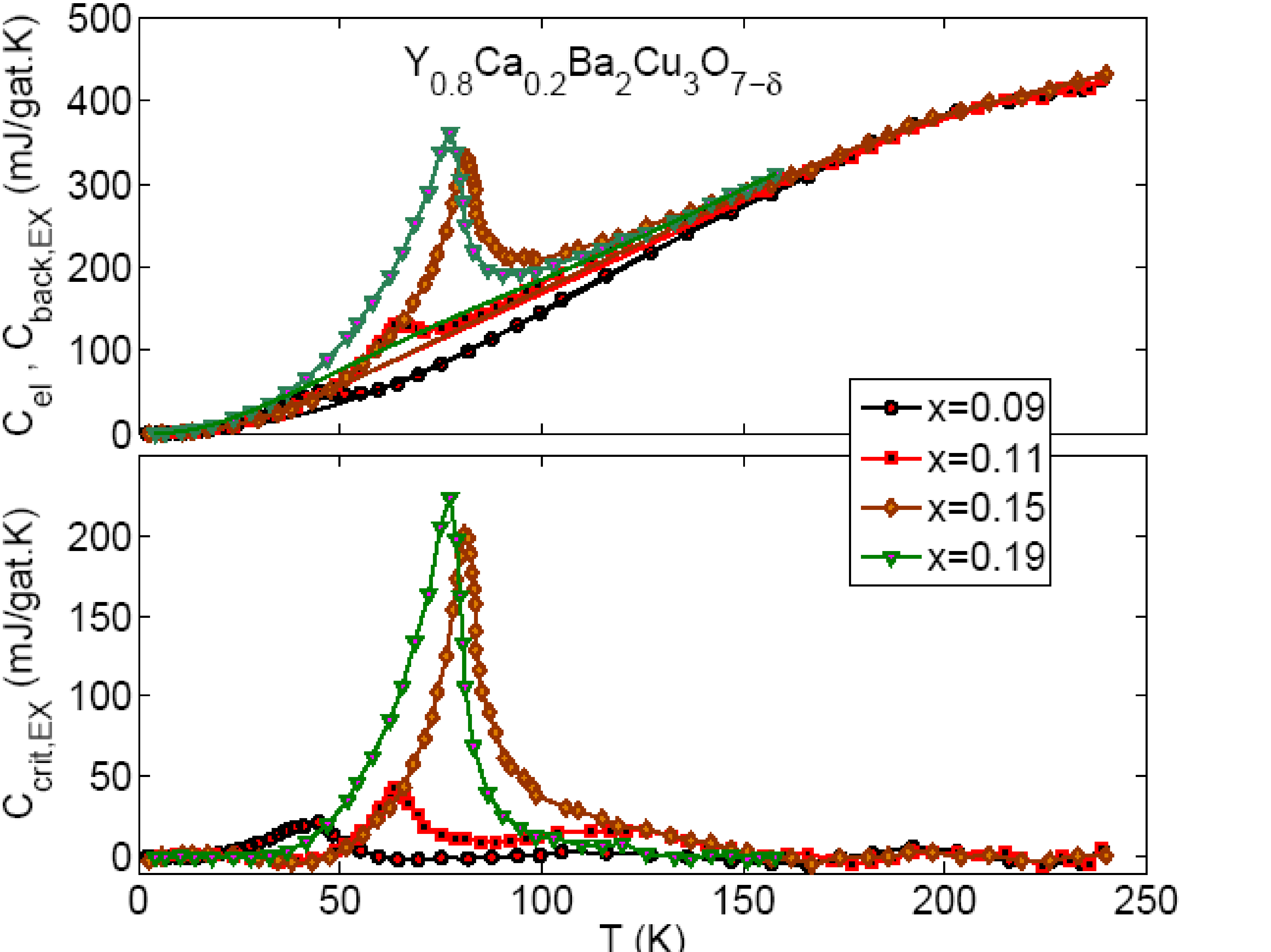
$T_C(x)$ or $T_{KT}(x)$ (MC)
$T_C(x)$ (MFT)
$\rho_s(T=0)$ (Exact)
$\rho_s(T=0.01)$ (MC)
$E_b(x)$
$E_c(x)$
$T_C, \rho_s(0), E_c, E_b$
x
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.1
0.2
0.3
0.4
0.5


