Bose-Einstein Condensate and External Trap

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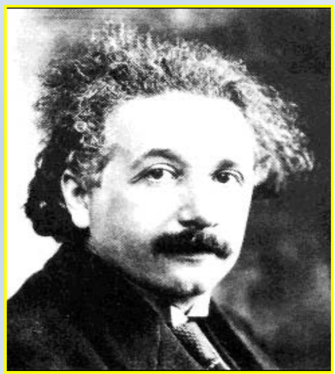
Outline of the Talk

- Introduction to the theory of Bose-Einstein Condensate
- Dynamics of BEC under a variety of external confinement
- Analytical Approach:
  - Unified Model of Exact Solution
  - BEC in Bi-chromatic Optical Lattice
- Numerical Approach:
  - Higher Harmonic Generation in BEC by Chirping
  - Production of Mesoscopic superposition States in BEC
  - Bose-Einstein condensate in a Toroidal Trap
- Conclusions

Bose Einstein Condensate

Theoretical Prediction, 1925

Experimental Realization, 1995



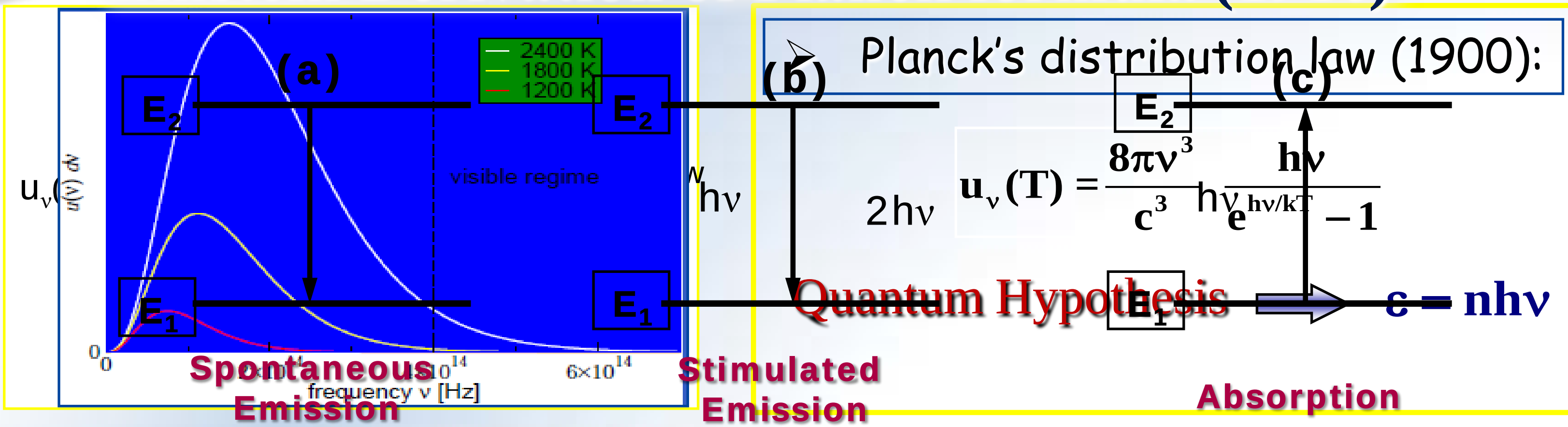
Satyendra Nath Bose & Albert Einstein

E. Cornell and C. Wieman   Wolfgang Ketterle

- "This discovery must be viewed as one of the most beautiful physics experiments of the 20th century" -Lev Pitaevskii

Background of BEC & Black body radiation spectrum

Einstein Derivation of Planck formula (1916)



- A_{21} , B_{21} , B_{12} ⇒ Einstein's A, B coefficients
- In equilibrium

$$A_{21}n_2 + B_{21}n_2u_\nu = B_{12}n_1u_\nu \quad \Rightarrow \quad u_\nu = (A_{21}/B_{21})\frac{1}{(B_{12}/B_{21})e^{h\nu/kT}-1}$$

- Einstein forced matching with Planck's formula

$$B_{12} = B_{21} \quad \text{and} \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

- It was not a full-proof new derivation
- He didn't know the concept of indistinguishability in the year 1916

Bose's derivation of Planck's law (1924)

- He was inspired by Compton's discovery and considered electromagnetic radiation as collection of quanta or photons with particle like energy hν and momentum p= hν /c.

- He computed the density of states in phase space that are available for radiation between frequency ν and ν + d ν.

$$g(\nu)d\nu = \frac{4\pi\nu^2}{c^3}\, d\nu$$

- For a photon gas

$$N(\nu) = \frac{g(\nu)d\nu}{e^{h\nu/kT} - 1} = \frac{4\pi\nu^2}{c^3}\frac{d\nu}{e^{h\nu/kT} - 1}$$

- He assumed the photons to be indistinguishable and any number of photons can be accommodated in a single state.

- Since the radiation can have two states of polarization, the result is multiplied by two

$$u(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$

S.N.Bose: Planck's Law and the Hypothesis of Light Quanta (1924)

- Three remarkable and bold steps for photons:

1. They are indistinguishable
2. Their number was not conserved
3. They have spin 1

- This statistical derivation laid the foundation of Quantum Statistics

Bose-Einstein condensate

- $$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT} - 1}$$ ➡ BE distribution

- **High temperature limit:**
  - Particle are distributed over a wide energy range

    $g_i >> N_i$ $\quad f(\varepsilon) = e^{-(\varepsilon-\mu)/kT} \rightarrow \mu << \varepsilon_{min}$

    Classical Maxwell-Boltmann distribution

- **Low temperature limit:**
  - As temperature decreases, μ rises and mean occupation number increases

    when $\mu = \varepsilon_{min}$

➡ The number of particles in single particle ground state becomes arbitrarily large

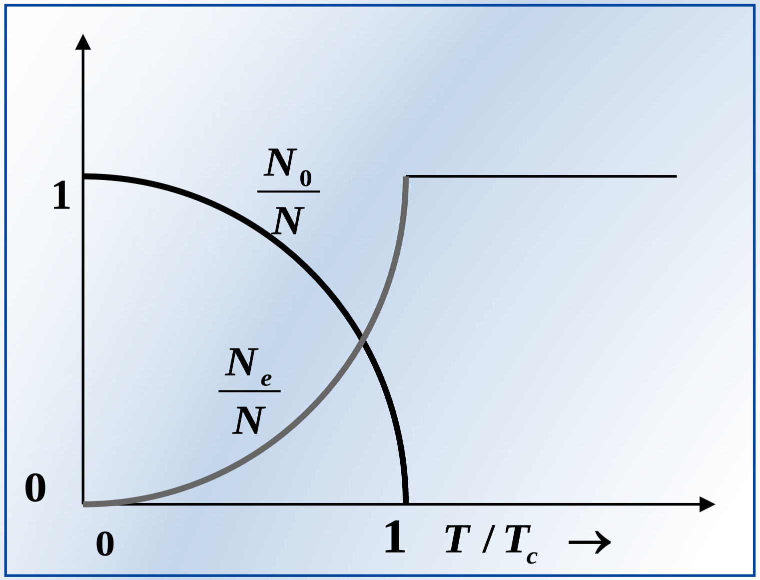
- At low temperature, a significant proportion of atoms in a gas condense in a state of lowest energy of the system.

⇩

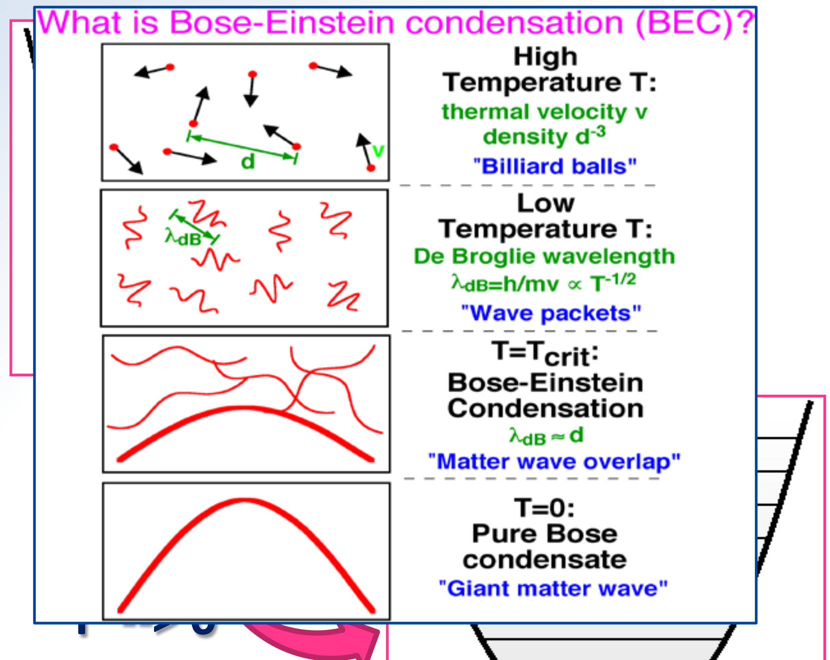
Bose-Einstein Condensation

☆ Bose-Einstein condensation occurred in momentum space and not in coordinate space

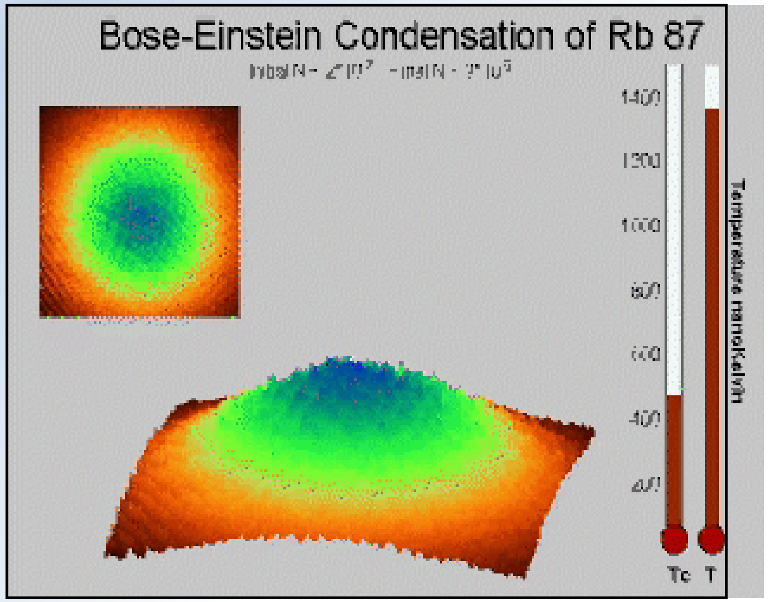
☆ Condensation is purely of quantum origin



$$N_0 = N\left(1-\left(\frac{T}{T_c}\right)^{\alpha}\right)$$



$\lambda \rightarrow$ De Broglie wavelength $= \sqrt{\frac{2\pi\hbar^2}{mk_bT}}$


Bose-Einstein Condensation of Rb 87
1400
1000
Temperature nanoKelvin
Tc
T

Dynamics of Bose-Einstein Condensate

- Bose field operator → $\Psi(r)$ and $\Psi^{\dagger}(r)$
- Second quantization Hamiltonian

$$H = \int \Psi^{\dagger}(r) H_0 \Psi(r) d^3r + \frac{1}{2}\iint \Psi^{\dagger}(r)\ \Psi^{\dagger}(r') U(r,r') \Psi(r') \Psi(r) d^3r d^3r'$$

where H_0 = Single particle hamiltonian; $U(r,r')$ = Interaction term

- The system is modeled by

$$U(r,r') = g\ \delta(r - r')$$

where $g = \frac{4\pi\hbar^2 a}{m}$, $a \Rightarrow$ scattering length

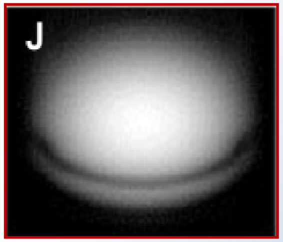
- $$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + g|\psi|^2\psi + V(r)\ \psi$$

⇓

Gross Pitäevskii Equation

Important Applications Towards BEC in External Confinement

Dark Soliton

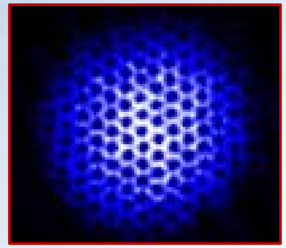


Science 287, 97 (2000)

