

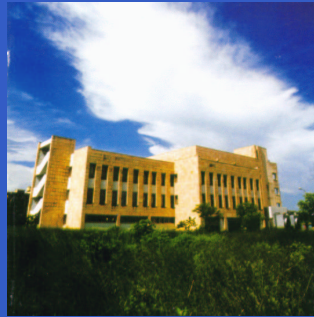
Growing smooth interfaces using moving,
non-uniform fields: Dynamical transitions and
Devil's staircases.

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Surajit Sengupta

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Devil's staircases.

P. A. Sreeram, Abhishek Chaudhuri

Growing smooth interfaces using moving, non-uniform fields: Dynamical transitions and Devil's staircases.

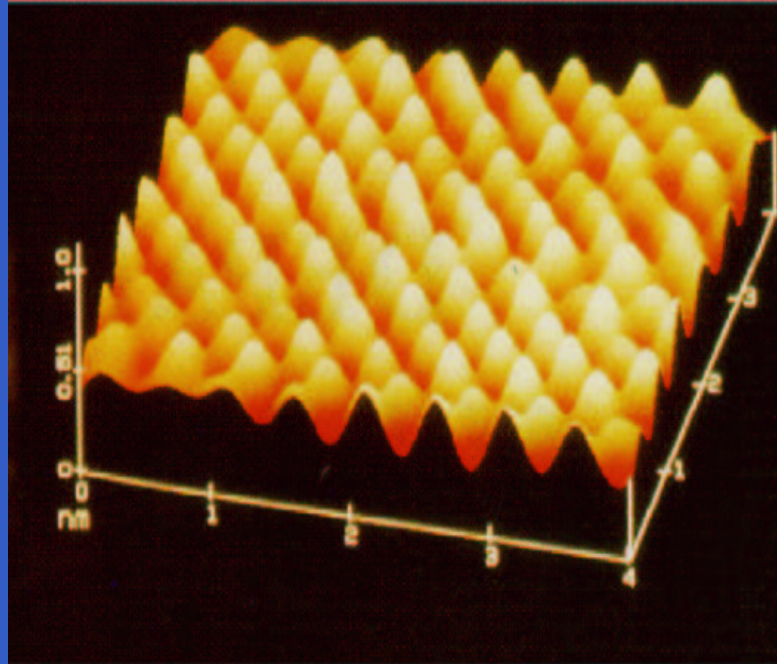


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Surfaces and Interfaces :

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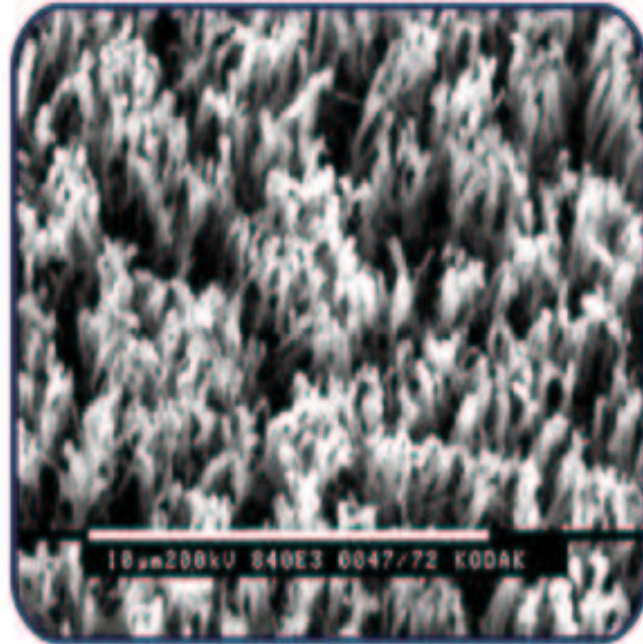
may be **smooth**



Picture of MoO3 single crystal by an inter-atomic-force microscopy

Surfaces and Interfaces :

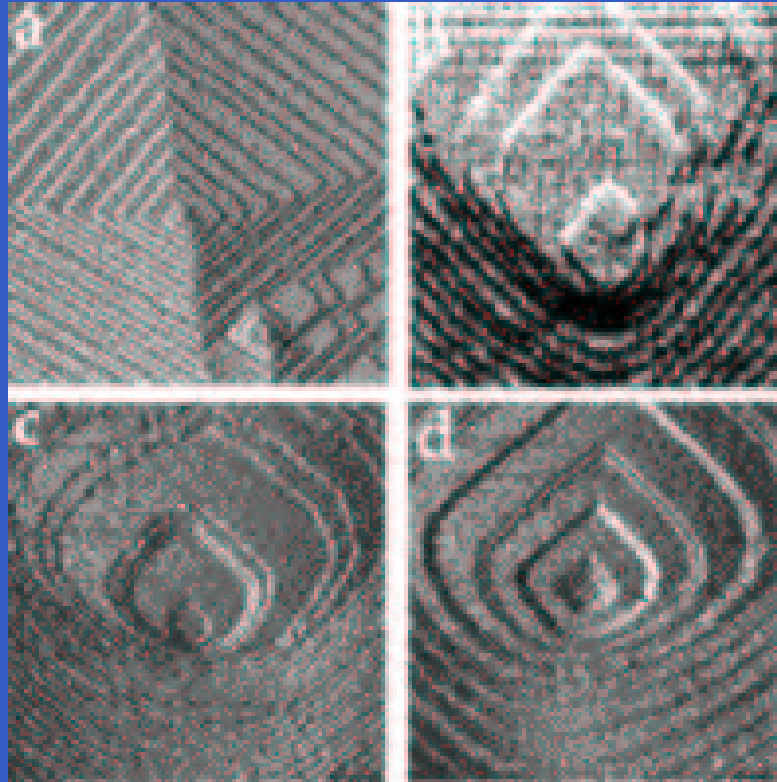
or rough



Scanning Electron Micrograph Of A Nanostructured
Electrode Material For Li-Ion Batteries