--- lines → Error fn. Calculation
Symbol + line → Monte Carlo (MC)
x=0.02
x=0.05
x=0.08
x=0.10
x=0.13
x=0.18
x=0.21
x=0.26
x=0.31
x=0.34
x=0.37
<Δ^2>
T
0
0.5
1
1.5
2
2.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

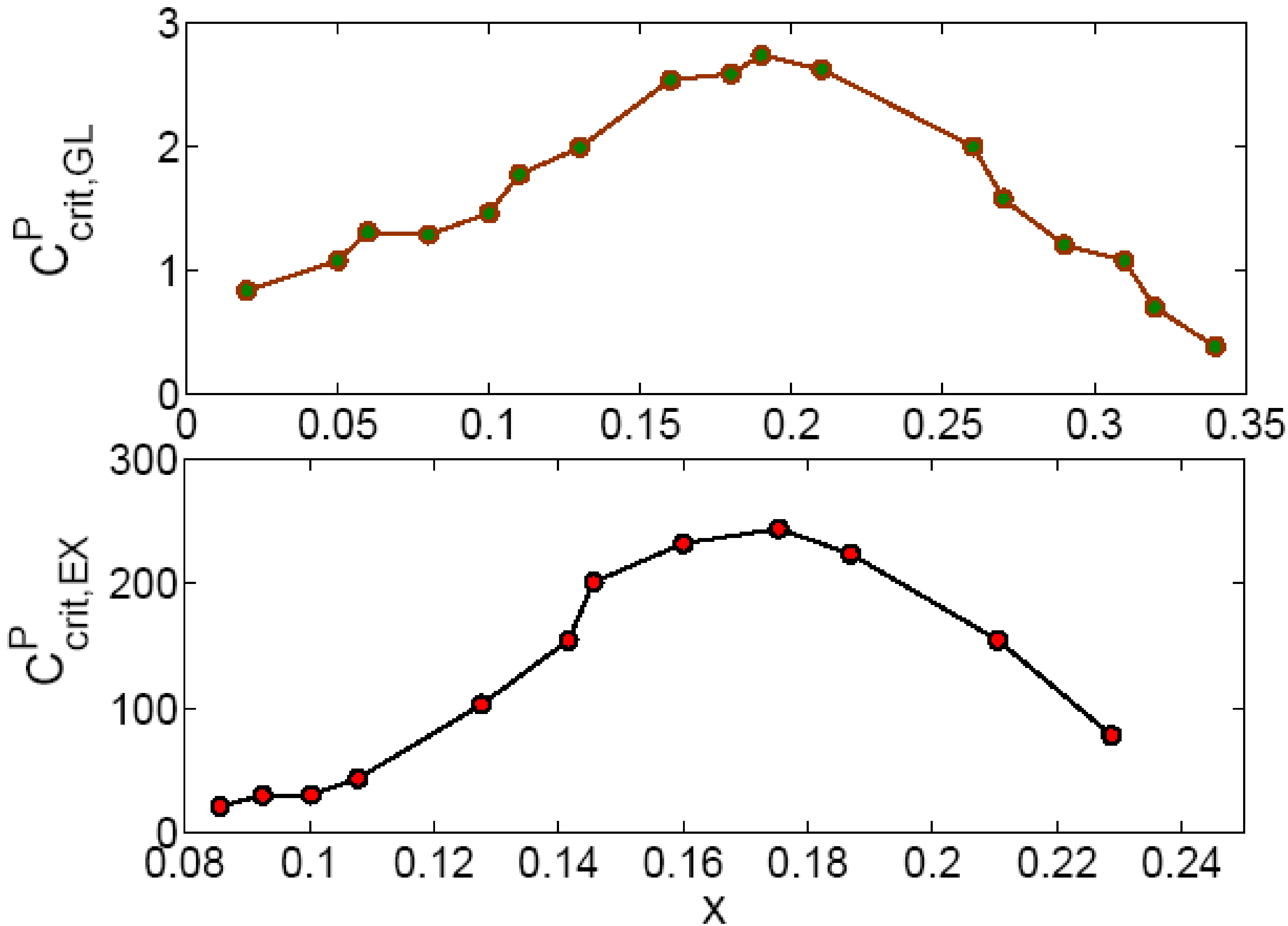

γ (= ρ_S)
T
Slope = 2/π
x=0.02
x=0.05
x=0.08
x=0.10
x=0.13