Bright Soliton

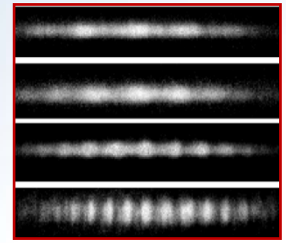
Science 296 1290 (2002)

Vortices



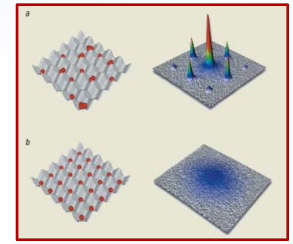
Phy. Rev. Lett. 87, 080402 (2001)

Faraday Waves



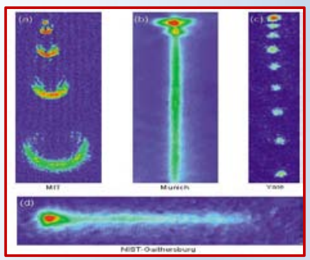
Phy. Rev. Lett. 98, 095301 (2007)

Superfluid-Mott -Insulator



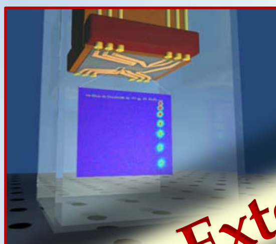
Nature 415, 39 (2002)

Atom Lasers



Phys. Rev. Lett. 79, 549 (1997)

BEC on atom chip

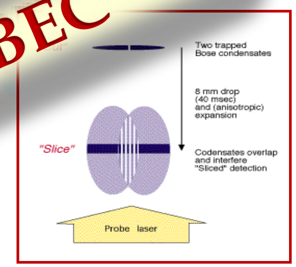


Natur[illegible]8-501 (2001)

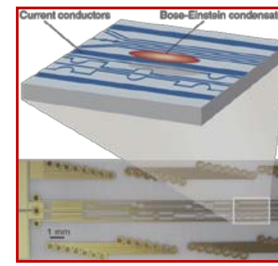
Anderson Localization

Nature Physics 453, 891 (2008)

Interf[illegible] of two BEC



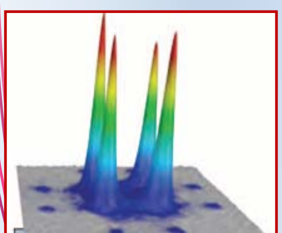
Atom chip



Science, 307 (2007)

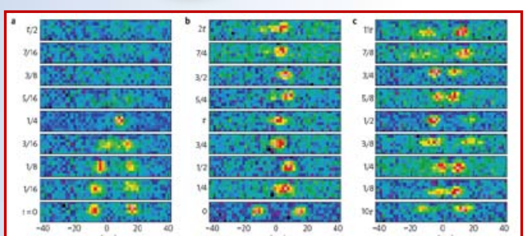
External Trap in BEC

Negative Temperature



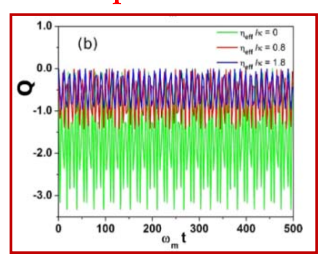
Science 339, 52 (2013)

Collision of BEC Solitons



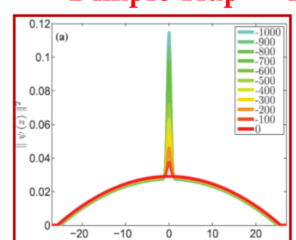
Nature Physics (Lett) 10, 918 (2014)

BEC-optomechanics



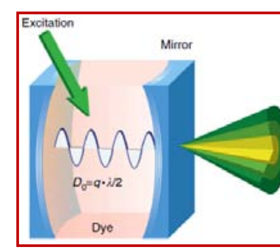
Sc. Reports 10612, 01 (2015)

Dimple Trap



Laser Phys. 26, 065501 (2016)

Photon-BEC in a Cavity



Nature Comm., 01, (2016)

Points worth Mentioning...

- There is tremendous progress of Experimental Research in this field
- Exact Theoretical method: A nontrivial task but an Essential Need
- BEC in an external trapping potential : A Trapped BEC
- Exact analytical methods are possible only for couple of well-known potentials
- A huge number of theoretical studies are required on Trapped BEC to pave future technology in this system

Analytical Approach

A Unified Method of Exact Solution !!

The generalized nonlinear Schrödinger equation with time- and space-modulated distributed coefficients:

$$i\frac{\partial\psi(x,t)}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x,t)\psi(x,t)+g(x,t)|\psi(x,t)|^2\,\psi(x,t)+i\Gamma(x,t)\psi(x,t)$$

The wave function in the form

$$\psi(x,t)=B(x,t)\Phi[X(x,t)]Exp[i\theta(x,t)+G(x,t)]$$

- Amplitude $B(x,t)$, external potential $V(x,t)$, phase $\theta(x,t)$, nonlinearity $g(x,t)$, gain/loss $\Gamma(x,t)$ are all real function of x and t.
- X denotes the travelling coordinate, a real function of x and t

$$G(x,t)=\int\Gamma(x,t)d\ t$$

$$A(x,t)=B(x,t)Exp[G(x,t)]$$

Conditions to be satisfied…

- $$\frac{\partial A(x,t)}{\partial x}\frac{\partial X(x,t)}{\partial x}+\frac{A(x,t)}{2}\frac{\partial^2 X(x,t)}{\partial x^2}=0$$
- $$-\frac{\partial^2\Phi(x,t)}{\partial X^2}+2g(x,t)A(x,t)^2/\left(\frac{\partial X(x,t)}{\partial x}\right)^2 Exp[-2G(x,t)]\,|\Phi|^2\,\Phi-\mu\,\Phi=0$$
- $$\frac{\partial X(x,t)}{\partial t}+\frac{\partial X(x,t)}{\partial x}\frac{\partial \theta(x,t)}{\partial x}=0$$
- $$\frac{\partial A(x,t)}{\partial t}+\frac{\partial A(x,t)}{\partial x}\frac{\partial \theta(x,t)}{\partial x}+\frac{A(x,t)}{2}\frac{\partial^2 \theta(x,t)}{\partial x^2}-\Gamma(x,t)A(x,t)=0$$
- $$\frac{1}{2}\frac{\partial^2 A(x,t)}{\partial x^2}-\frac{A(x,t)}{2}\left(\frac{\partial \theta(x,t)}{\partial x}\right)^2-V(x,t)A(x,t)-A(x,t)\frac{\partial \theta(x,t)}{\partial t}$$
$$-\frac{\mu A(x,t)}{2}\left(\frac{\partial X(x,t)}{\partial x}\right)^2=0$$

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Solutions…

- The potential is solved as a general expression, dependent on other variables of the system

$$V(x,t) = -\frac{1}{4}\frac{a'''(x)}{a'(x)} + \frac{3}{8}\left(\frac{a''(x)}{a'(x)}\right)^2 + \frac{3}{8}\gamma(t)^2 a'(x)^2\left(\frac{X''(\gamma(t)a(x)+\delta(t))}{X'(\gamma(t)a(x)+\delta(t))}\right)^2 - \frac{1}{4}\gamma(t)^2 a'(x)^2\left(\frac{X'''(\gamma(t)a(x)+\delta(t))}{X'(\gamma(t)a(x)+\delta(t))}\right)$$
$$-\frac{1}{2}\left(\frac{\gamma'(t)a(x)+\delta'(t)}{\gamma(t)a'(x)}\right)^2 + \left(\frac{\gamma''(t)\gamma(t)-\gamma'(t)^2}{\gamma(t)^2}\right)\int\frac{a(x)}{a'(x)}dx$$
$$+\left(\frac{\gamma(t)\delta''(t)-\delta''(t)\gamma'(t)^2}{\gamma(t)^2}\right)\int\frac{dx}{a'(x)} - \alpha'(t) - \frac{1}{2}\mu X'\left(\gamma(t)a(x)+\delta(t)\right)^2\gamma(t)^2 a'(x)^2$$

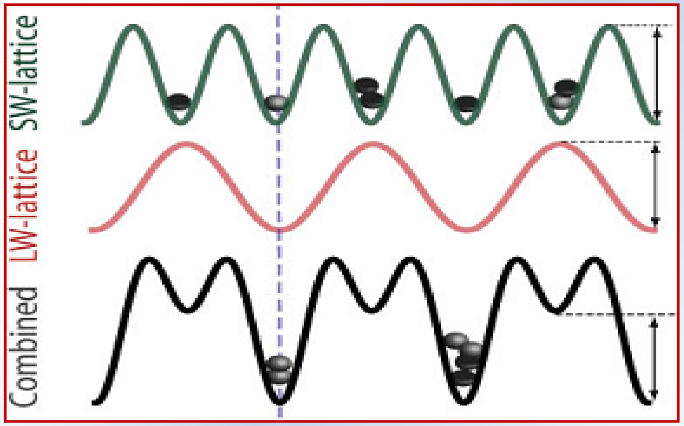
Unified Model for External Trap

$$\psi(x,t) = \sqrt{\frac{c(t)}{\gamma(t)X'\left(a(x)\gamma(t)+\delta(t)\right)a'(x)}}\,Sech\left[X\left(a(x)\gamma(t)+\delta(t)\right)\right]$$
$$\times\ Exp\left[i\left(-\frac{\gamma'(t)}{\gamma(t)}\int\frac{a(x)}{a'(x)}dx - \frac{\delta'(t)}{\delta(t)}\int\frac{dx}{a'(x)} + \alpha(t)\right)\right]$$
$$\times\ Exp\left[\int\left(\frac{c'(t)}{2c(t)} - \frac{\gamma'(t)}{\gamma(t)} + \frac{a''(x)}{a'(x)^2}\left[\frac{\gamma'(t)a(x)}{\gamma(t)} + \frac{\delta'(t)}{\gamma(t)}\right]\right)dt\right]$$

A. Nath and Utpal Roy, Journal of Physics A: Mathematical and Theoretical 47, 415301 (2014)

BEC in Bi-chromatic Optical Lattice

Bichromatic Optical Lattices

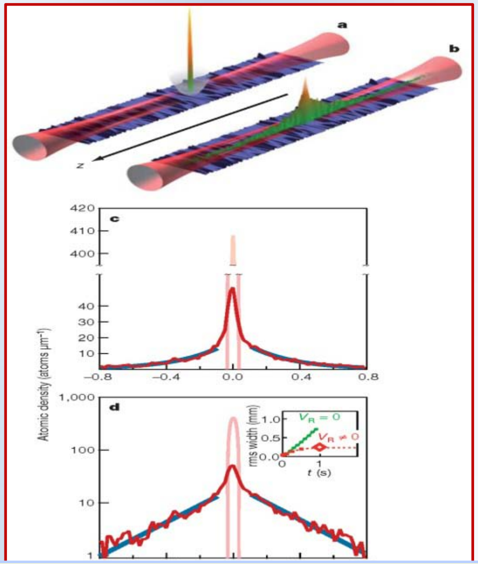


$$\frac{s_1 h^2}{2m\lambda_1^2}\cos\left(\frac{2\pi}{\lambda_1}z\right)$$

$$\frac{s_2 h^2}{2m\lambda_2^2}\cos\left(\frac{2\pi}{\lambda_2}z\right)$$

$$\frac{s_1 h^2}{2m\lambda_1^2}\cos\left(\frac{2\pi}{\lambda_1}z\right)+\frac{s_2 h^2}{2m\lambda_2^2}\cos\left(\frac{2\pi}{\lambda_2}z\right)$$

Phys. Rev. Lett. 108, 045305 (2012)



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Roati et al., Nature (London) 453, 895 (2008)
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Beitia et al., Phys. Rev. Lett 100, 164102 (2008)
J. Struck, Science 333, 996 (2011)
Phys. Rev. Lett. 108, 045305 (2012)
Adhikari et al., Phys. Rev. A 80, 023606 (2009)

Analytical Approach for Localized Solution

$$i\frac{\partial\psi(z,t)}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi(z,t)}{\partial z^2}+V(z,t)\psi(z,t)+g(z,t)|\psi(z,t)|^2\,\psi(z,t)+i\Gamma(z,t)\psi(z,t)$$

