

$$\hat{a}^+ |n\rangle = \sqrt{n+1} |n+1\rangle ; \hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

# Boson subtraction and addition

Due to the symmetrical nature, the probability of a transition in which a photon is created into (subtracted from) state  $x$  is proportional to the number of bosons originally in state  $x$ .

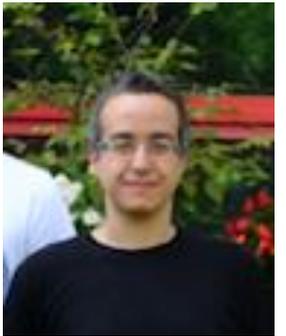
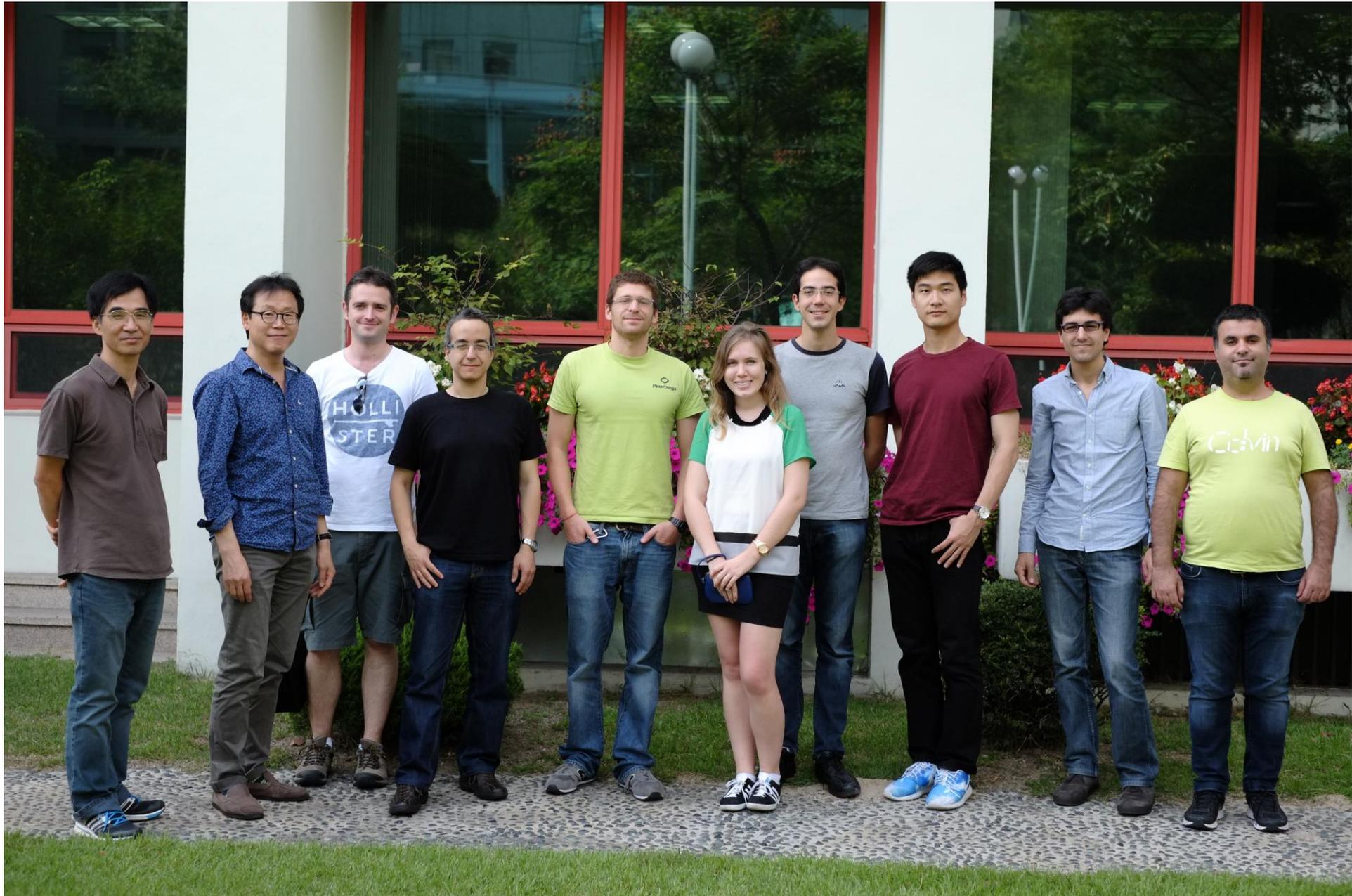
$$\hat{a}^+ |n\rangle = \sqrt{n+1} |n+1\rangle ; \hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