x=0.18
x=0.21
x=0.26
x=0.31
x=0.34
x=0.37
0
0.05
0.1
0.15
0.2
0.25
0.3
La2-xSrxCuO4
x=0.10
x=0.15
x=0.20
x=0.22
x=0.24
x=0.10 μSR
x=0.15 μSR
1/λ_ab^2 (μm^-2)
(a)
0
5
10
15
20
25
40



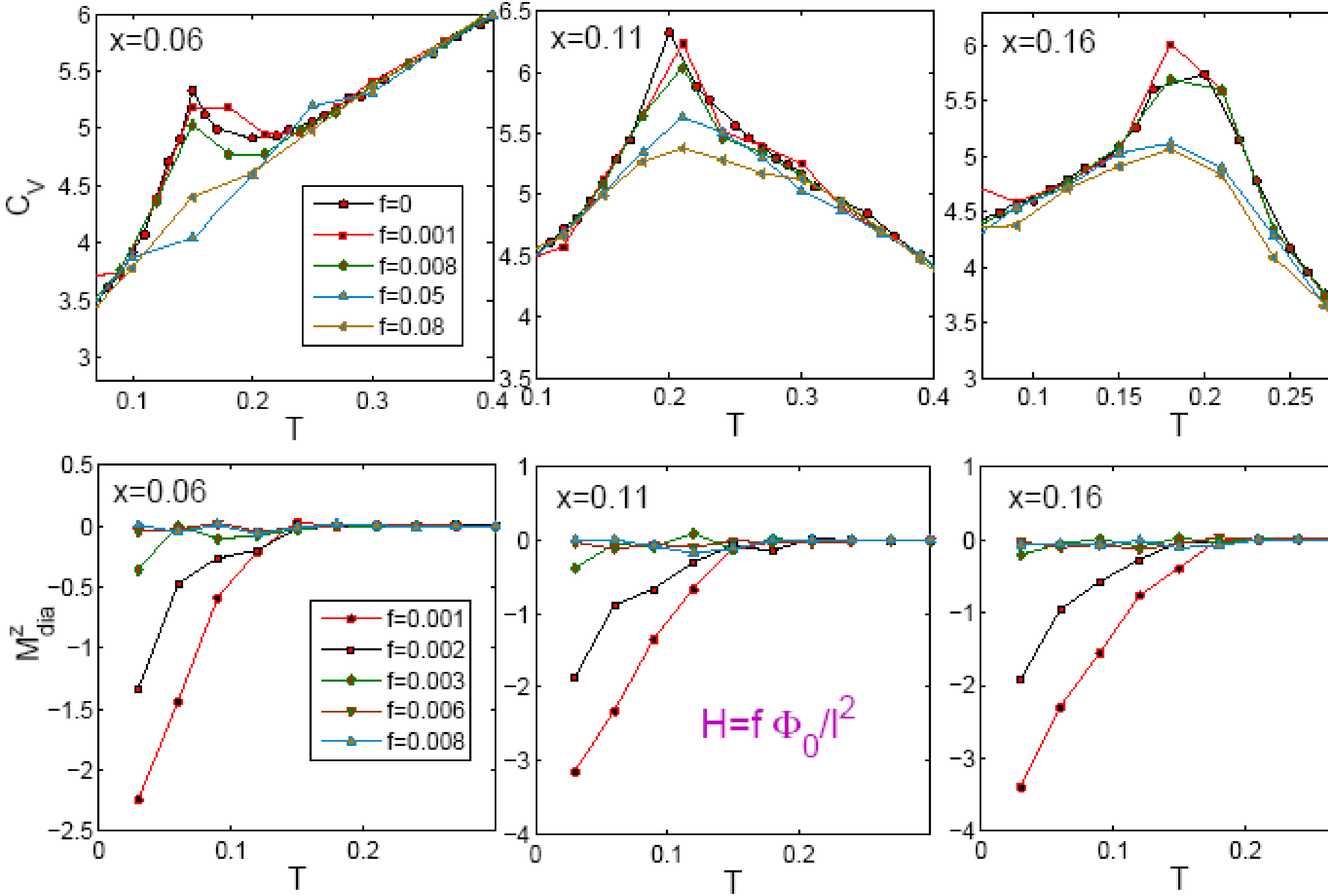
C, C_back,GL
C_crit,GL
T
x=0.06
x=0.11
x=0.16
x=0.21
0
2
4
6
8
0
1
2
2.5
0.1
0.2
0.3
0.4
0.5
0.6