The Bichromatic Optical Lattice Potential

$$V(z)=V_1\cos(2lz)+V_2\cos(lz)$$

The wave function in the form

$$\psi(z,t)=A(z,t)F[Z(z,t)]Exp[i\theta(z,t)]$$

- Amplitude $A(z,t)$, external potential $V(z,t)$, phase $\theta(z,t)$, nonlinearity $g(z,t)$, gain/loss $\Gamma(z,t)$ are all real function of z and t.
- Z denotes the travelling coordinate, a real function of z and t

Exact Analytical Form of Variables :

$$V(z) = V_1 \cos(2lz) + V_2 \cos(lz)$$

$$Z(z) = \gamma \int_0^z e^{\beta \cos(\ell z')} dz' \qquad V_1 = -\frac{\beta^2 l^2}{16}, \quad V_2 = \frac{\beta l^2}{4}$$

$$A(z,t) = \sqrt{\frac{c(t)}{\gamma e^{\beta \cos(\ell z)}}}, \quad \theta(t) = -\int \frac{\beta^2 \ell^2}{16} \partial t, \quad g(z,t) = \frac{G\gamma^3}{c(t)} e^{3\beta \cos(\ell z)}, \quad \tau(z,t) = \frac{1}{2} \frac{c'(t)}{c(t)}$$

Exact Wavefunction

$$\psi(z,t) = \sqrt{\frac{c(t)}{\gamma e^{\beta \cos(\ell z)}}} \, cn[\gamma \int e^{\cos(\ell z')} dz', m] e^{-\int \frac{\beta^2 \ell^2}{16} \partial t} \qquad (G<0)$$

$$\psi(z,t) = \sqrt{\frac{c(t)}{\gamma e^{\beta \cos(\ell z)}}} \, sn[\gamma \int e^{\cos(\ell z')} dz', m] e^{-\int \frac{\beta^2 \ell^2}{16} \partial t} \qquad (G>0)$$

→ β, ℓ, γ and m could be meticulously modulated for studying various relevant scenarios.

→ We will study both bright and dark solitary waves with $G = -1$ (attractive) and $G = 1$ (repulsive) nonlinearity.

Utilize the Tunability of BOL

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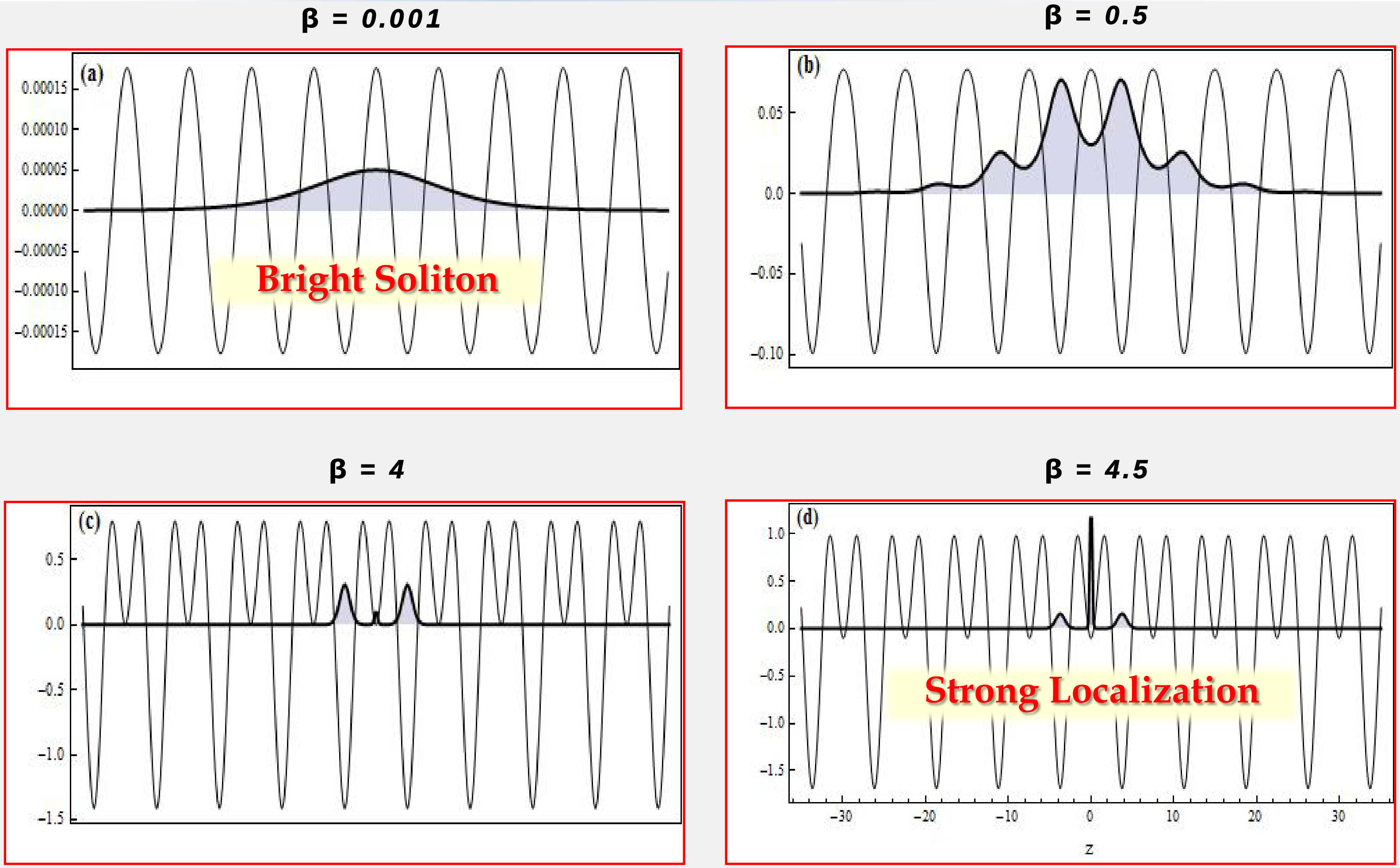
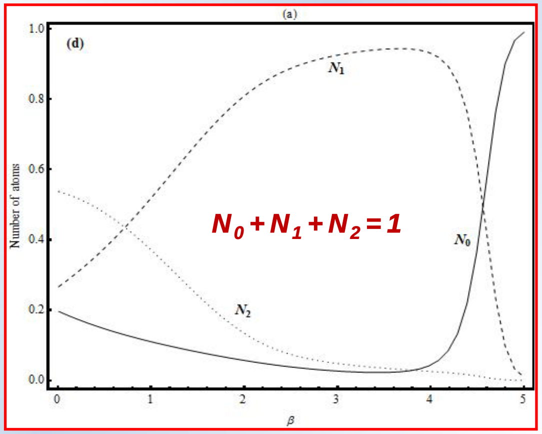
Condensate density in Attractive regime



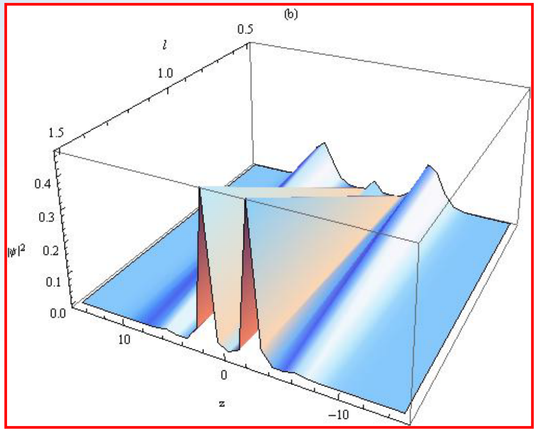
Condensate density for $l = 0.84,\ c(t) = c = 0.1,\ \gamma = 0.1,\ G = -1$

Localization of Condensate density & Energy per particle

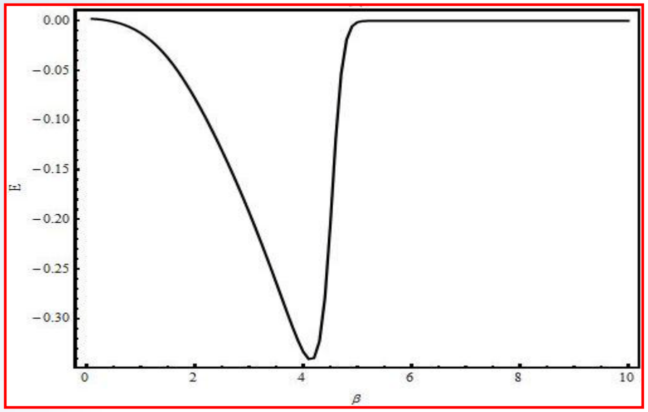
Variation with β



Variation with ℓ

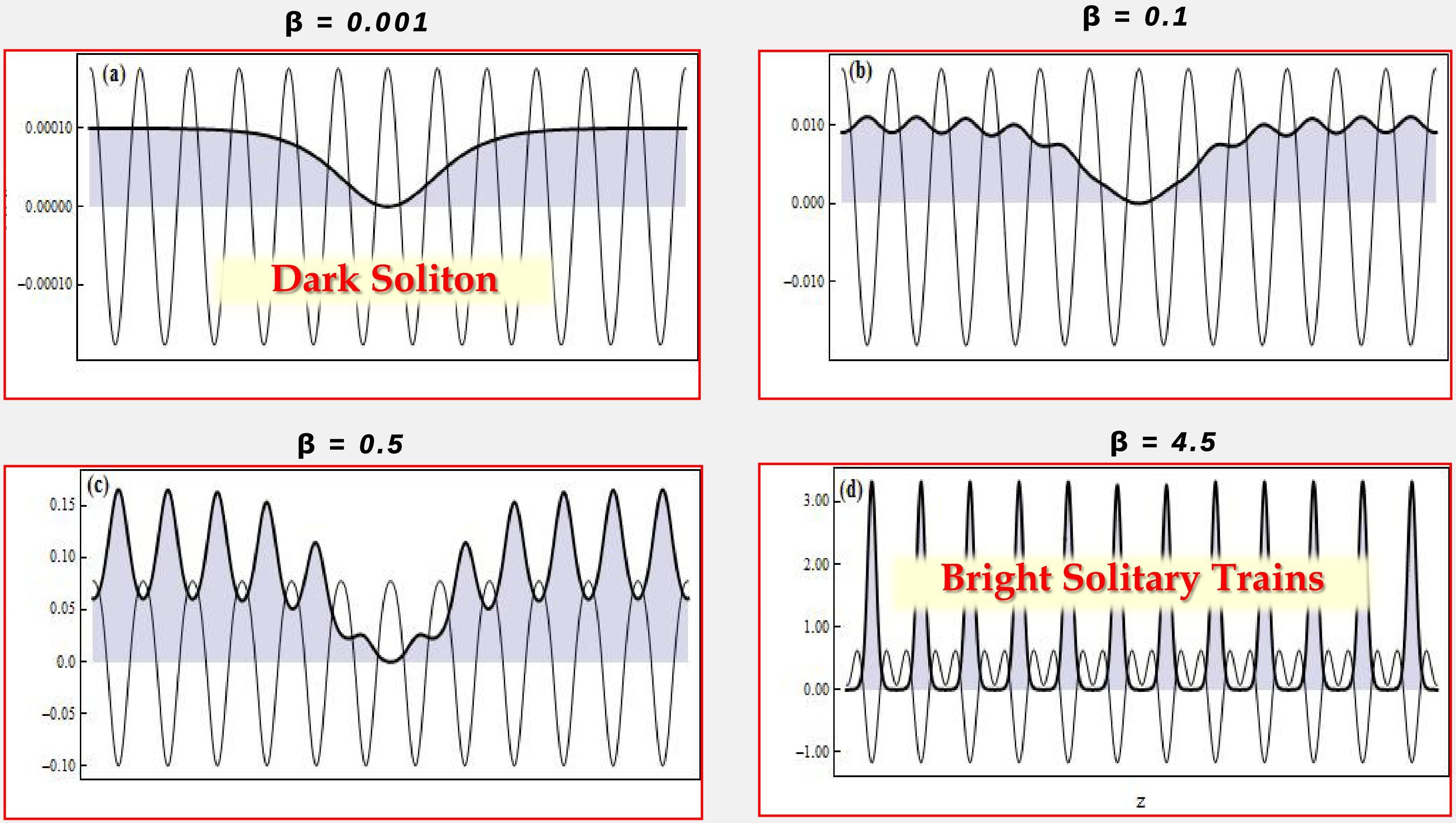


ℓ=0.84, c=0.1, Y=0.1, G=-1 and m = 1



ℓ=0.84, c=0.1, Y=0.1, G=-1 and m = 1

What happens in Repulsive domain ?