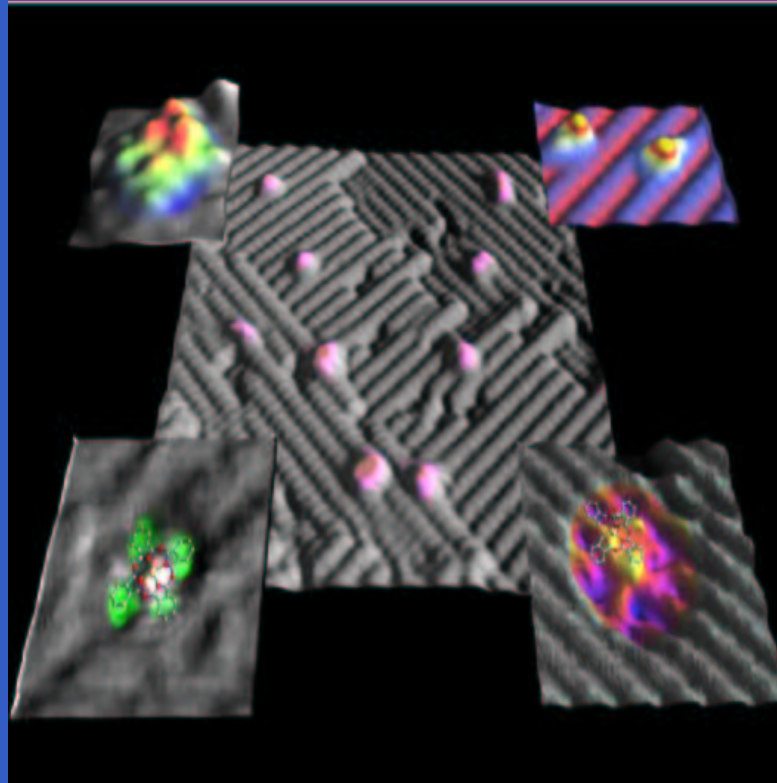
Surfaces and Interfaces :

or **stepped**



Surfaces and Interfaces :

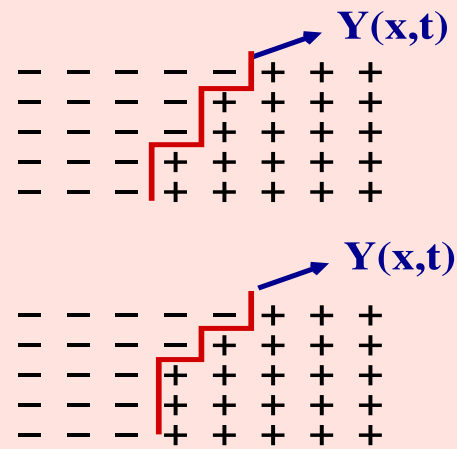
or maybe **designed** using Lithography (FCL).



Can one **design** surfaces and interfaces using
external fields ?

Ising interfaces in 2-d

Ising interfaces in 2-d



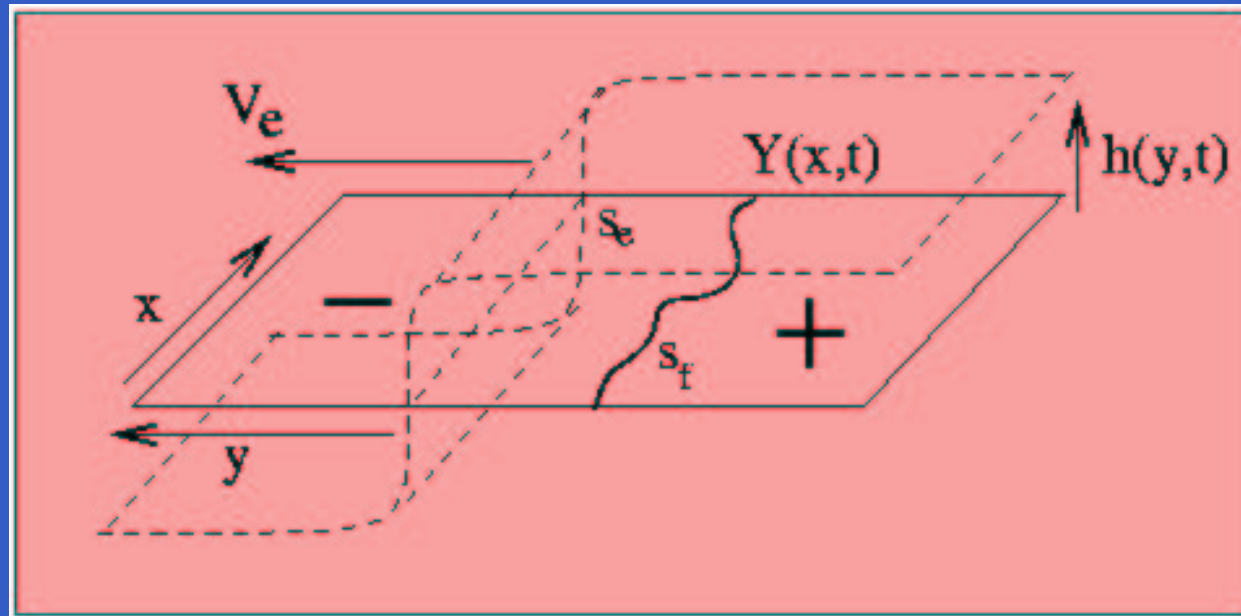
Ising interfaces in 2-d

Results for interface structure and dynamics for a uniform field h :