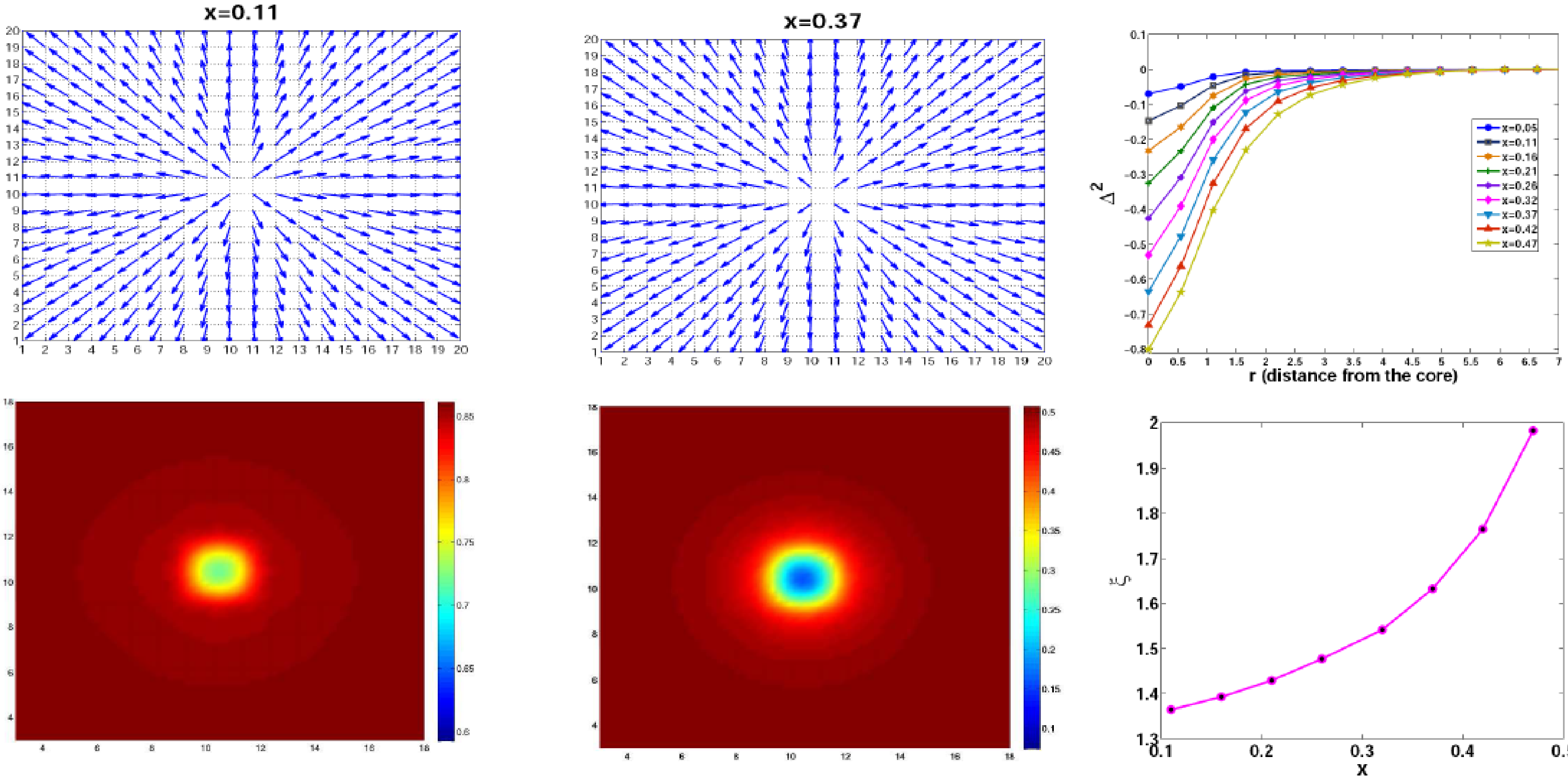


$Y_{0.8}Ca_{0.2}Ba_2Cu_3O_{7-\delta}$
C_{el} , $C_{back,EX}$ (mJ/gat.K)
$C_{crit,EX}$ (mJ/gat.K)
T (K)
x=0.09
x=0.11
x=0.15
x=0.19
0
100
200
300
400
500
0
50
100
150
200
0
50
100
150
200
250



$C^{P}_{crit,GL}$
3
2
1
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
$C^{P}_{crit,EX}$
300
200
100
0
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
X


x=0.06
x=0.11
x=0.16
C_V
T
f=0
f=0.001
f=0.008
f=0.05
f=0.08
M^z_{dia}
f=0.001
f=0.002
f=0.003
f=0.006
f=0.008
$H=f\,\Phi_0/l^2$


x=0.11
x=0.37
r (distance from the core)
x=0.05
x=0.11
x=0.16
x=0.21
x=0.26
x=0.32
x=0.37
x=0.42
x=0.47
x

Electron moving in a medium of bond pairs

'AF' or d wave short range order ($T > T_c$)

Long range order and residual thermal or quantum fluctuations ($T < T_c$)

Electron (self energy) Σ in this medium :

$$\Sigma \sim P\,D\,G_o$$

P : form factor (reduces to $\{\cos(k_x a) - \cos(k_y a)\}$ for long range d wave superconductivity)

D : pair-pair correlation function (for small momentum transfer q with respect to the 'AF' ordering wavevector G)

G_o : Electron propagator

Above T_c :

