

**From Eden to Invasion:
Domain Growth in Field-driven
Random-Field Ising Model**

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August 12, 2004

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- Hamiltonian for RFIM :

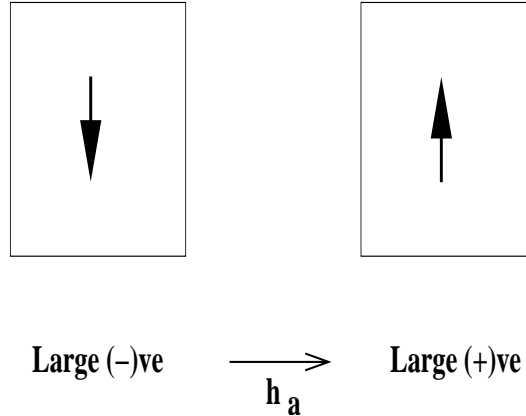
$$H = -J \sum_{\langle ij \rangle} s_i s_j - \sum_i h_i s_i - h_a \sum_i s_i$$

h_i is the quenched random-field on site i with $p(h_i)$

h_a is the applied external field

- RFIM driven by an external field is used to study:
 - * Driven disordered systems
 - * Hysteresis, Barkhausen noise
 - * Effect of disorder on first order transitions in general

Methodology:

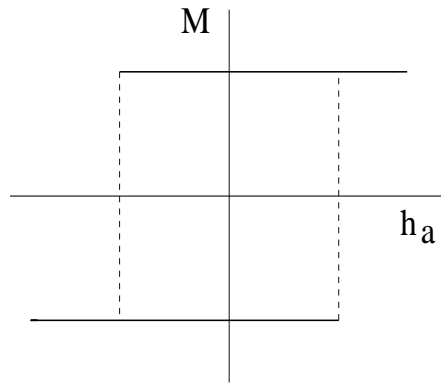


Spin Flip Rule: T=0 Glauber dynamics

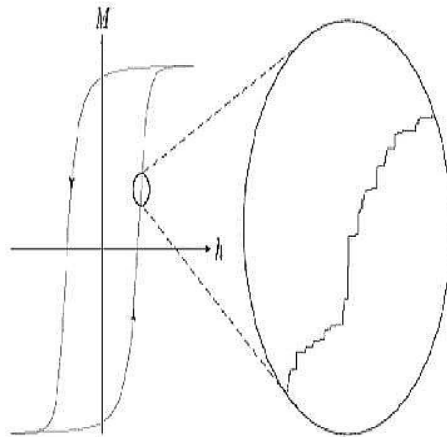
- Spin direction is given by the sign of local field at that site

$$s_i = \text{sign}(l_i) = \text{sign}\left(J \sum_{j=1}^z s_j + h_i + h_a\right)$$

h_a is incremented only when all flippable spins have been flipped for a certain h_a .



M vs. h_a for a pure-system ($h_i = 0$)



M vs. h_a for a disordered system

- Sharp first order transition is smeared out
- change in M occurs in small jumps/avalanches

Mean-Field Analysis :

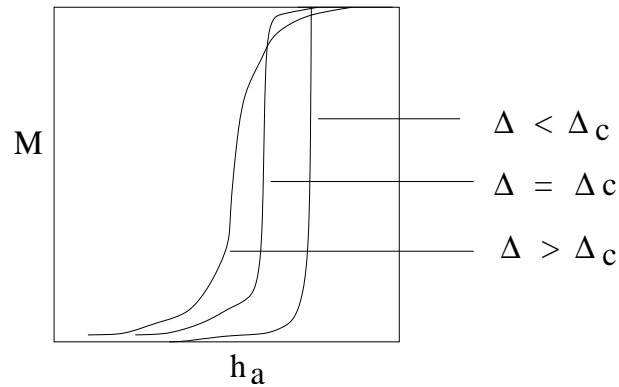
- Every spin is coupled to all other spins
- The effective field acting on a site

$$JM + h_i + h_a, M = \sum_i s_i / N$$

- Spins with $h_i < -JM - h_a$ will point down, rest points up

$$M(h_a) = 1 - 2 \int_{-\infty}^{-JM(h_a) - h_a} p(h_i) dh_i$$

$M(h_a)$ as a function of h_a can be found self-consistently.