- interface $Y(x, t)$ moves in the direction of the minority phase with a constant **velocity** $v_f(h)$.
- interface **coarsens** with the width $\sigma^2(t) = \int dx \langle (Y(x, t) - \langle Y(x, t) \rangle)^2 \rangle / L$ going as L^α and t^β ($\alpha = 1/2, \beta = 1/3$ for a **KPZ** interface).

Ising interfaces in 2-d

What happens for a **non-uniform** field ?



Ising interfaces in 2-d

- Magnetization $\phi(x,y,t)$
- Field $h = h_{\max} f(y,t)$,

$$f(y, t) = \tanh[(y - v_e t) / \chi]$$

where χ is the **width** of the **field -profile**.

$h = h_{\max}$ in the positive ϕ region

$h = -h_{\max}$ in the negative ϕ region.

Ising interfaces in 2-d

$S_e \rightarrow$ Edge of the Field.

$S_f \rightarrow$ The interface, (Given by $Y(x,t)$).

$\phi(x,y,t)$ is **constant** far away from the interface.

$\phi(x,y,t) = \phi[y - Y(x,t)]$ in the region of the interface.

Relative position of the edge w.r.t. interface \rightarrow
a **force** on the interface.

Interface dynamics

TDGL Equation of motion :

$$\frac{\partial \phi}{\partial t} = -\Gamma \frac{\partial H_T}{\partial \phi} + \xi(\vec{r}, t)$$

where,

$$H_T = \int d\vec{r} \left[a_1 \phi^2 + a_2 \phi^4 + a_3 (\vec{\nabla} \phi)^2 - h\phi \right]$$

$\xi \rightarrow$ Gaussian white noise.

Interface dynamics

Equation of motion for the interface :

$$\frac{\partial Y}{\partial t} = \lambda_1 \frac{\partial^2 Y}{\partial x^2} - \lambda_2 \left(\frac{\partial Y}{\partial x} \right)^2 f(y, t) - \lambda_3 f(y, t) + \xi(\vec{r}, t)$$

$\lambda_1, \lambda_2, \lambda_3$ are parameters.

Note that for $f(y, t) = \text{constant}$, this reduces to the standard KPZ (Kardar-Parisi-Zhang) equation for interface motion in $2 + 1$ dimensions.

Mean - field results

Mean - field results

No spatial fluctuations: $Y = Y(t)$

For $t \rightarrow \infty$, $Y \rightarrow v_f t$

$v_f \rightarrow$ Average velocity of the interface.

Mean - field results

Solution :

$$v_f = \lim_{t \rightarrow \infty} \{-\lambda_3 \tanh[(v_f - v_e)t/\chi]\}$$

For large times i.e. $t \rightarrow \infty$ or $\chi \rightarrow 0$ (sharp edge),

$$v_f = -\lambda_3 \operatorname{sign}(v_f - v_e)$$

Mean - field results

Stuck Phase :

$$v_f = v_e \text{ for } v_e < v_\infty (= \lambda_3)$$

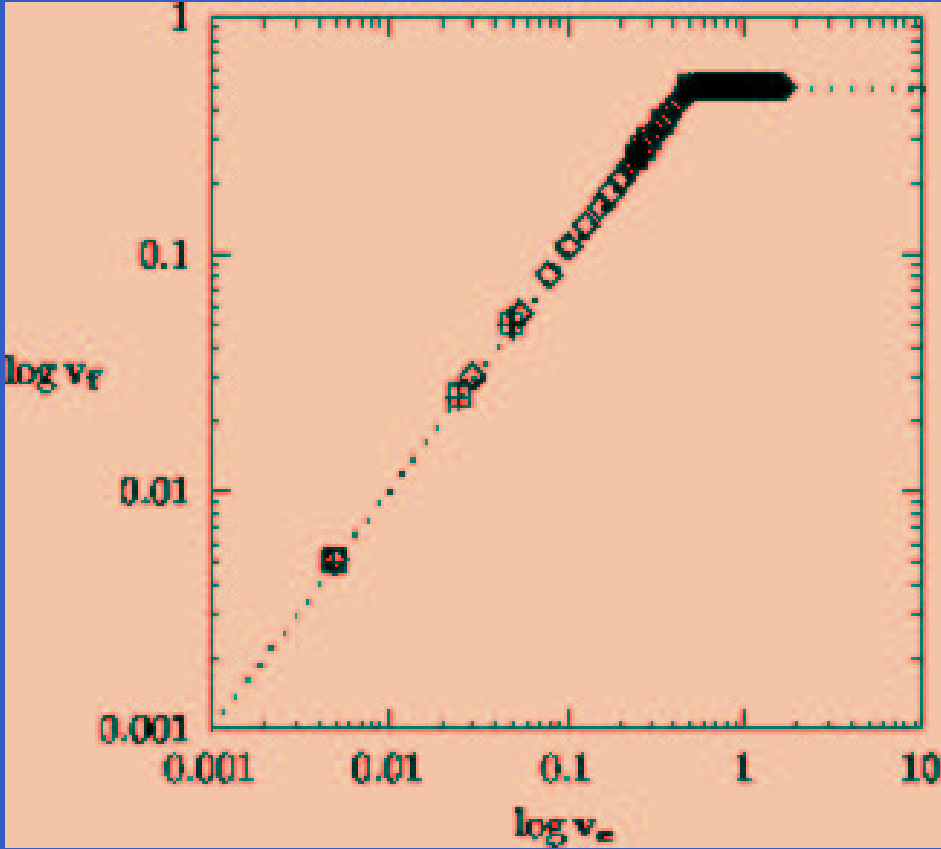
Detached Phase :