This provides a connection to a  
harmonic oscillator

# Nonclassicalities in multiphoton interference

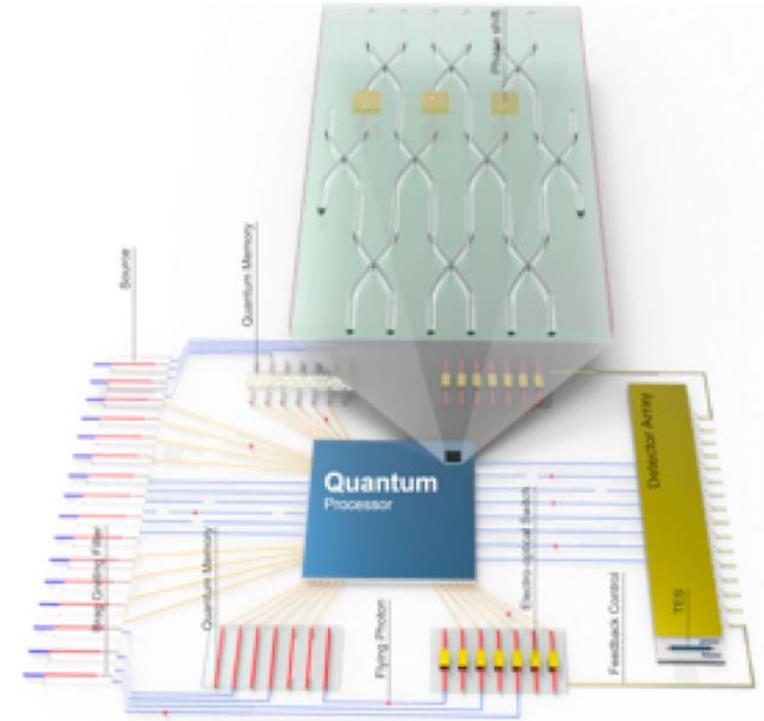
Luca Rigovacca, Carlo Di Franco and Myungshik Kim  
Imperial College London