Condensate density for l=0.84, c(t)=c=0.1, γ=0.1, G=1

A. Nath and U. Roy, Laser Physics Letters 11, 115501 (2014)

Selected for free access and the Article of the Year

Higher H … Condensate

by Tuning External Confinement

Numerical Approach

Model Under Study

- Quasi-one dimensional Bose-Einstein condensate
- Weak inter-atomic interaction is considered
- Harmonic trap is used as a basic external confinement
- An Optical Lattice potential is applied in addition
- Depth of the optical lattice introduces time dependent Chirping

$$i\frac{\partial}{\partial t}\varphi(x,t)=\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x,t)+g_{nd}\left|\varphi(x,t)\right|^2\right]\varphi(x,t)$$

$$V(x,t)=\gamma x^2+v_0\left[1+\alpha\cos(\omega t)^2\right]\sin(2\pi x/\lambda)^2$$

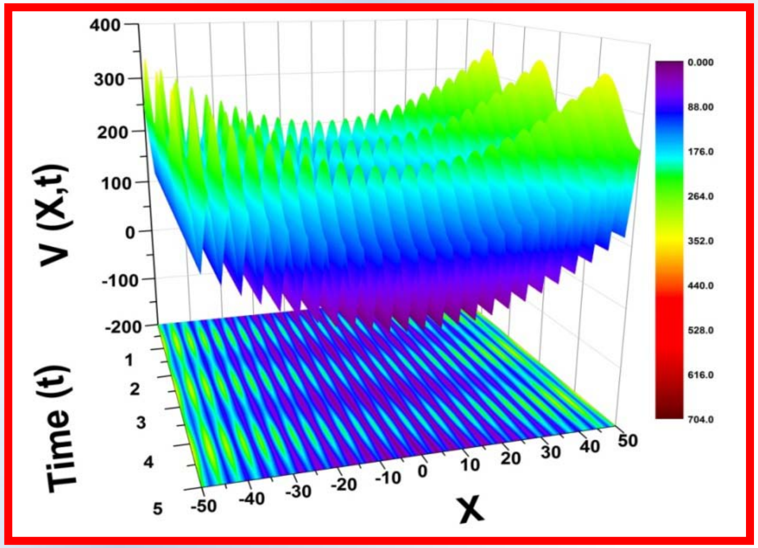
- **Under harmonic trap, both condensate density as well as RMS size oscillate with time.**

- **This oscillation is commensurate with the trap frequency.**

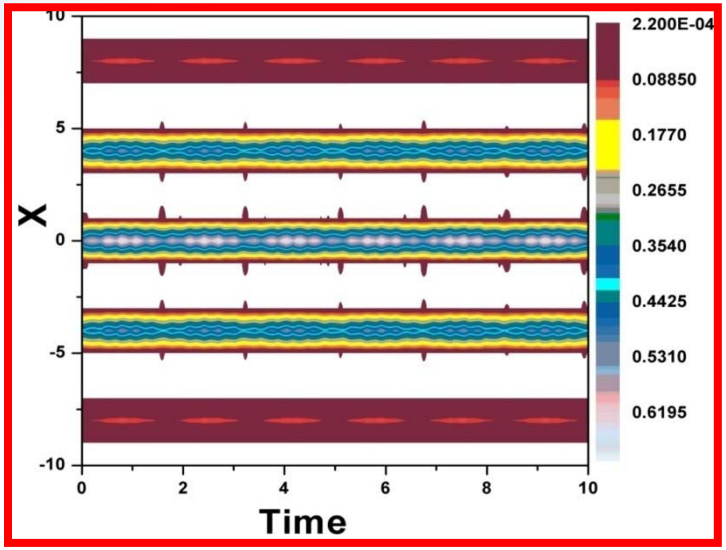
- **It is worth to see oscillation of condensate density in presence of both the trap, simultaneously.**

- **We solve the equation numerically and observe the system in time and frequency planes.**

Variation of Trapping Potential and Corresponding Condensate Density in Space and Time

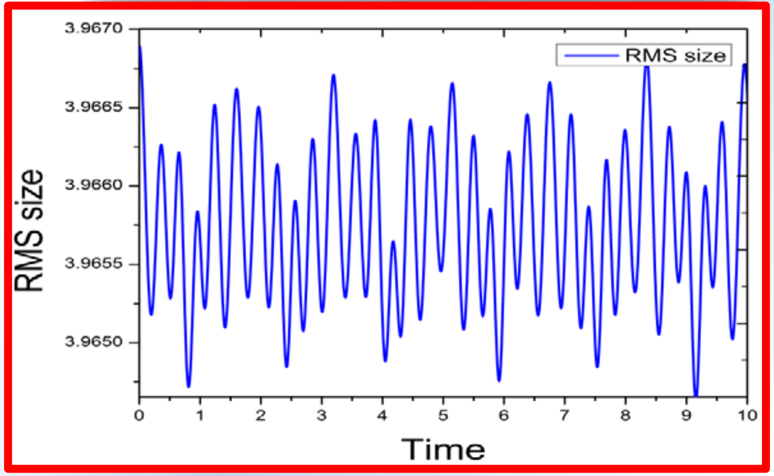


Potential

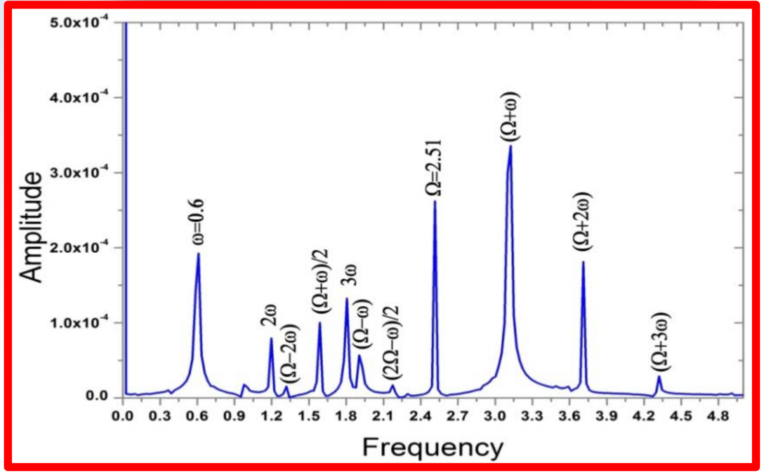


Condensate Density

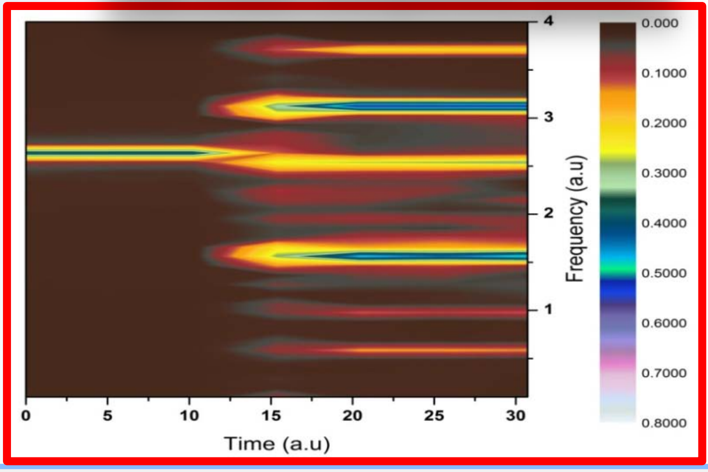
RMS size variation with time



In Frequency plane



In Time- Frequency plane



Production of Quantum Mesoscopic Superposition states by utilizing tunneling and reflection in BEC

ARTICLE

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Controlled formation and reflection of a bright solitary matter-wave

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Bright solitons are non-dispersive wave solutions, arising in a diverse range of nonlinear, one-dimensional systems, including atomic Bose–Einstein condensates with attractive interactions. In reality, cold-atom experiments can only approach the idealized one-dimensional limit necessary for the realization of true solitons. Nevertheless, it remains possible to create bright solitary waves, the three-dimensional analogue of solitons, which maintain many of the key properties of their one-dimensional counterparts. Such sc
many potential applications and provide a rich testing ground for theoret
many-body quantum systems. Here we report the controlled formation o
matter-wave from a Bose–Einstein condensate of ^{85}Rb, which is observed tc
distance of ~1.1 mm in 150 ms with no observable dispersion. We demonst
of a solitary wave from a repulsive Gaussian barrier and contrast this
repulsive condensate, in both cases finding excellent agreement with theo
using the three-dimensional Gross–Pitaevskii equation.

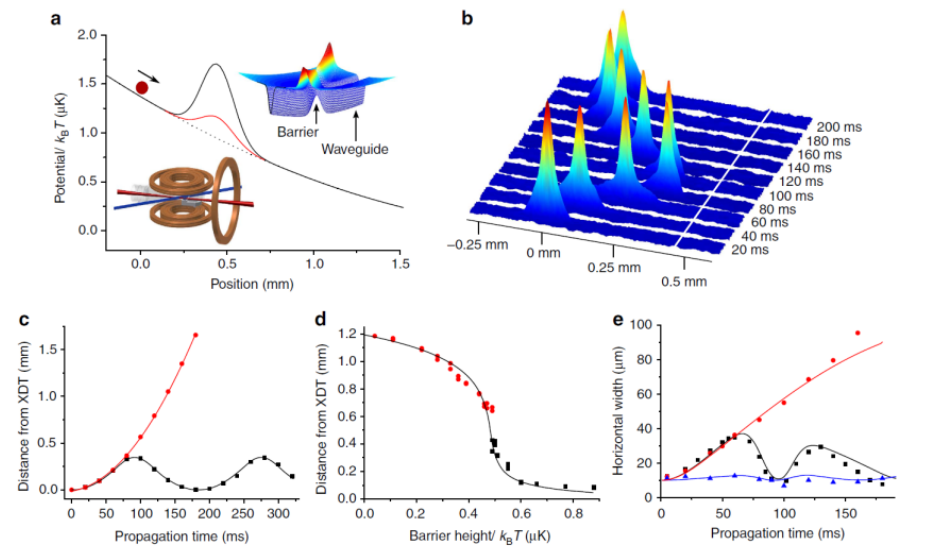


Figure 3 | Reflection from a repulsive Gaussian barrier. (**a**) Potential in the axial direction along the waveguide in the presence of the repulsive barrier. (Inset, upper: combined waveguide and Gaussian barrier potential. Lower: experimental setup.) (**b**) False colour images of a solitary wave reflecting from the barrier. The white line shows the location of the barrier centre. (**c**) Horizontal position, relative to the crossed dipole trap (XDT), of a solitary wave propagating in the waveguide in the absence (red) and presence (black) of the repulsive barrier. (**d**) The position of a solitary wave after 150 ms propagation time as a function of the barrier height. Red (black) points correspond to the solitary wave travelling over (being reflected from) the barrier. Solid lines in (**c**,**d**): theoretical trajectory calculated using a classical particle model with no free parameters. (**e**) Condensate width following reflection from the barrier. In the absence of a barrier, a repulsive BEC ($a_s = 58\,a_0$, $N = 3.5 \times 10^3$) will expand as it propagates (red). With the barrier in place, an oscillation in the condensate width is set up following the strong compression of the condensate at the barrier due to the shape of the potential (black). A solitary wave ($a_s = -11\,a_0$, $N = 2.0 \times 10^3$) undergoing the same collision emerges unaltered (blue). Solid lines are the theoretical condensate widths calculated by solving the 3D (cylindrically symmetric) GPE.