$$v_f = v_\infty \text{ for } v_e \geq v_\infty (= \lambda_3)$$

Sharp transition.

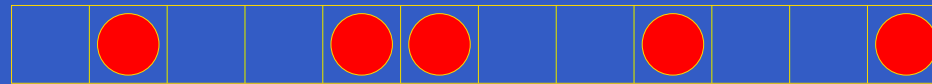
Simulation shows mean field results to be correct.

Mean - field results



Mapping to 1-D exclusion process

Interface coordinates \rightarrow a set of integers $\{y_i\}$ (position of the i^{th} particle), $1 < i < N_p$, where N_p is the number of particles.



x	1	2	3	4	5
y	1	4	5	8	10

Mean slope of the interface : $\tan(\theta) = 1/\rho$.

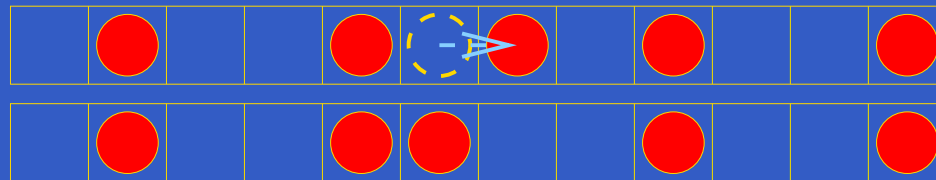
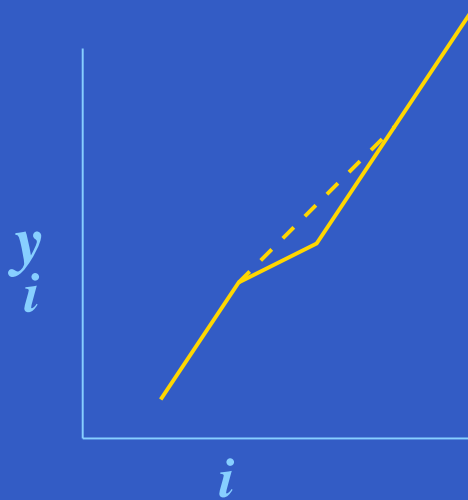
Mapping to 1-D exclusion process

Motion of the interface \rightarrow hopping of the particles subject to the “hard sphere” constraint.

$p \rightarrow$ Right jump probability

$q \rightarrow$ Left jump probability

$$p+q = 1$$



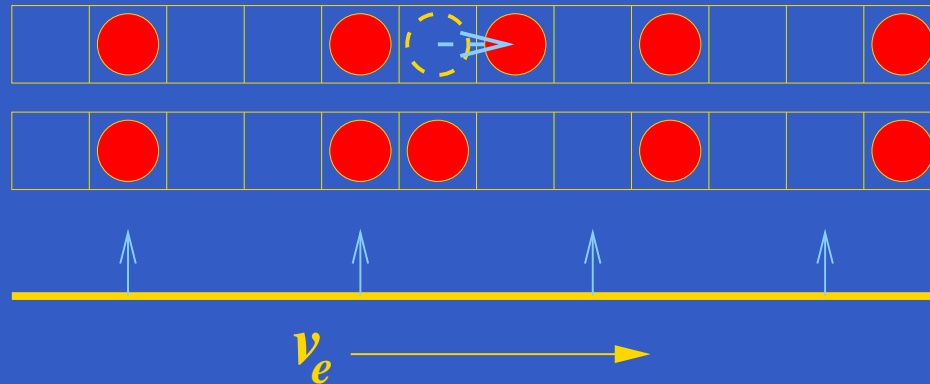
Mapping to 1-D exclusion process

In our case:

p and q are position dependent.