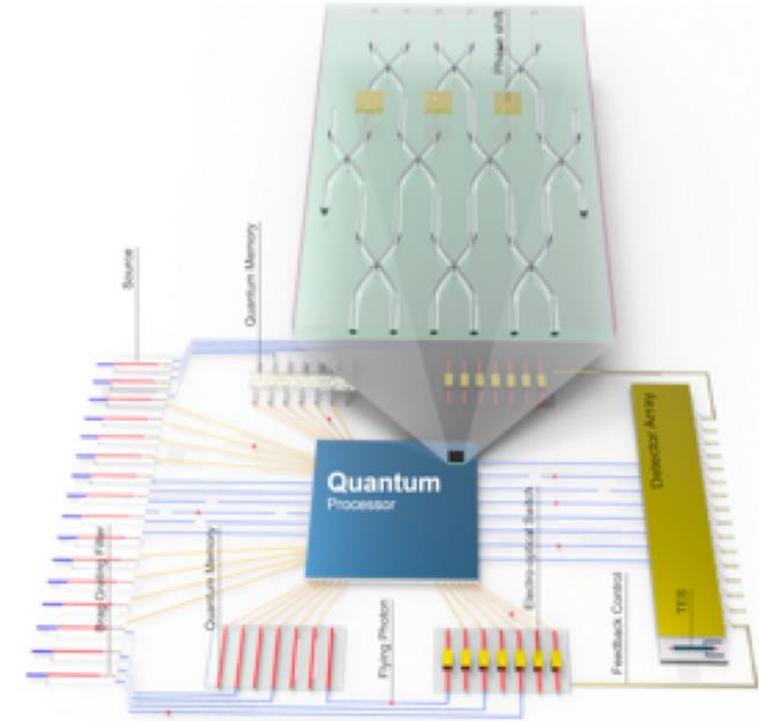
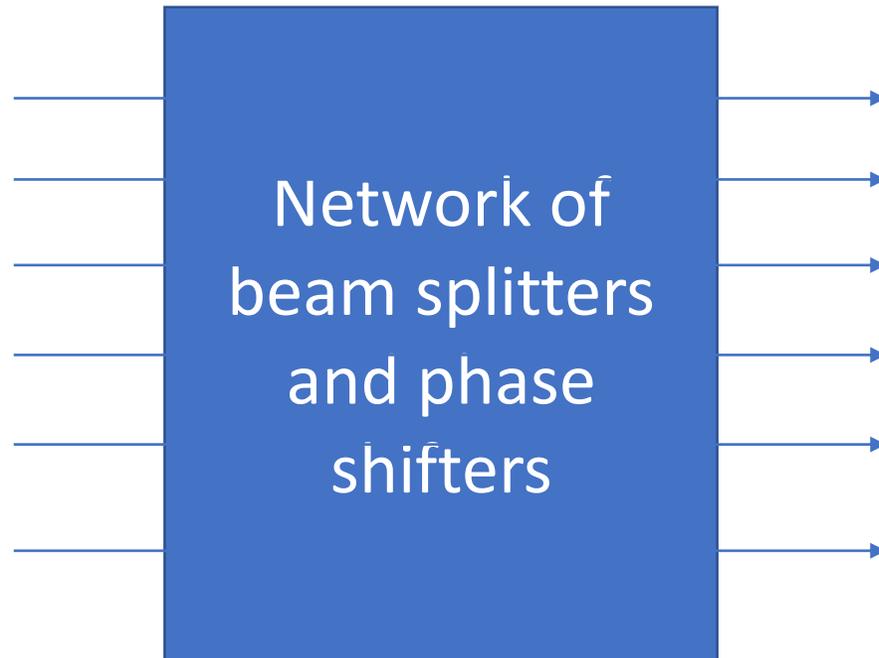
# Motivation