Typical $M - h_a$ curve for $p(h_i)$ -
Gaussian with width Δ

- Avalanche size distribution $D(s, h_a)$
at critical values Δ_c and $h_a = h_c$

$$D(s, h_c) \sim s^{-3/2}$$

- At Δ_c , cumulative avalanche size
distribution $D(s)$ upto h_c

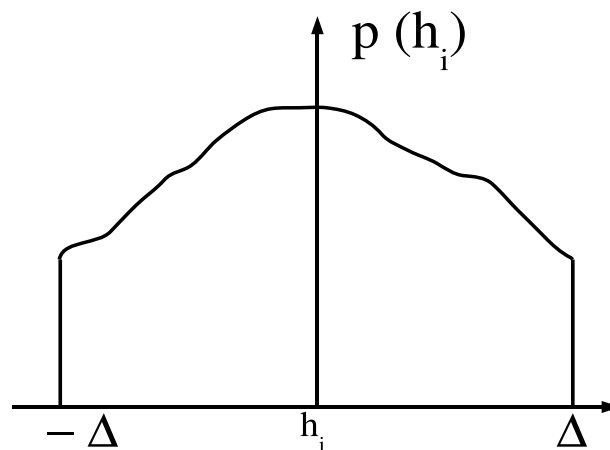
$$D(s) = \int_{-\infty}^{h_c} D(s, h_a) dh_a$$

$$D(s) \sim s^{-5/2}$$

Simulations with $p(h_i)$ gaussian
(unbounded) in 3d support mean-field
results.

Our Study :

- To check the universality of the mean-field behavior for bounded distribution and at low dimensions



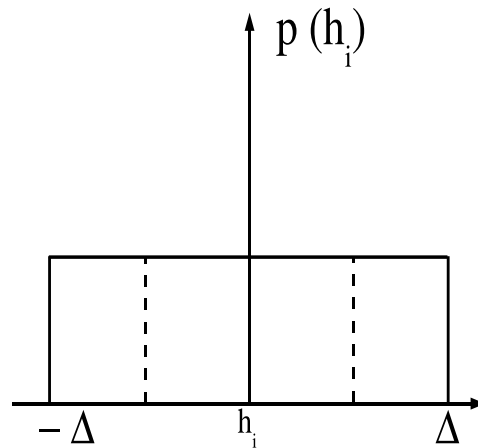
The probability of flipping of a spin at h_a having n flipped nearest neighbors is

$$P_n(h_i) = \int_{(z-2n)J-h_a}^{\Delta} p(h_i) dh_i$$

z = co-ordination number

$$h_a = zJ - \Delta$$

For a uniform distribution of h_i :

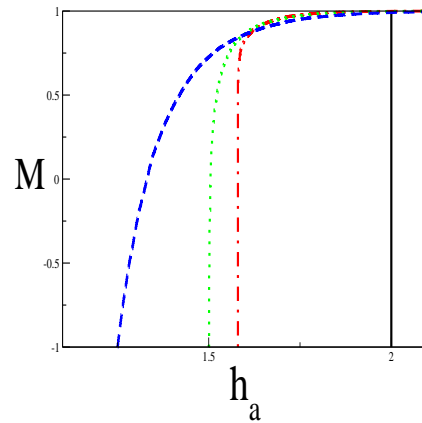


$$P_n(h_i) = n \frac{J}{\Delta}$$

- Rule for growth of Eden clusters
- Flipping probability is proportional to the number of nn flipped spins

Our Simulation:

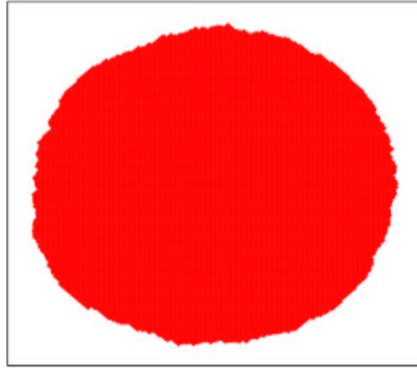
- **2d**: square, honeycomb, triangular (500^2 to 7000^2)
- **3d**: simple cubic lattices (50^3 to 200^3)
- Results averaged over 100 different random-field configurations



Features of magnetization curves:

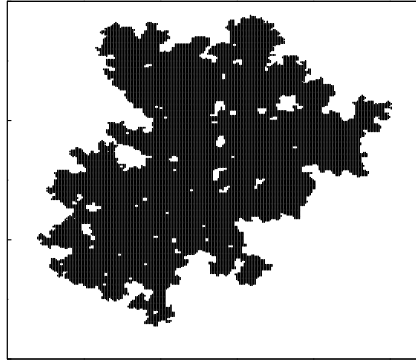
- i) For low Δ , M jumps from -1 to $+1$ at some value of h_a (black line)
- ii) For intermediate Δ , M jumps from -1 to a fixed value $M' < M$ at some h_a - value and then increases in small jumps with h_a
- iii) At critical Δ_c , magnetization jumps occur in all sizes
- iv) For $\Delta_d > \Delta_c$, M changes in small jumps with increase of h_a