$$\Delta_i(v_e t) = p - q = \Delta_0 \text{sign}(y_i - i/\rho - v_e t)$$

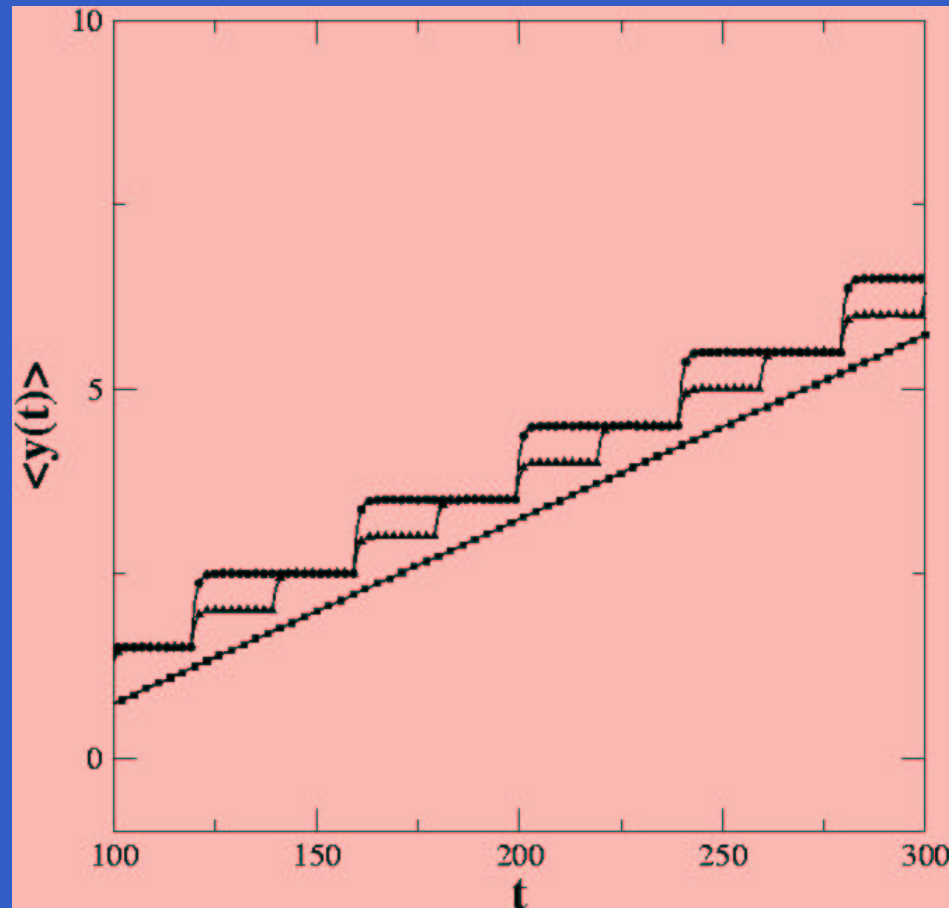
$$\Delta_0 = 1$$



Mapping to 1-D exclusion process

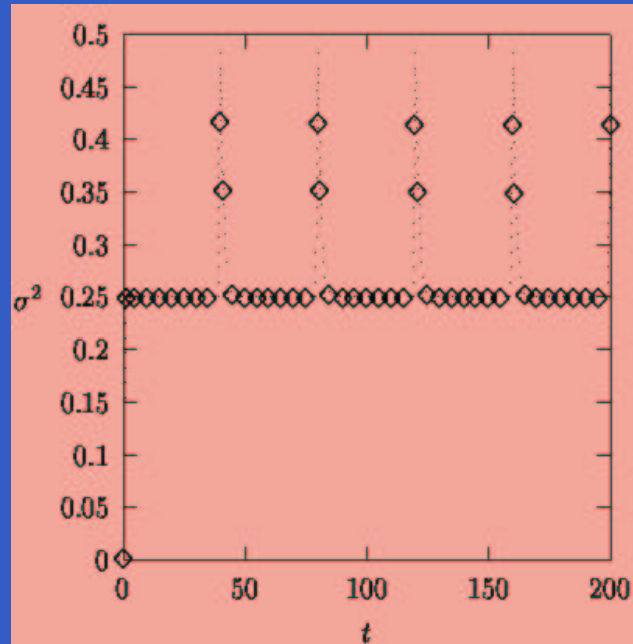
Average position of the interface : »

$$\langle y(t) \rangle = (1/N_p) \sum_i y_i(t)$$



Mapping to 1-D exclusion process

Average width of the interface : $\sigma^2(t) = (1/N_p) \sum_i \langle (y_i - \langle y_i(t) \rangle)^2 \rangle$



Mapping to 1-D exclusion process

Suggests **structure** for the **stuck** phase.

Structure of the stuck phase

Structure of the stuck phase

Local slope \equiv local particle density. $\tilde{\rho}$

Structure for $v_e = 0 \rightarrow$ ground states of:

$$E = 1/N_p \sum_i (y_i - i/\rho - c)^2$$

• $c \rightarrow$ constant shift of the edge which minimizes the energy.

Structure of the **stuck** phase

Density $\rho = m/n$ m, n are integers.