- Linear photonic quantum computation/simulation



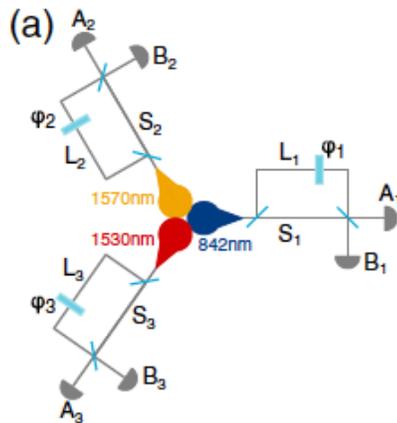
# Motivation

- Nonclassical features of multiphoton interference

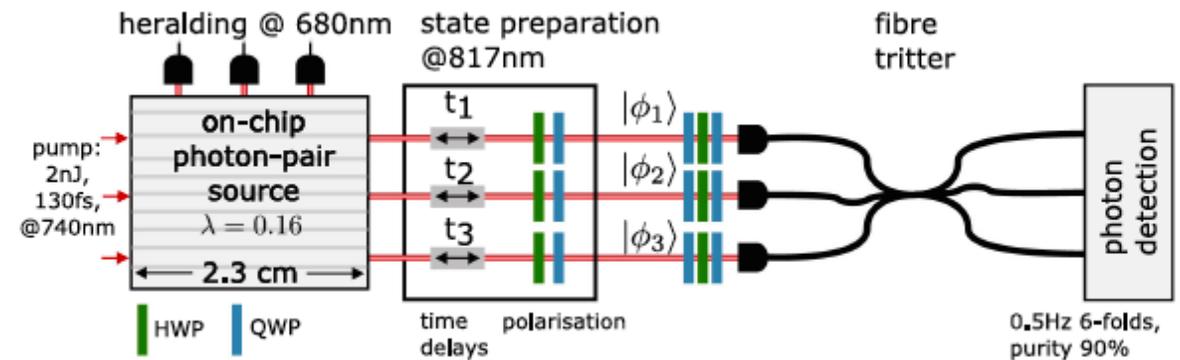


# Flow of the talk

- Recent experiments of three photon interference



S. Agne *et al.* PRL 118, 153602 (17)



A. J. Messen *et al.*, PRL 118, 153603 (17)

-- Remarks on nonclassicalities

- Generalisation of Hong-Ou-Mandel interferometer – Nonclassical criteria



# Franson interferometer

VOLUME 62, NUMBER 19

PHYSICAL REVIEW LETTERS

8 MAY 1989

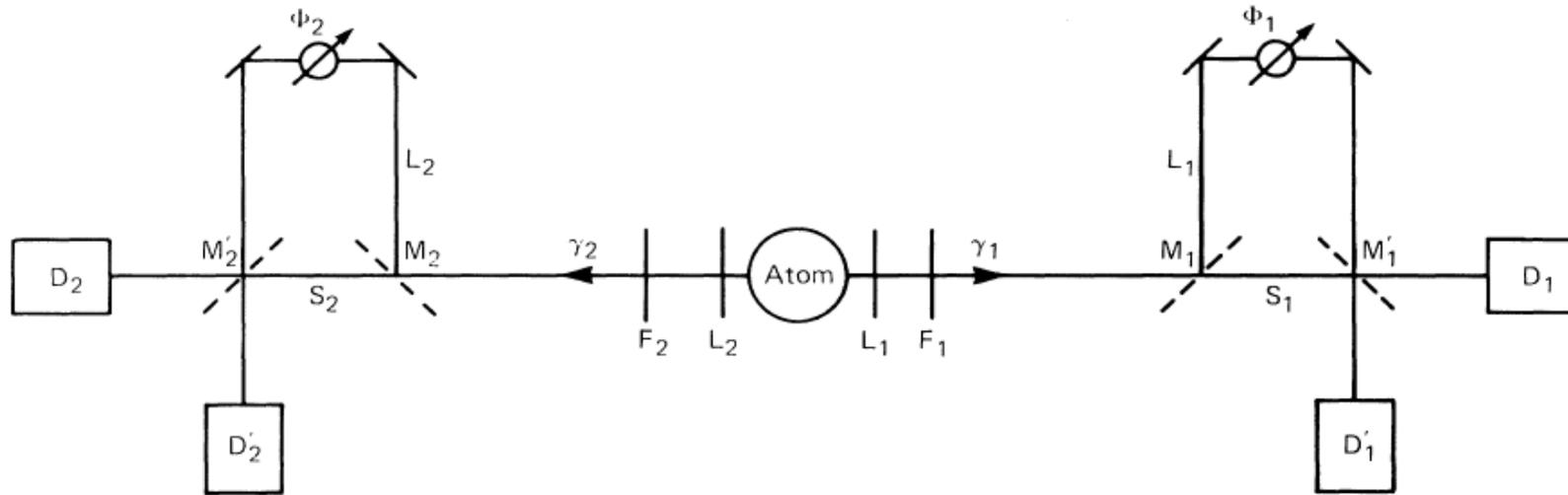
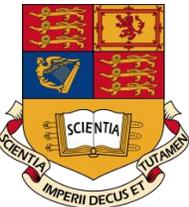