Domain growth corresponding to the various magnetization curves:



a

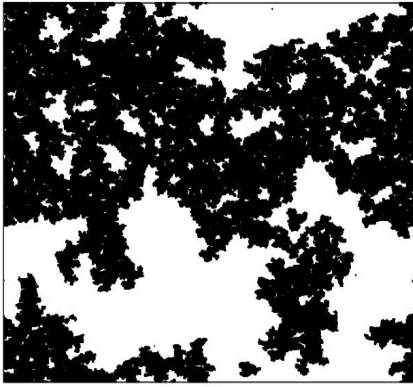
- i) Fig.(a) - a snapshot of the flipped spin domain for low Δ . M goes from -1 to +1. Domain is compact.



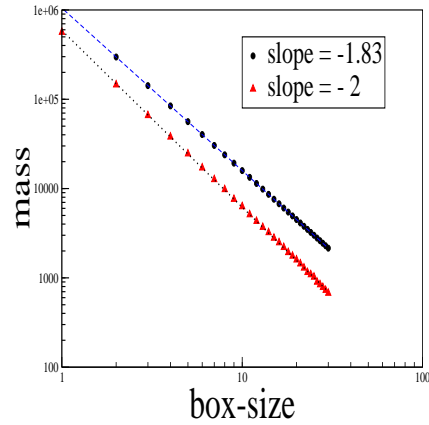
b

ii) Fig.(b) - domain of flipped spins for intermediate Δ .

- * Domain is interspersed with clusters of unflipped spins, fractal dimension of the cluster $d_f = 2$
- * Unflipped spins responsible for M going to M'
- * Domain is system spanning

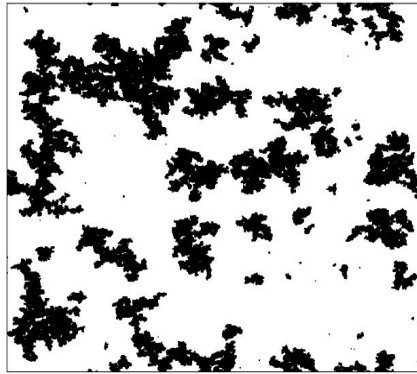


c



iii) Fig.(c) - Flipped spin domain at Δ_c when it has just spanned the system on few increments of h_a .

- Fractal dimension of the cluster is same as that for a 2d Invasion Percolation cluster with traps (black circles).
- Integrated avalanche size distribution has a power-law form with exponent $-3/4$



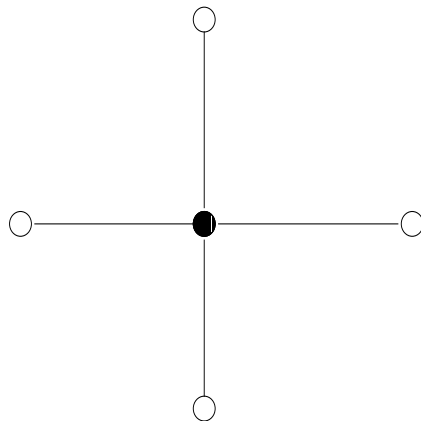
d

- iv) Fig.(d) - a snapshot of the flipped spin domains at large Δ after a few increments of h_a
- Clusters of small domains are scattered all over the lattice
 - * shows that spin flip can occur anywhere on the lattice in small avalanches with increment of h_a

Similar results are obtained in 3d.

At Δ_c , the fractal dimension d_f of the flipped spin region is 2.54 (same as that of site percolation cluster in 3d)

Growth model studied:



$$P_1(= P) : P_2(= 2P) : P_3(= 3P)$$

We find at P_c , $d_f = 1.83$

$1/P_c$ corresponds to Δ_c of RFIM

Conclusion:

- i) $M - h_a$ curve depends crucially on $p(h_i)$ specially at low dimensions
- ii) For uniform bounded distribution, the critical point corresponds to Invasion percolation
- iii) For uniform distribution, RFIM provides an interesting growth model where one can go from compact Eden growth to Invasion Percolation by changing the parameter Δ