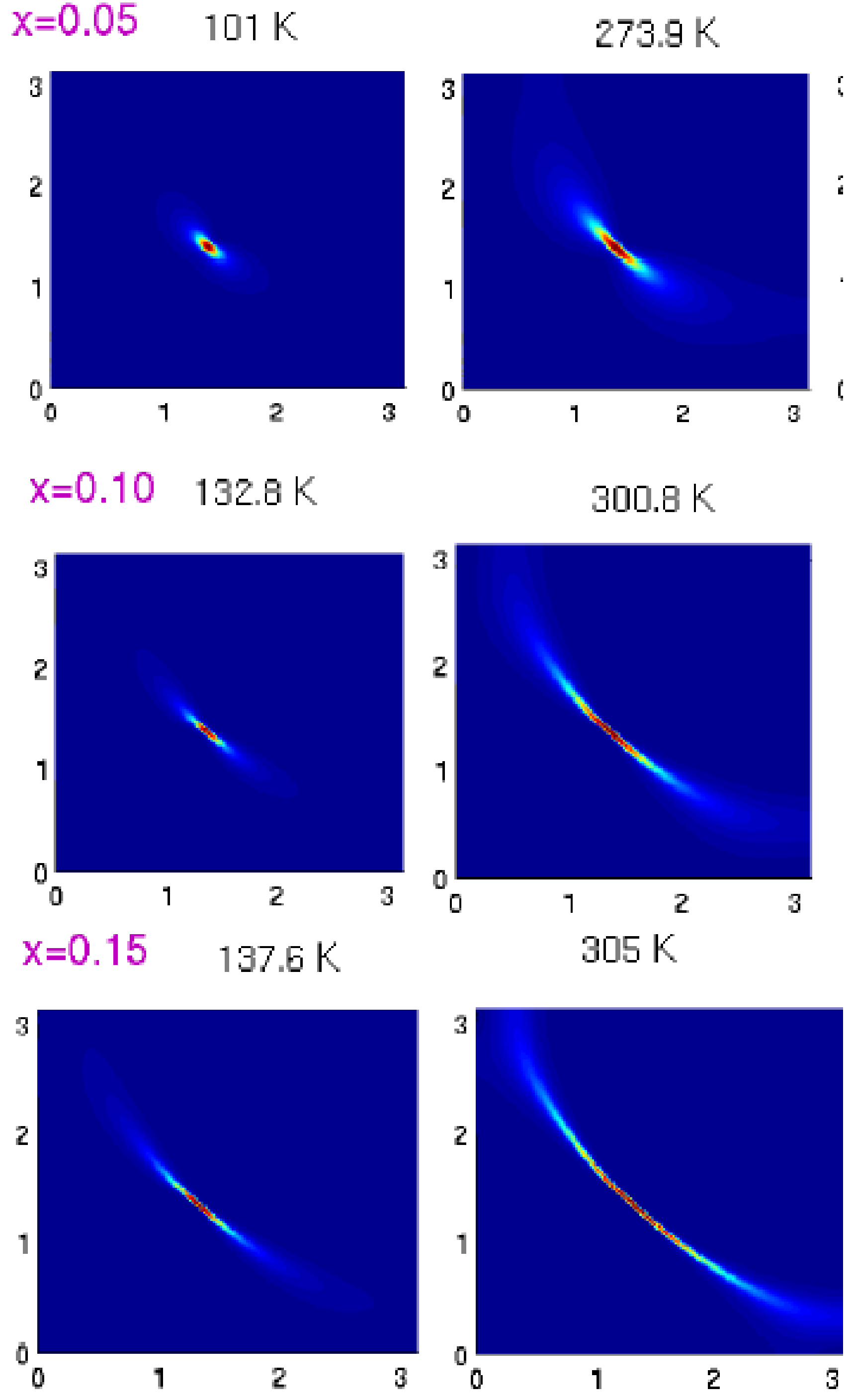
Arcs near the nodal point , with length ~ (T/T*)

(Pseudo)gap near the antinode fills in as (T/T*)