Minimizing with respect to $\{y_i\}$ and c ,

$$E(\rho) = (1/6)(1/2 - 1/m)(1 - 1/m) + 1/(4m) - 1/(4m^2)$$

for even m

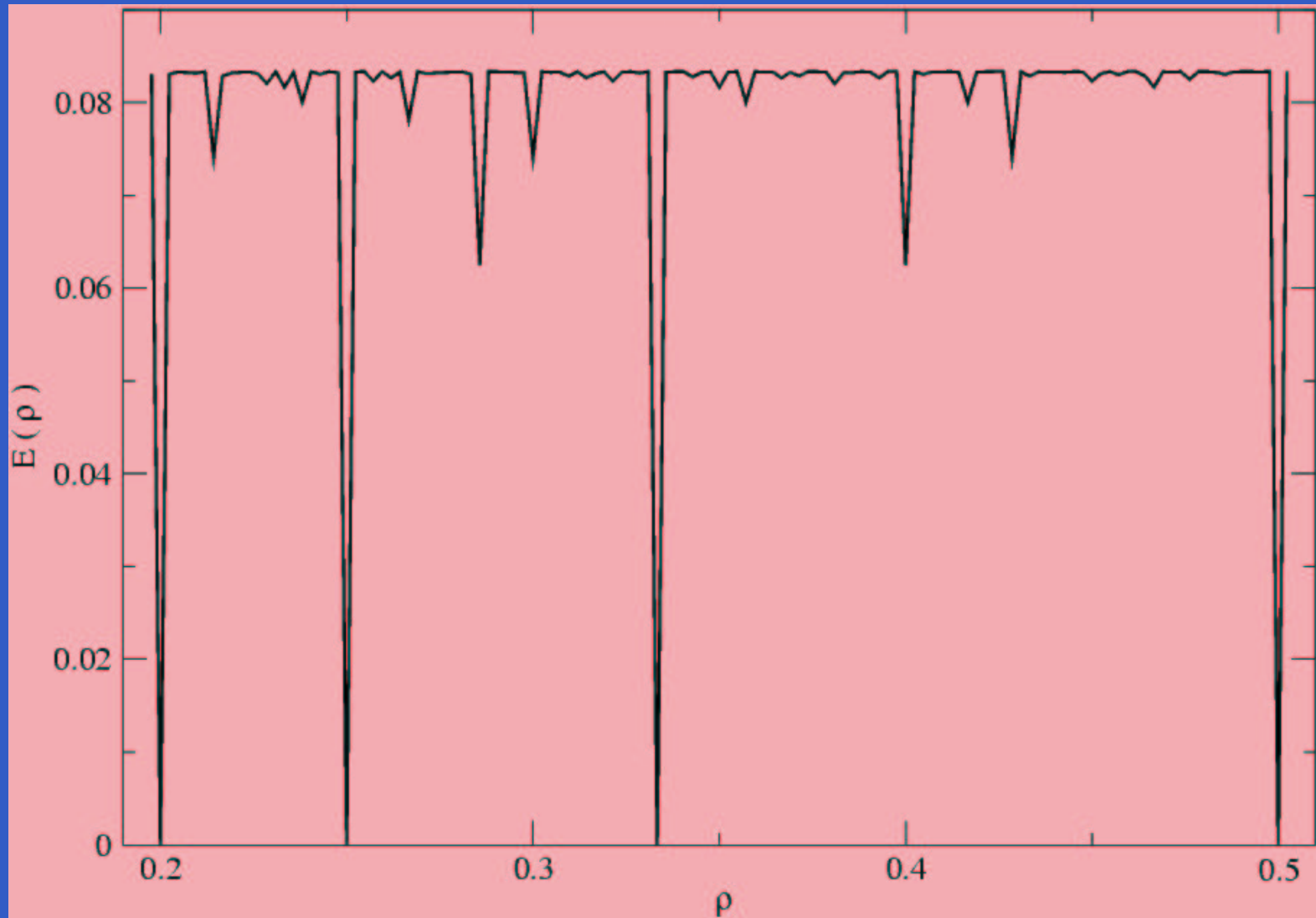
$$E(\rho) = (1/12)(1 - 1/(m^2))$$

for odd m .

$$0 [E(1/n)] < E(\rho) < 1/12 [\lim_{m \rightarrow \infty} E(m/n)]$$

Upper bound is for irrational density.

Structure of the stuck phase



Structure of the **stuck** phase

Results :

- For small v_e , for any ρ , system distorts locally to conform to the nearest low lying rational slope $1/\tilde{\rho}$, interspersed with an ordered array of a finite number of
- "Discommensurations" \rightarrow Rippled Structure \rightarrow Devils staircase in the plot of $\tilde{\rho}$ vs ρ .
- As the edge velocity is increased, the structure can no longer support dis-commensurations and local density goes over to the average density.
- For densities close to $\rho = 1/n$, the rippled structure is stable even for large velocities v_e .