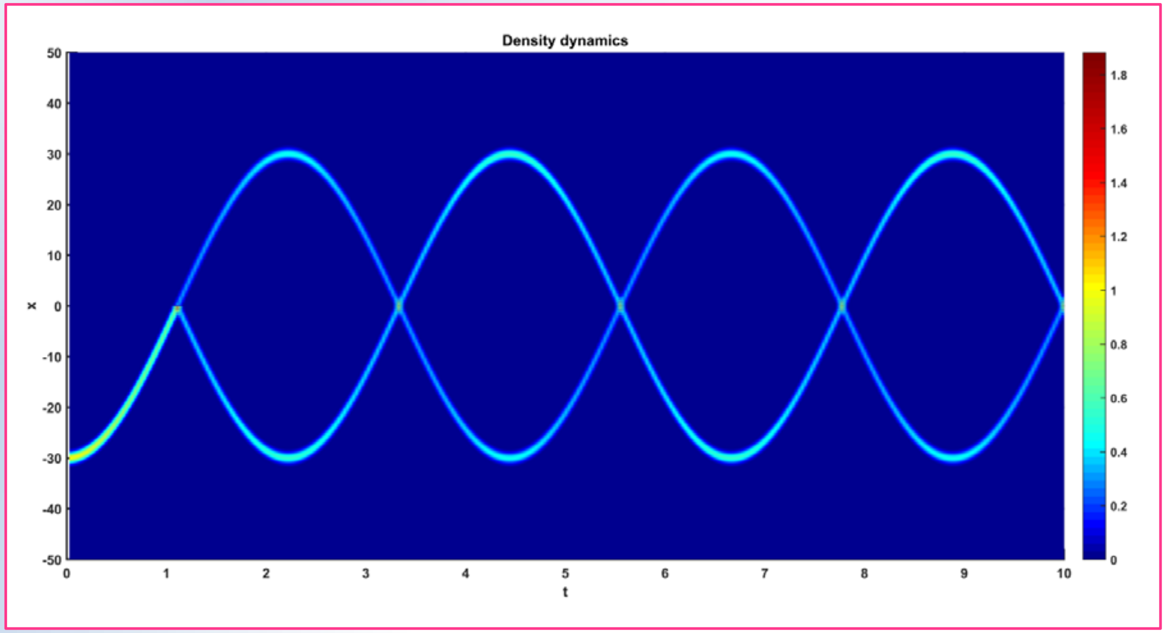
Model Under Study

- Quasi-one dimensional Bose-Einstein condensate
- Weak inter-atomic interaction is considered
- Harmonic trap is used as a basic external confinement
- In addition, an Gaussian peak potential is applied
- Height and width of the peak can be controlled with time to observe higher order quantum superpositions

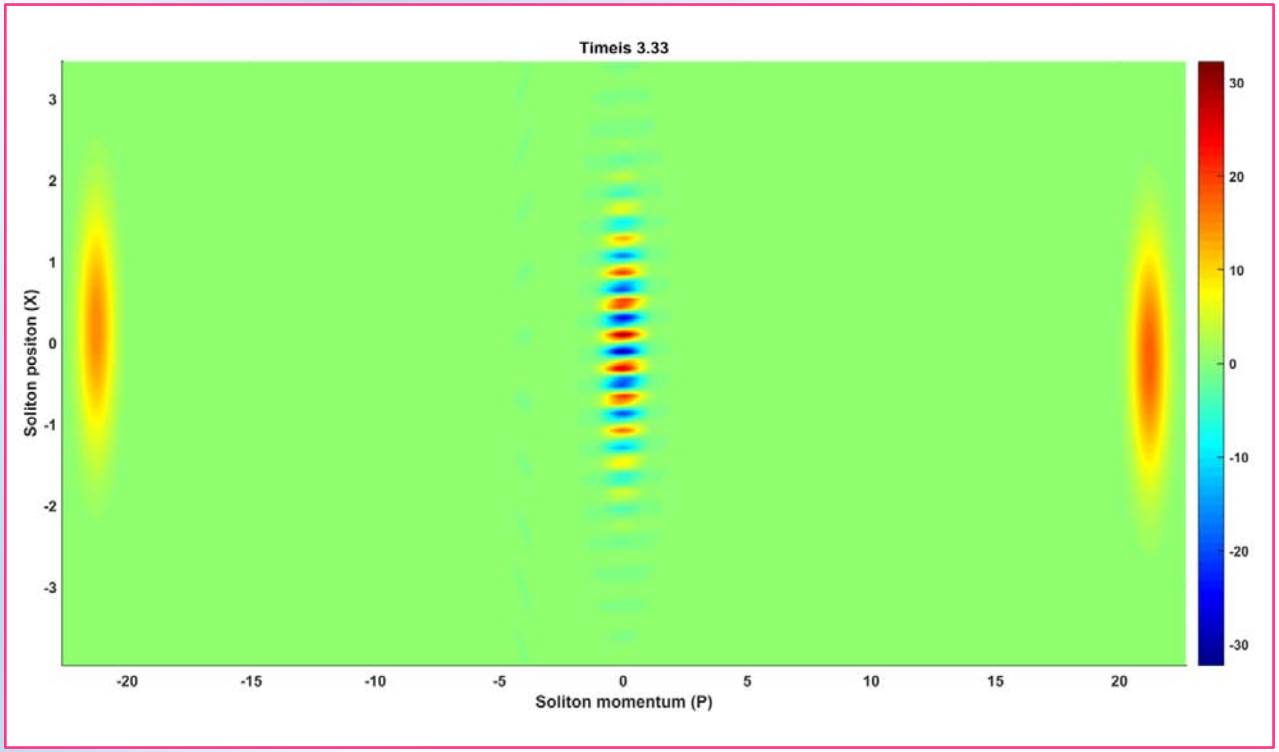
$$i\frac{\partial}{\partial t}\varphi(x,t)=\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x,t)+g_{nd}\left|\varphi(x,t)\right|^2\right]\varphi(x,t)$$

$$V(x,t)=\gamma x^2+v_0(t)\,\mathrm{Exp}\left[-\frac{x^2}{2\sigma^2}\right]$$

Periodic Oscillation of Single Soliton & single potential spike

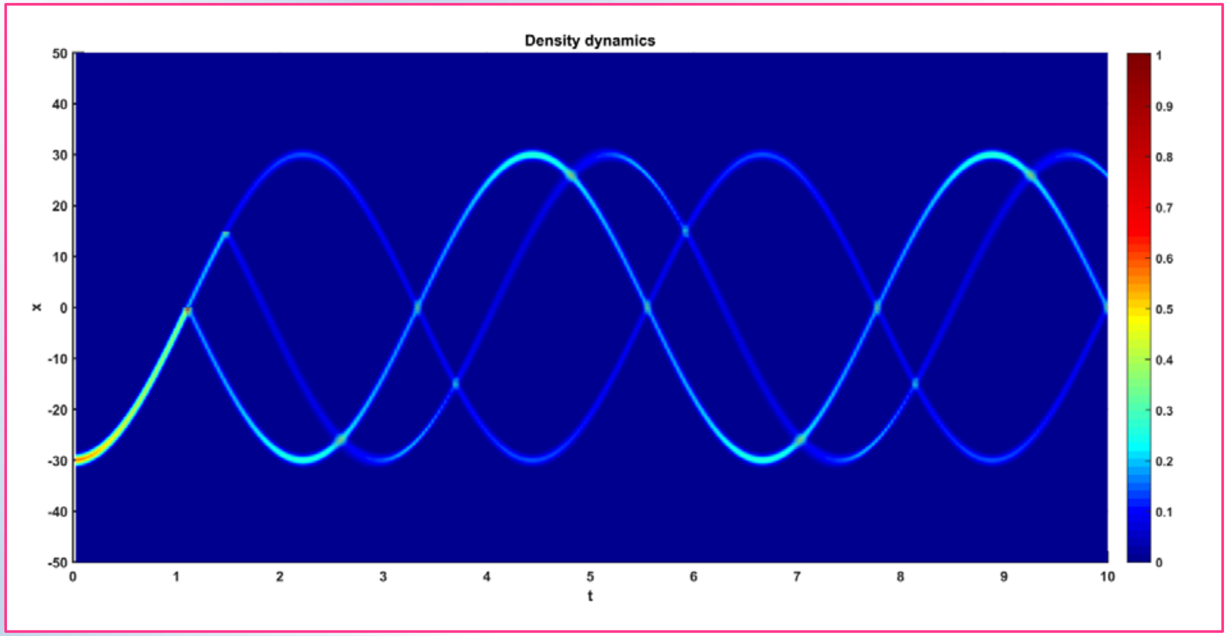


Schrödinger-cat State in Momentum



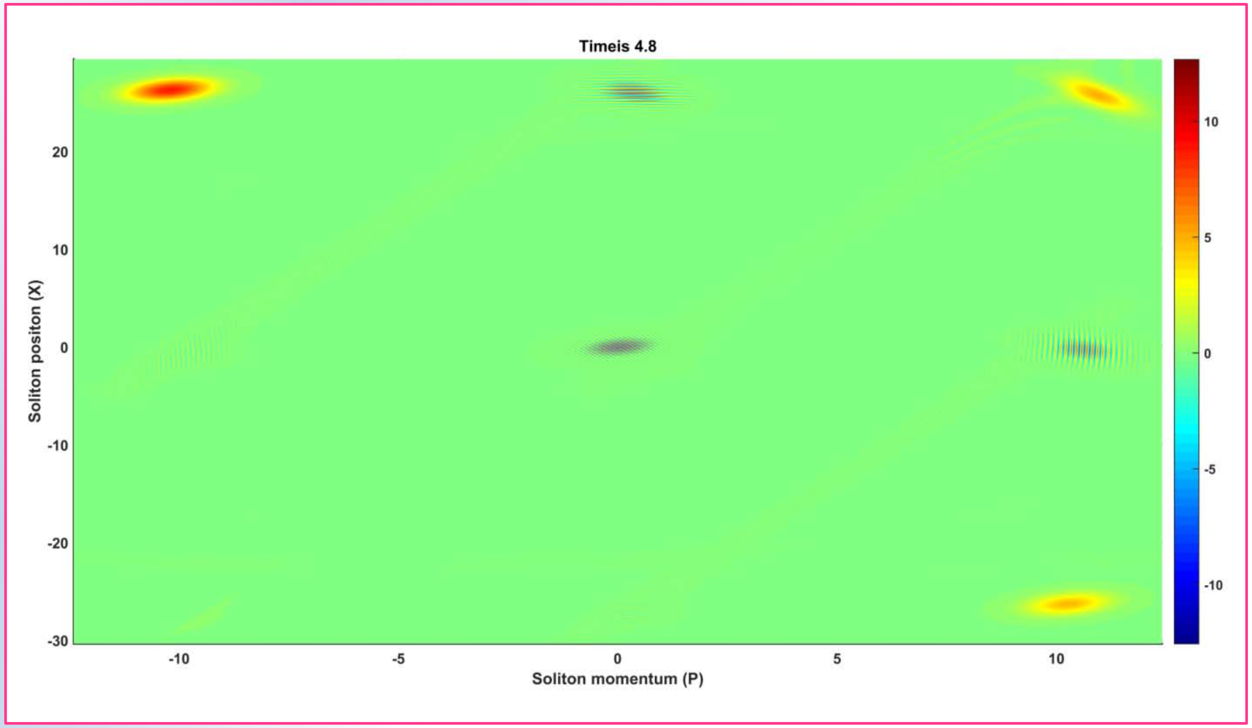
- One can obtain Position cat-state and also a cat-state, rotated in an arbitrary direction in phase space

Periodic Oscillation of Single Soliton & two repulsive potentials



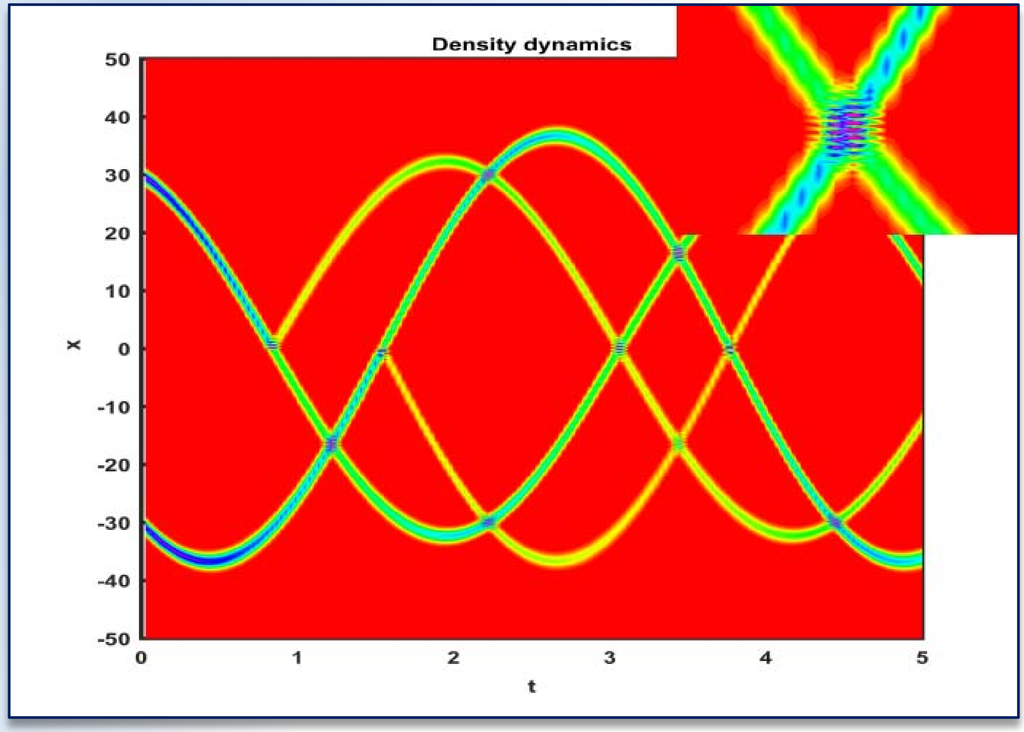
$$V(x,t) = \gamma x^2 + v_{01}(t)\left(\text{Exp}\left[-\frac{x^2}{2\sigma_1^2}\right] + v_{02}(t)\text{Exp}\left[-\frac{(x-a)^2}{2\sigma_2^2}\right]\right)$$