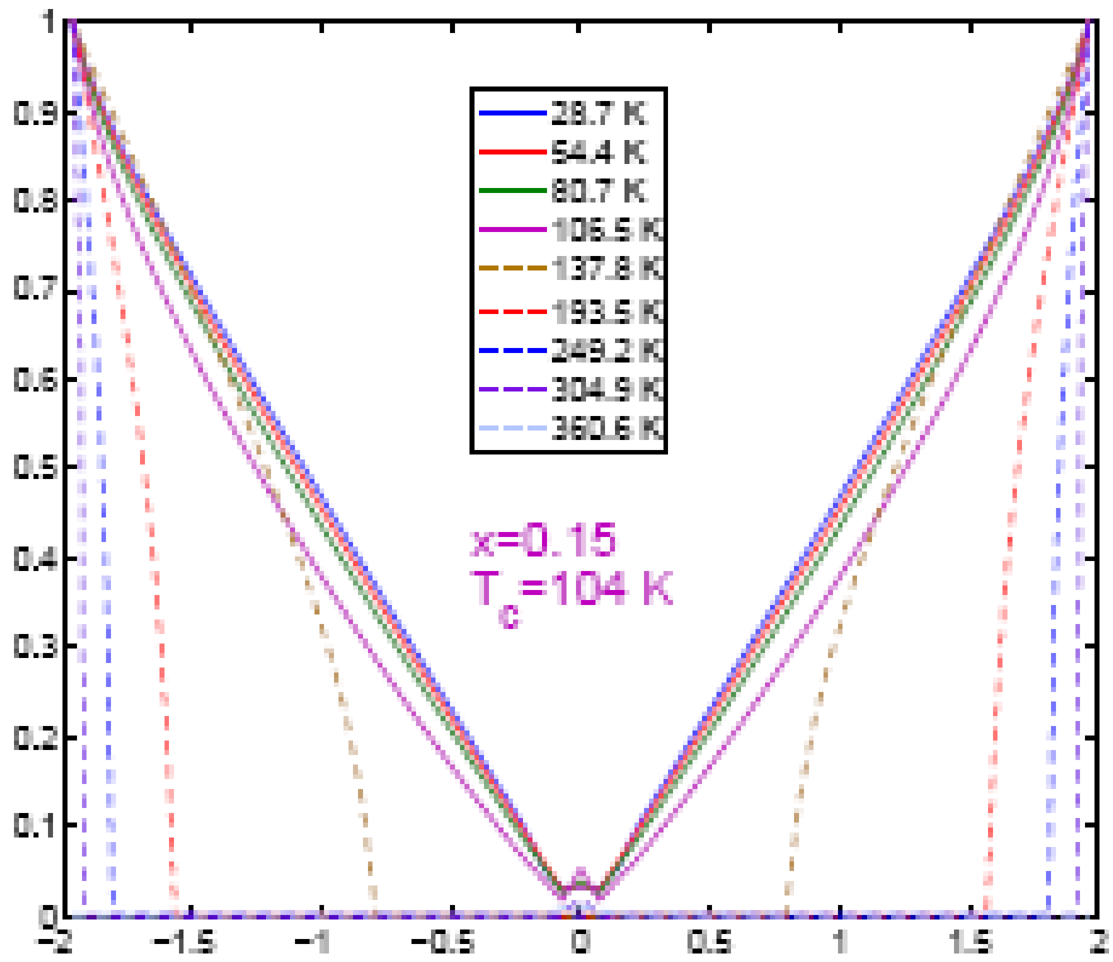
Below T_c :

Sharp quasiparticle with strength ~ $\rho_s(T)$

'Bending' of Δ_k due to fluctuations



x=0.05
101 K
273.9 K
x=0.10
132.8 K
300.8 K
x=0.15
137.6 K
305 K


28.7 K
54.4 K
80.7 K
106.5 K
137.8 K
193.5 K
249.2 K
304.9 K
360.6 K
x=0.15
T_c=104 K

- Attempt to describe how (d wave) superconducting phase coherence develops in hole doped cuprates
- Done via a (phenomenological) theory of

free energy as a functional of complex pair amplitude $\psi_{ij} = \psi_m = \Delta_m \exp(i\varphi_m)$

- A number of results for equilibrium properties obtained
- Other properties in the same picture(eg transport)?
- Other phenomena (eg stripes, ‘nematic’ correlations, 4x4 superstructure, quantum oscillations) needing changes/additions to the picture?
- Where does this picture come from? (microscopic theory)

Thank you