Structure of the **stuck** phase

$$y(x) = x/\rho$$

$$\rho = 1/n$$



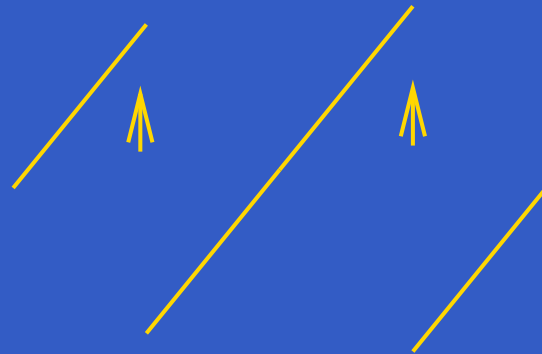
$$\rho = 2/n$$



ρ **irrational**

dis-commensurations

$$\rho_d = |\rho - \tilde{\rho}|$$

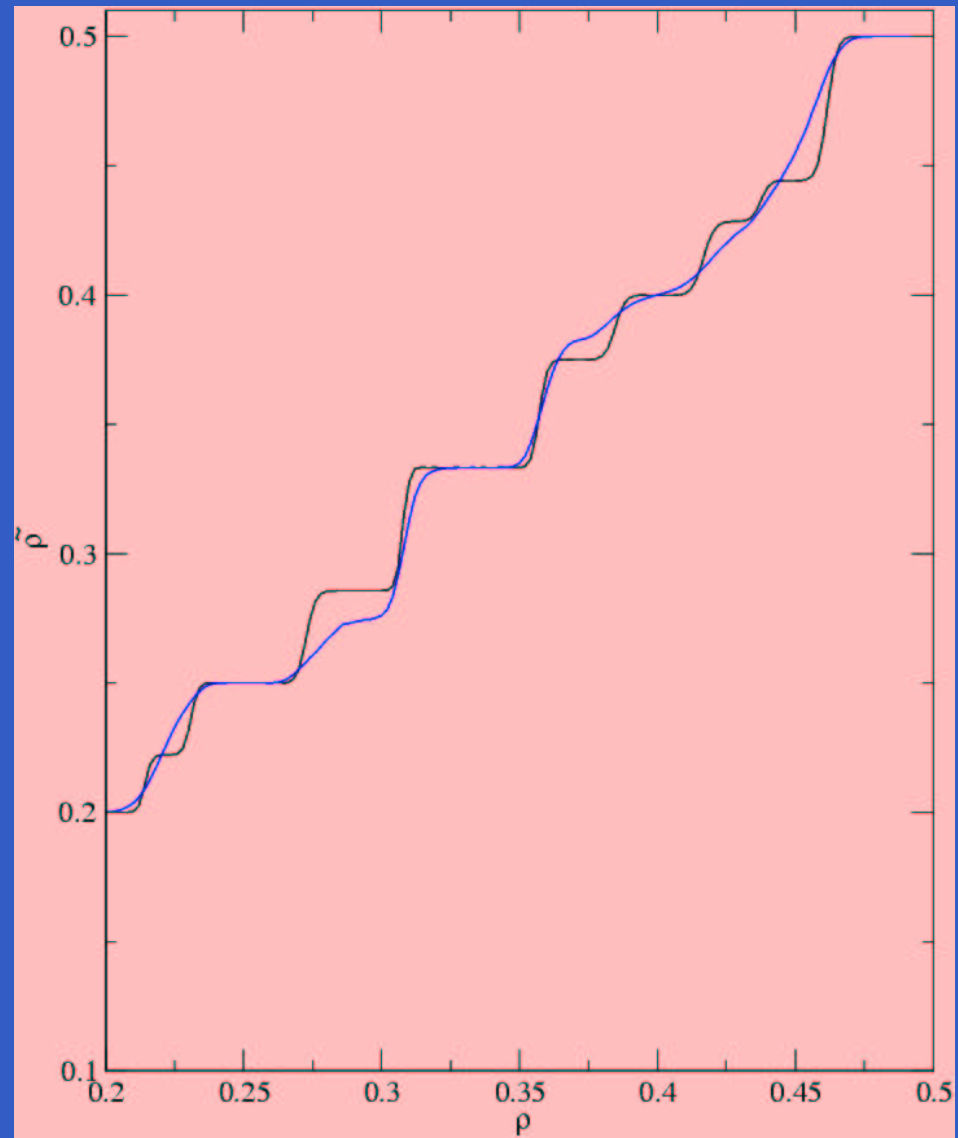


Structure of the **stuck** phase

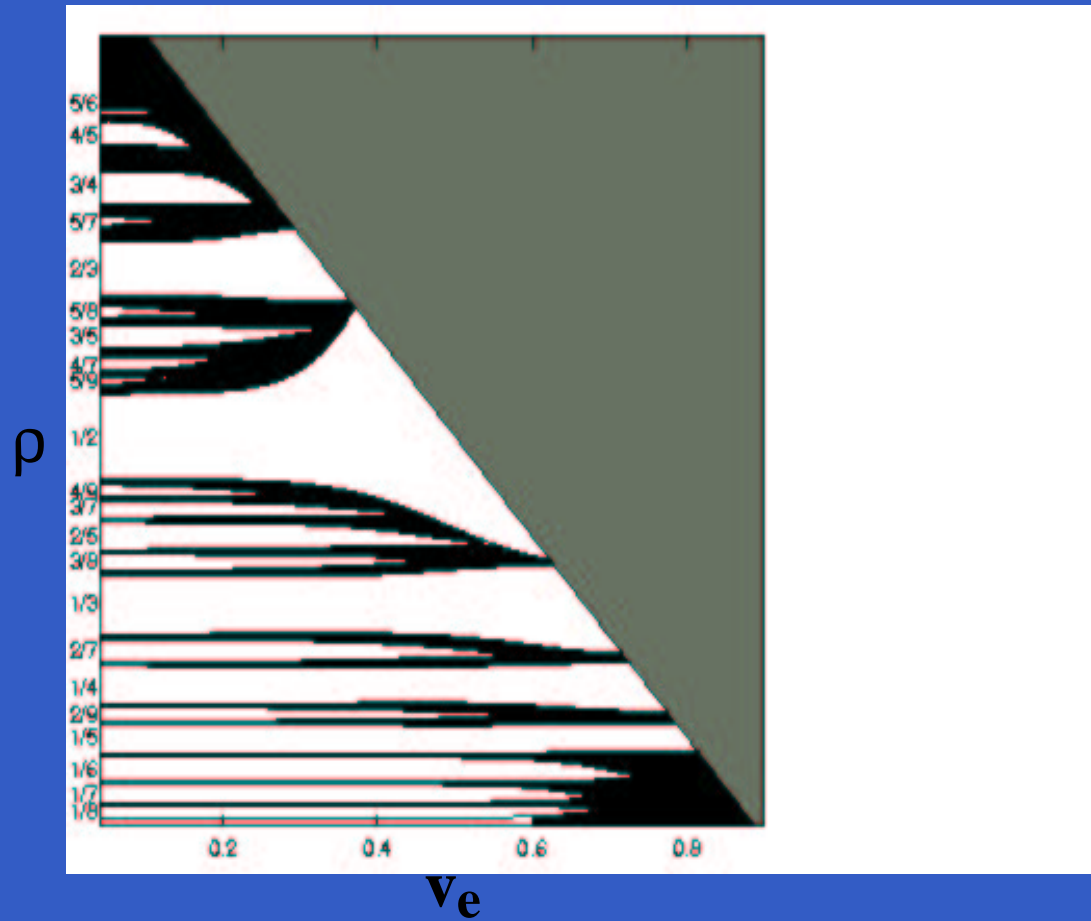
- **small** v_e , $\rho = 1/3$
no dis-commensurations. [animation 1](#)
- **small** v_e , $\rho = 1/3 + \delta$
with dis-commensurations. [animation 2](#)
- **large** v_e , $\rho = 1/3 + \delta$
with dis-commensurations. [animation 3](#)