Triangular Mesoscopic State

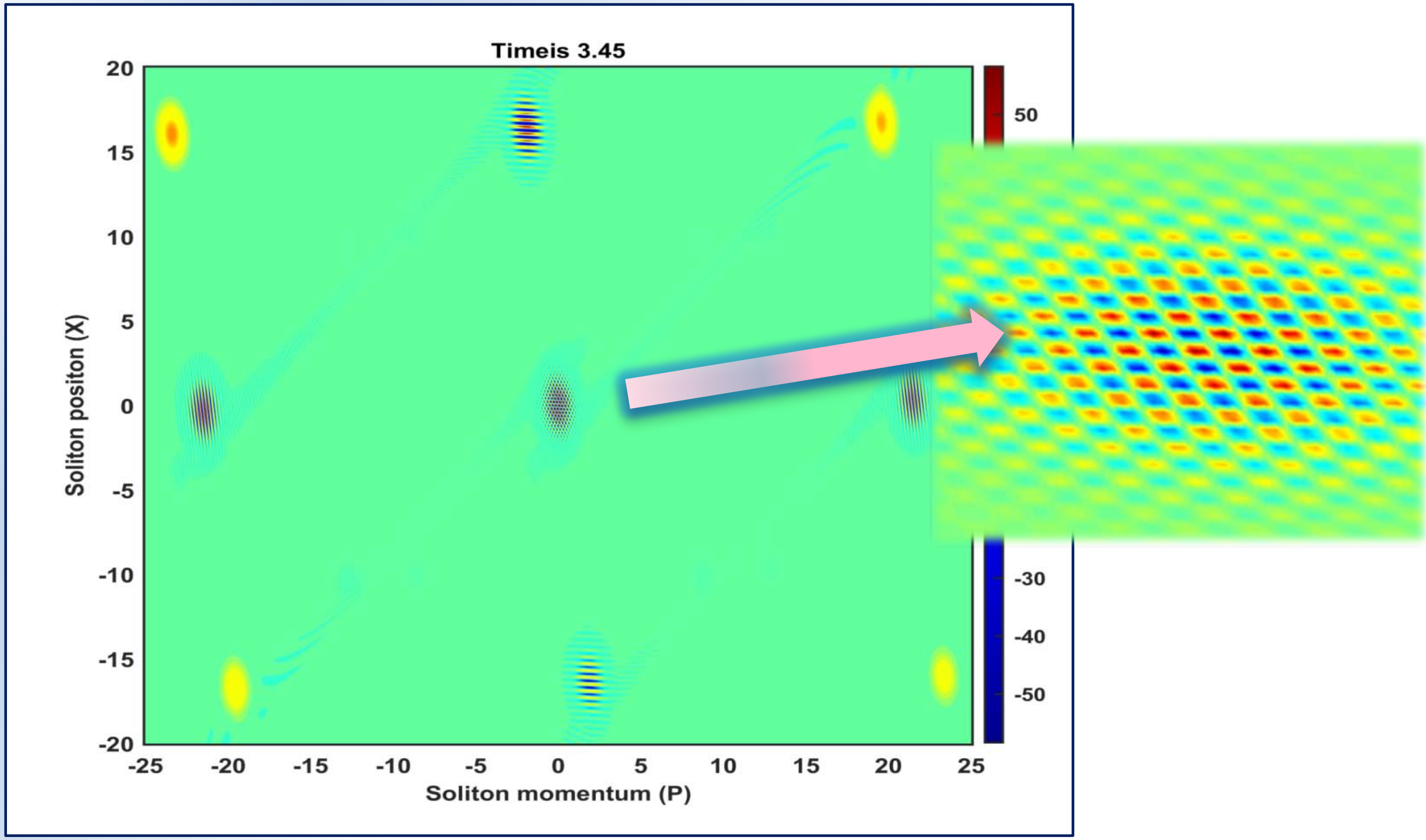


❖ One can also obtain a triangular state, rotated in an arbitrary direction in phase space

The Trajectory of the Condensate for Two Soliton & one potential spike



Compass-like state



Bose-Einstein condensate in a Toroidal Trap

First Experimental Realization of BEC in Toroidal Trap

PRL 95, 143201 (2005) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2005

Bose-Einstein Condensation in a Circular Waveguide

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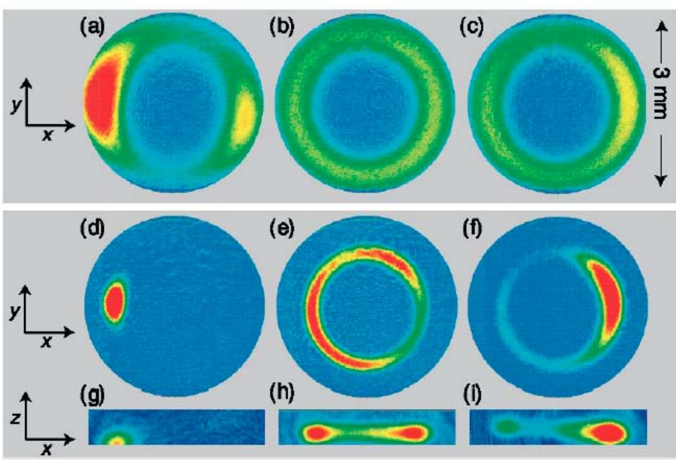
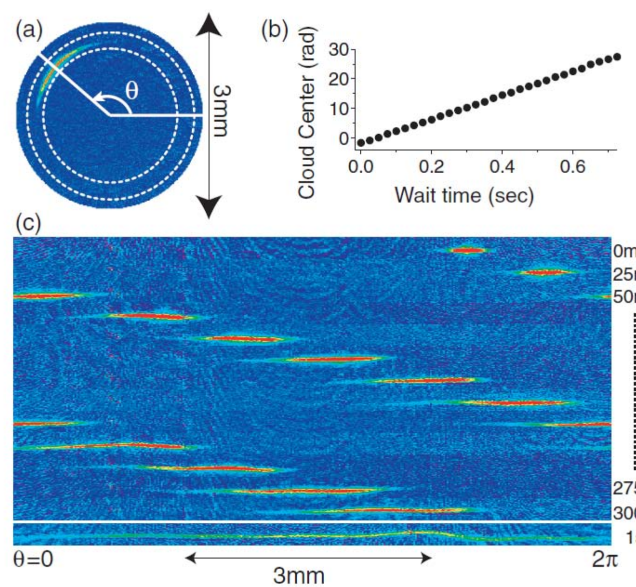
Department of Physics, University of California, Berkeley, California 94720, USA
(Received 27 April 2005; published 29 September 2005)

We have produced Bose-Einstein condensates in a ring-shaped magnetic waveguide. The few-millimeter diameter, nonzero-bias ring is formed from a time-averaged quadrupole ring. Condensates that propagate around the ring make several revolutions within the time it takes for them to expand to fill the ring. The ring shape is ideally suited for studies of vorticity in a multiply connected geometry and is promising as a rotation sensor.





PRL 99, 260401 (2007) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2007

Observation of Persistent Flow of a Bose-Einstein Condensate in a Toroidal Trap

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(Received 31 August 2007; published 28 December 2007)

We have observed the persistent flow of Bose-condensed atoms in a toroidal trap. The flow persists without decay for up to 10 s, limited only by experimental factors such as drift and trap lifetime. The quantized rotation was initiated by transferring one unit $\hbar$ of the orbital angular momentum from Laguerre-Gaussian photons to each atom. Stable flow was only possible when the trap was multiply connected, and was observed with a Bose-Einstein condensate fraction as small as 20%. We also created flow with two units of angular momentum and observed its splitting into two singly charged vortices when the trap geometry was changed from multiply to simply connected.

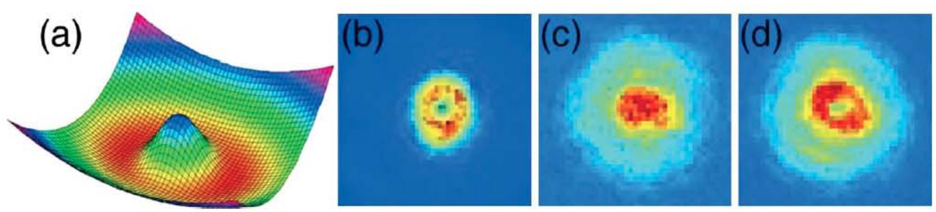


FIG. 1 (color). (a) Toroidal trap from the combined potentials of the TOP trap and Gaussian plug beam. (b) *In situ* image of a BEC in the toroidal trap. (c) TOF image of a noncirculating BEC released from the toroidal trap. (d) TOF image of a circulating BEC, released after transfer of $\hbar$ of OAM.

❑ Autocorrelation Function :

$$A(t) = | < \psi(x,y,t=0)|\psi(x,y,t=t) > |^2$$
$$= | \int\int [\psi(x,y,t=0)^* \psi(x,y,t=t)] dx dy|^2.$$

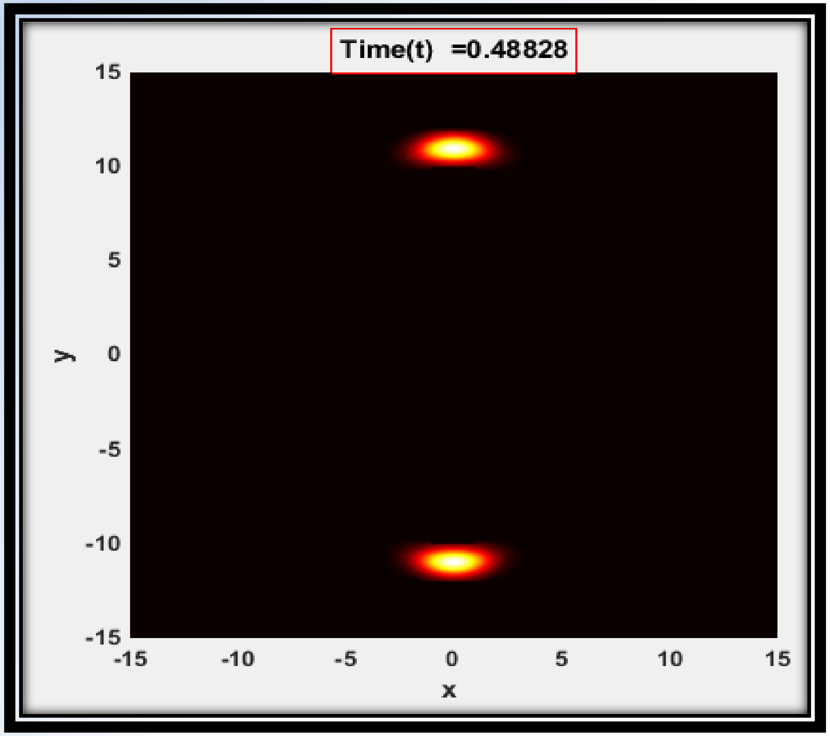
❑ Theoretical Counts of the number of petals:

$$n = \frac{2\pi r}{\Delta r} = \frac{rtd}{\omega_0^4 + t^4}.$$

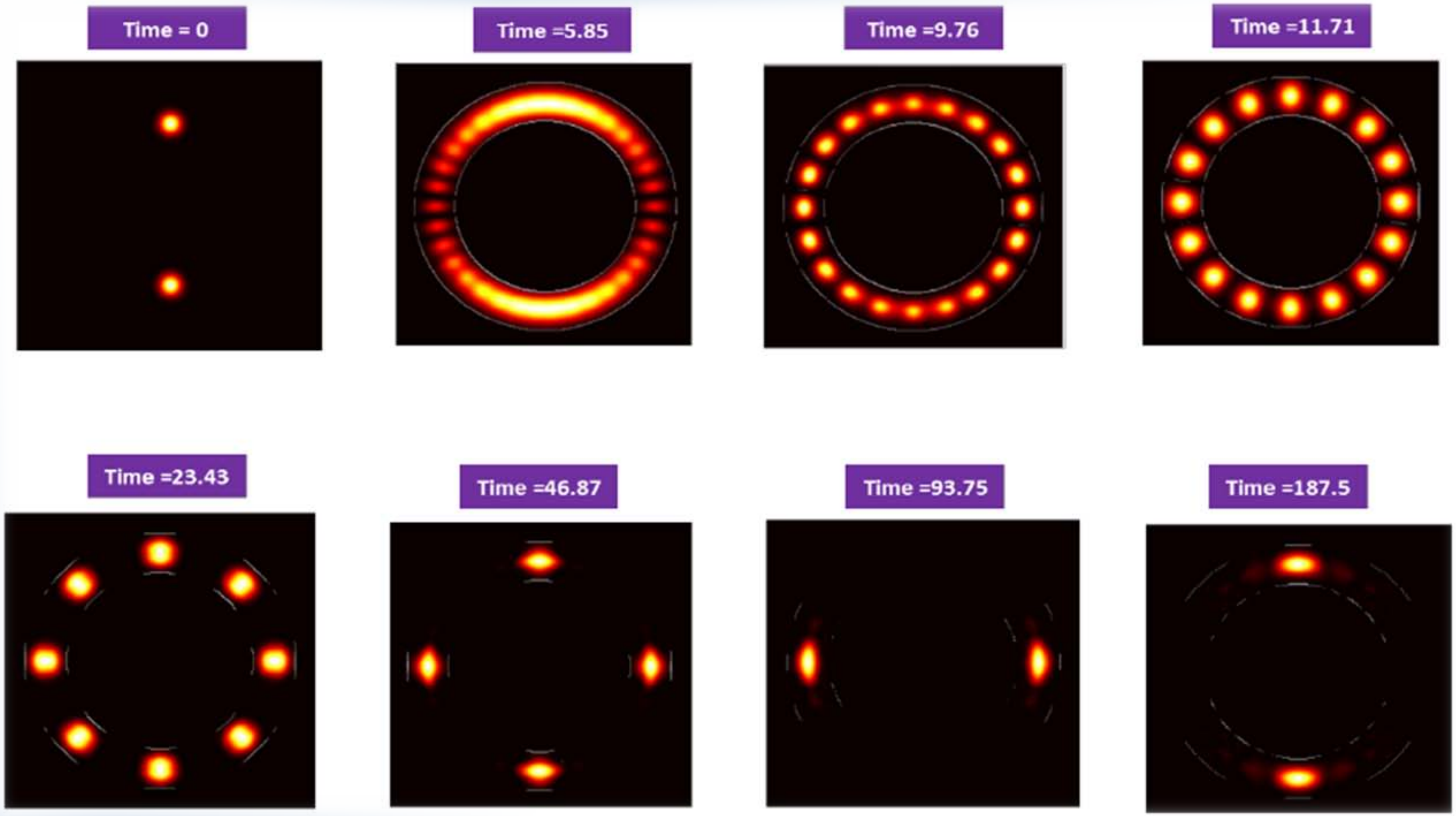
$$\Delta r = 2\pi \frac{\omega_0^2 \omega_t^2}{td}$$

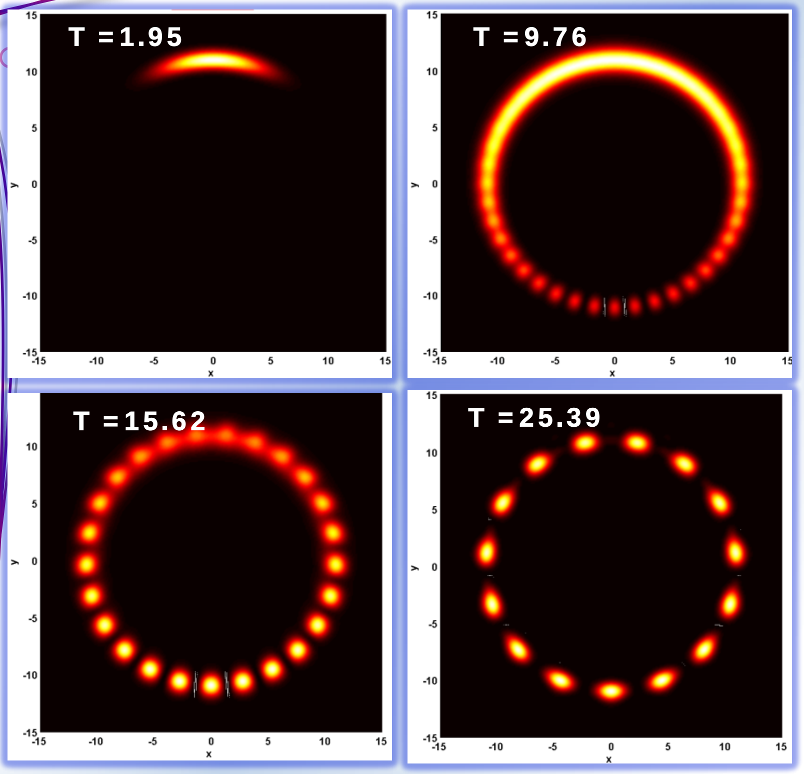
$$\omega_t = \sqrt{\omega_0 + (t/\omega_0)^2}$$

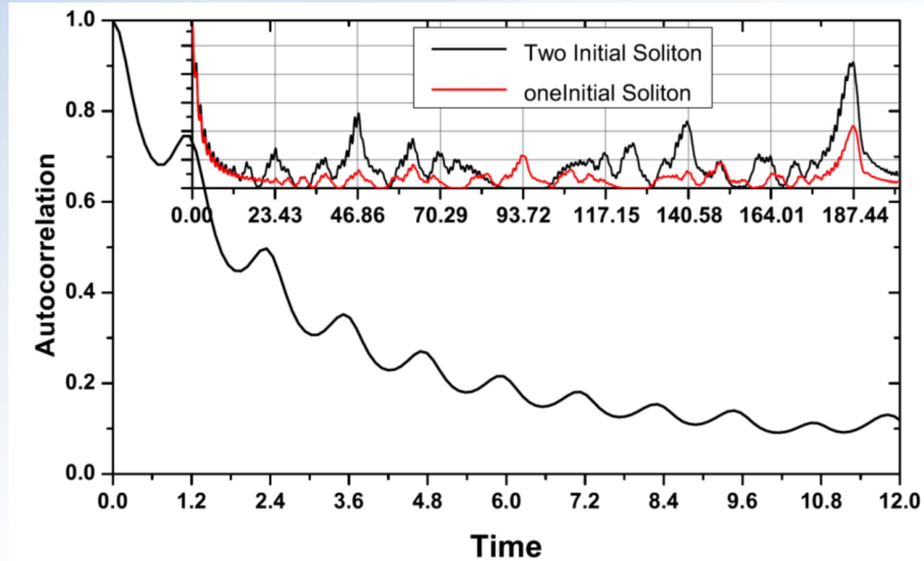
Wave Packet with Nonlinear Energy Spectrum: Evolution of the Condensate…