«.

Structure of the stuck phase



Structure of the stuck phase



Structure of the stuck phase

- For $\rho = m/n$, \rightarrow Independent motion of "m" particles each separated by a time lag $\tau_L = 1/(m v_e)$.
- For irrational fractions, $m \rightarrow \infty$, $\tau_L = 0$.
- As v_e is increased, for $\tau > \tau_L$, instantaneous values of ρ make excursions to nearby rational fractions. Eventually, the interface becomes disordered (conforming with the average density).

Structure of the stuck phase



Langevin Dynamics

Energy surface \rightarrow 1-D **Trap Model**

Project $\{y_i\}$ to a single variable ρ .

Langevin Dynamics

Energy surface \rightarrow 1-D **Trap Model**

Project $\{y_i\}$ to a single variable ρ .

Dynamics of a single particle with co-ordinate ρ' diffusing on the energy surface.

Free energy :

$$F(\rho') = E(\rho') + \kappa (\rho' - \rho)^2,$$

kicked by a Gaussian white noise of strength T ($\propto v_e^2$).

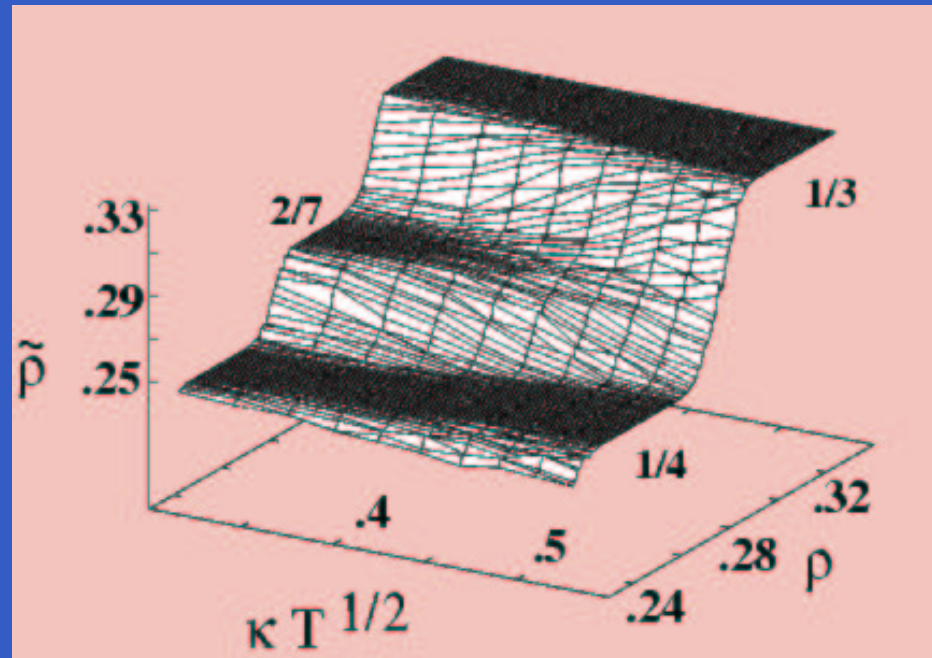
System gets trapped in a nearby minimum with $\rho' = \tilde{\rho}$.

$\uparrow T \rightarrow$ a **noise induced C-I transition**.

Langevin Dynamics

Energy surface \rightarrow 1-D **Trap Model**

Project $\{y_i\}$ to a single variable ρ .



Conclusions

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- We have studied the dynamics of a 1-D interface in the presence of an inhomogeneous, moving field by mapping it on to a 1-D exclusion problem.

Conclusions

- The macroscopic features are well understood within mean field theory.

There is a sharp transition from a region where the interface follows the edge closely to a region where the interface is detached from the edge and moves with a constant velocity, independent of the edge velocity.

Conclusions

- The local dynamics shows a rich phase diagram depending upon the velocity of the edge of the inhomogeneous field.

For small velocities, and for incommensurate densities, the interface is rippled, with a finite number of discommensurations, leading to a Devil's Staircase.

As velocity is increased, the discommensurations are no longer supported and the local density becomes equal to the average density.