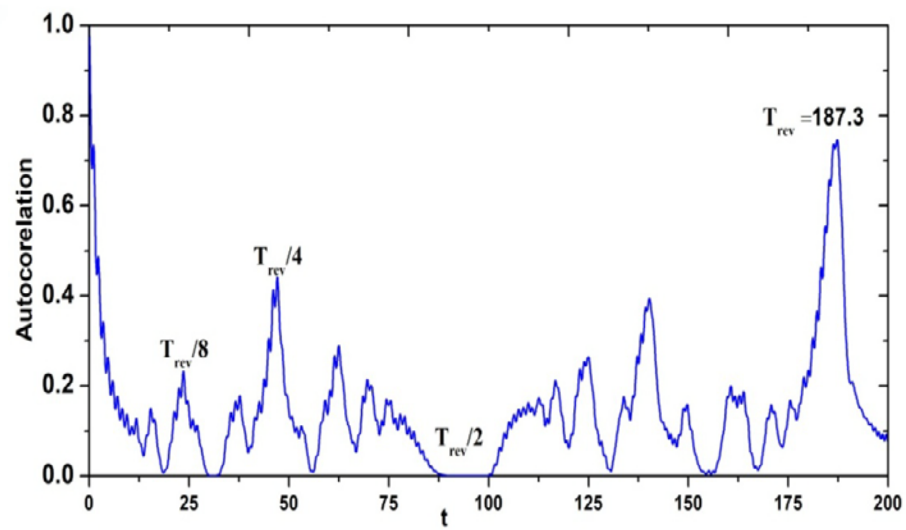


Time Spanshots of the Density Patterns

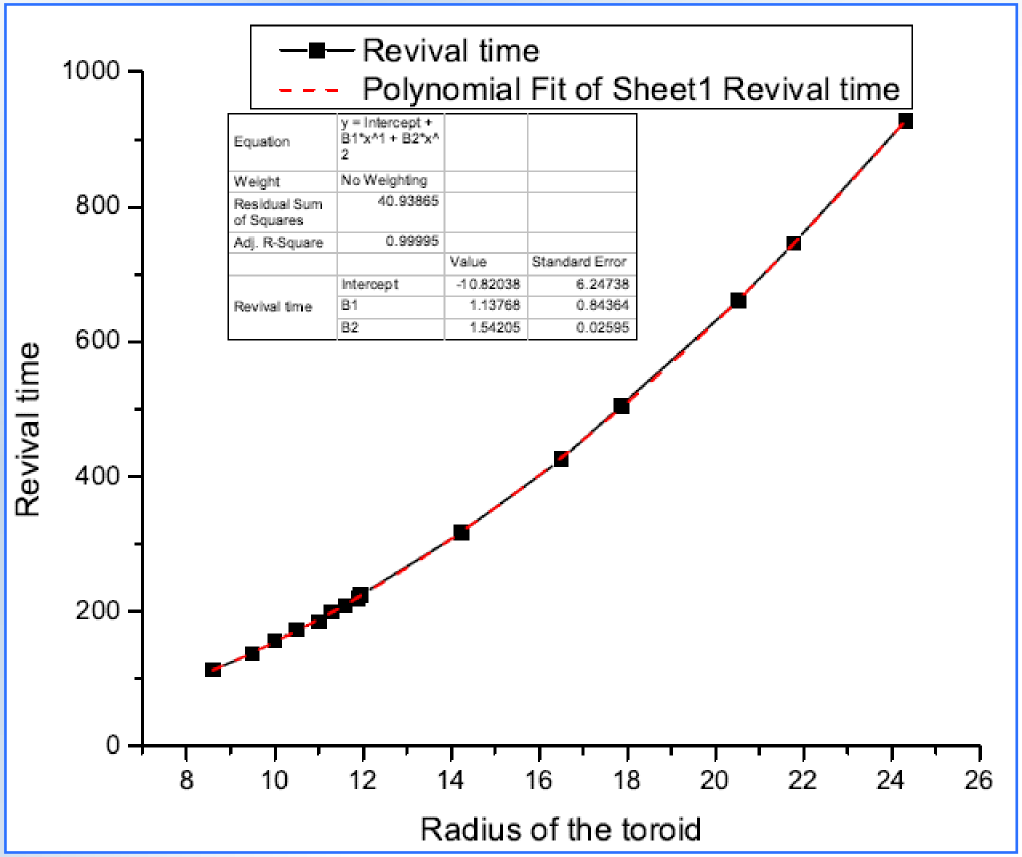






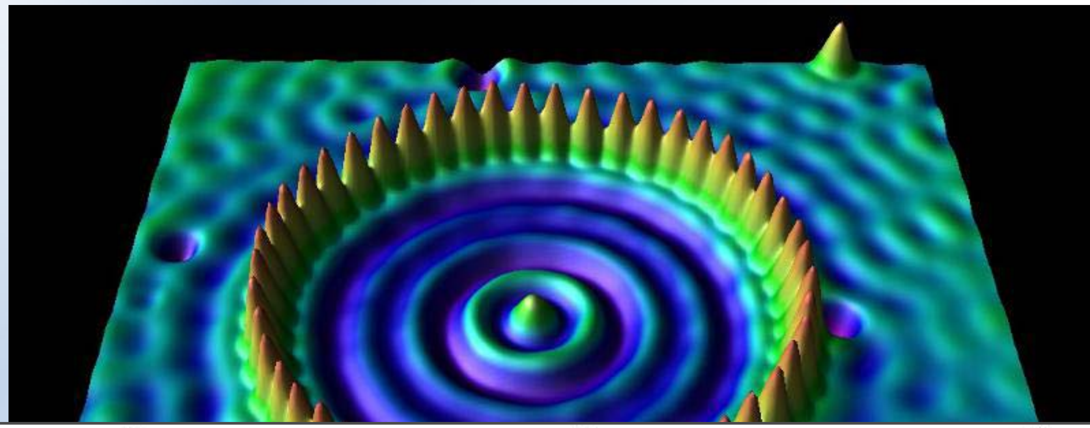


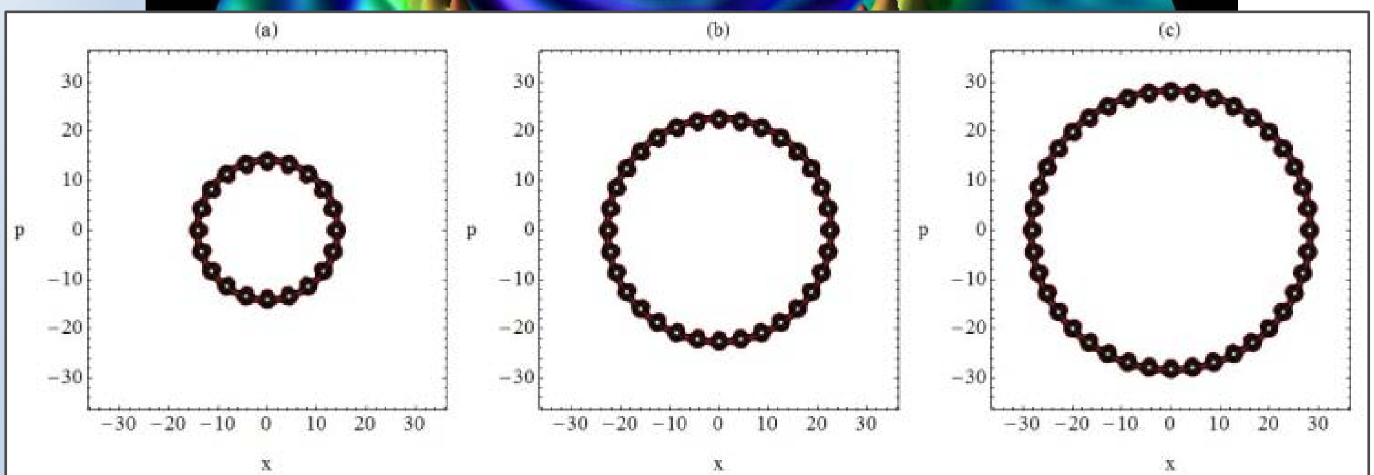
Autocorrelation Function



Revival time
Polynomial Fit of Sheet1 Revival time
Equation
y = Intercept + B1*x^1 + B2*x^2
Weight
No Weighting
Residual Sum of Squares
40.93865
Adj. R-Square
0.99995
Value
Standard Error
Revival time
Intercept
-10.82038
6.24738
B1
1.13768
0.84364
B2
1.54205
0.02595
Revival time
0
200
400
600
800
1000
8
10
12
14
16
18
20
22
24
26
Radius of the toroid

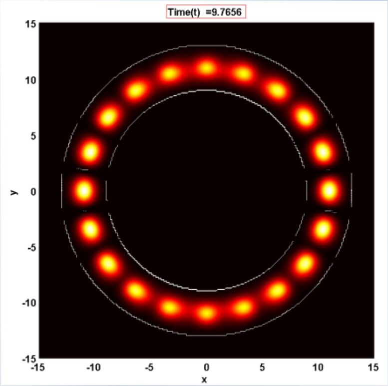
What about Interference Patterns ?

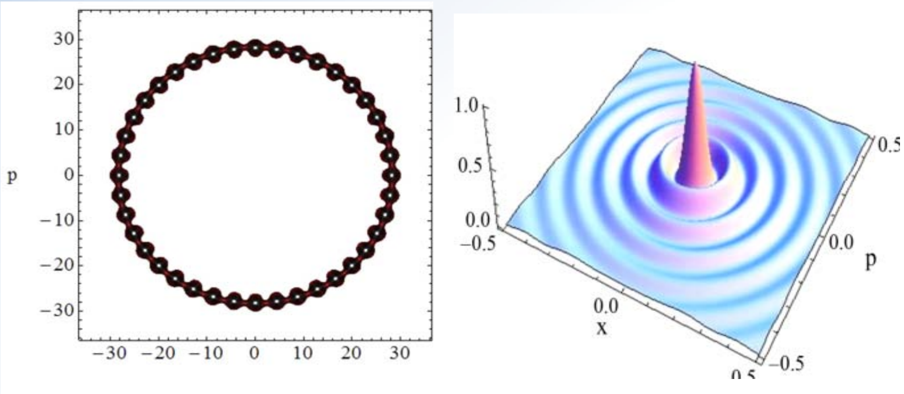




PHYSICAL REVIEW A 92, 053819 (2015)

Actual Interference Patters

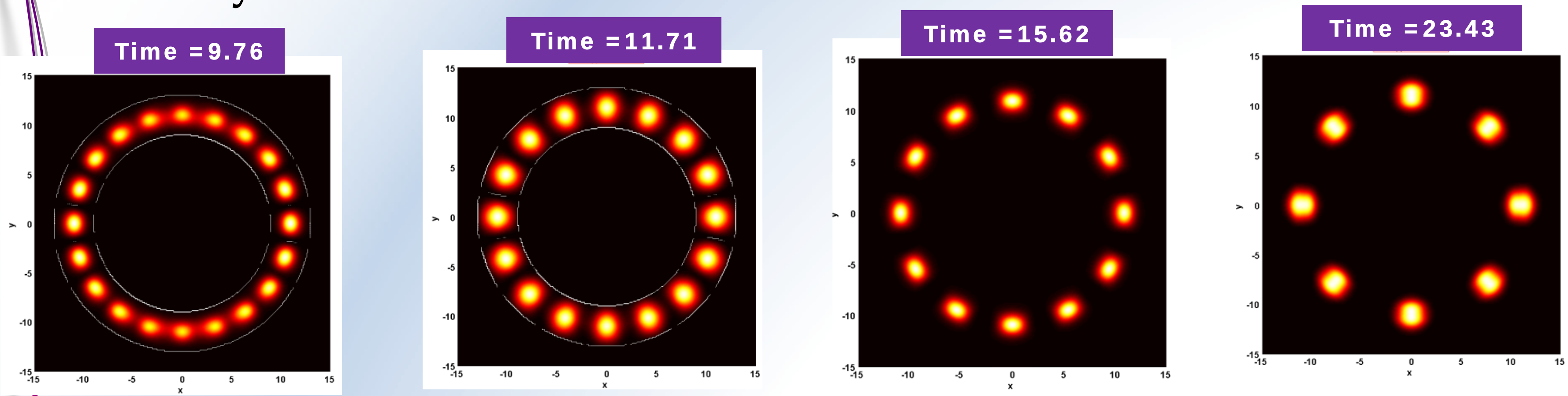




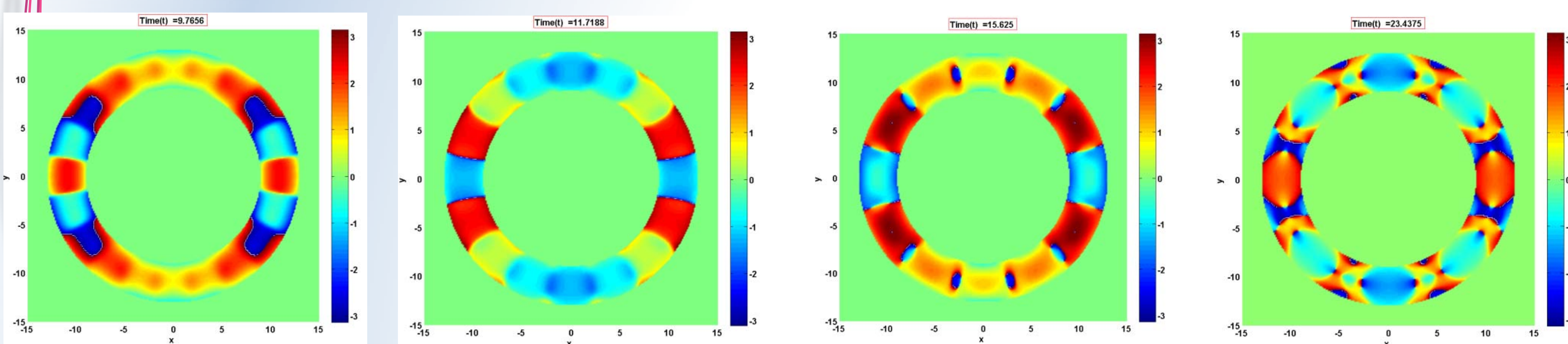
PHYSICAL REVIEW A 92, 053819 (2015)

Phase Dynamics

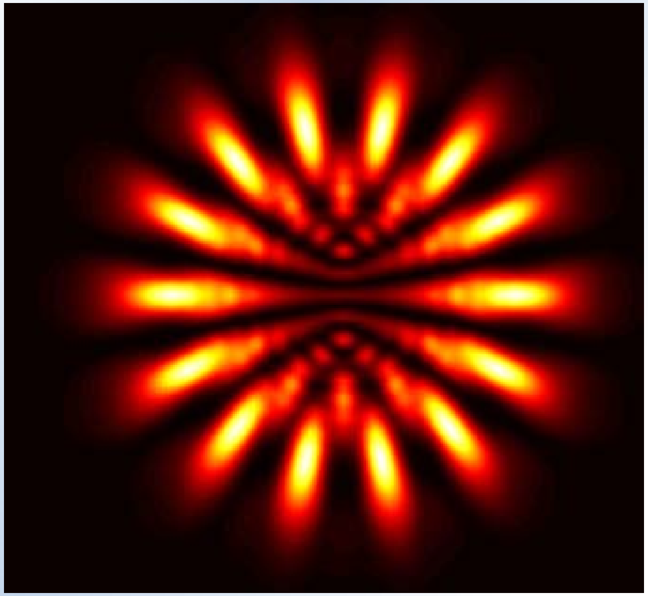
- Density at different time :

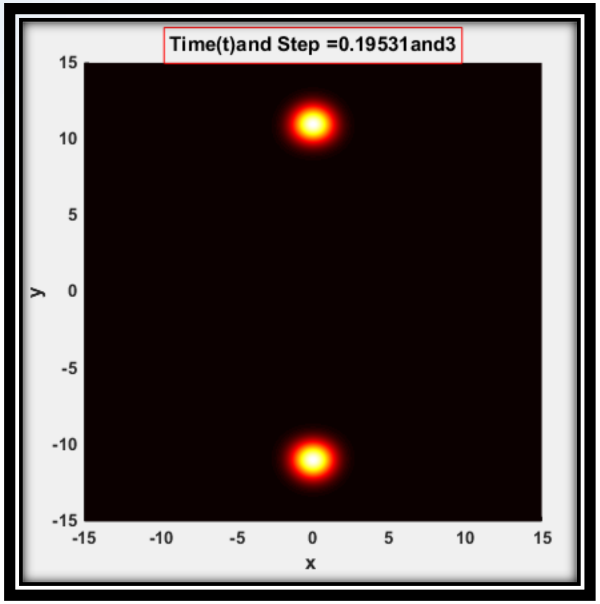


- Corresponding Phase at different time :



Spatial Interference





Thanks

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