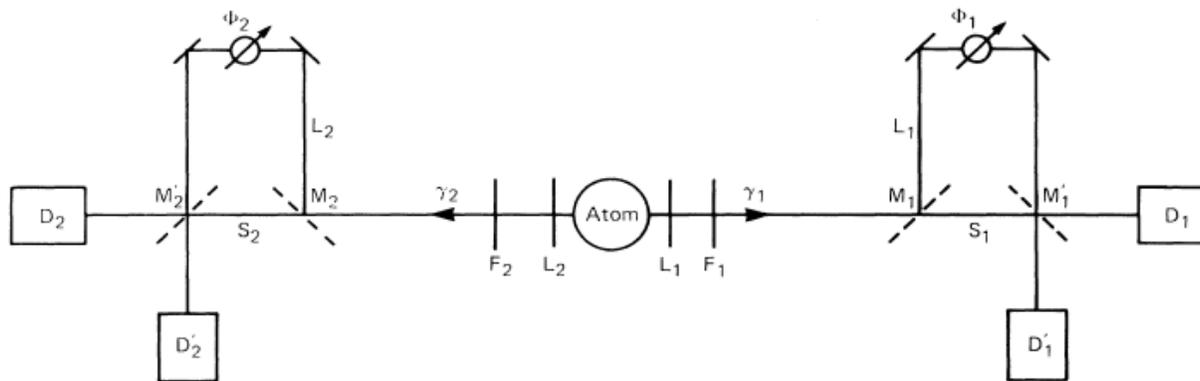
FIG. 2. Photon coincidence measurements including interference between the amplitudes along the shorter paths,  $S_1$  and  $S_2$ , and the longer paths,  $L_1$  and  $L_2$ .

Source emits time-correlated photons

$$\int dt \hat{a}^\dagger(t) \hat{b}^\dagger(t) |00\rangle_{ab}$$



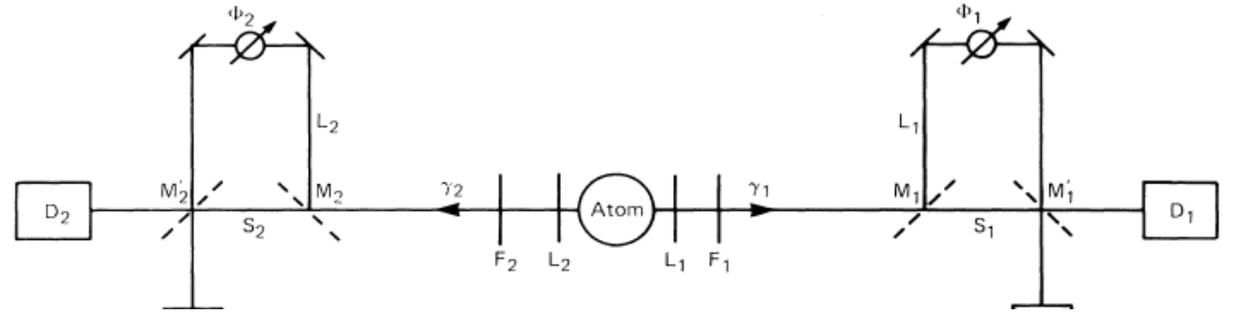
At the output ports



$$\frac{1}{4} \int dt \left[ \hat{a}^\dagger(t) - \hat{c}^\dagger(t) + e^{i\phi_1} (\hat{c}^\dagger(t - \Delta) + \hat{a}^\dagger(t - \Delta)) \right]$$

$$\left[ \hat{b}^\dagger(t) - \hat{d}^\dagger(t) + e^{i\phi_2} (\hat{d}^\dagger(t - \Delta) + \hat{b}^\dagger(t - \Delta)) \right] |0\rangle$$

At the output ports (if there is no



$$\frac{1}{4} \int dt dt' \left[ \hat{a}^\dagger(t) - \hat{c}^\dagger(t) + e^{i\phi_1} (\hat{c}^\dagger(t - \Delta) + \hat{a}^\dagger(t - \Delta)) \right]$$

$$\left[ \hat{b}^\dagger(t') - \hat{d}^\dagger(t') + e^{i\phi_2} (\hat{d}^\dagger(t' - \Delta) + \hat{b}^\dagger(t' - \Delta)) \right] |0\rangle$$

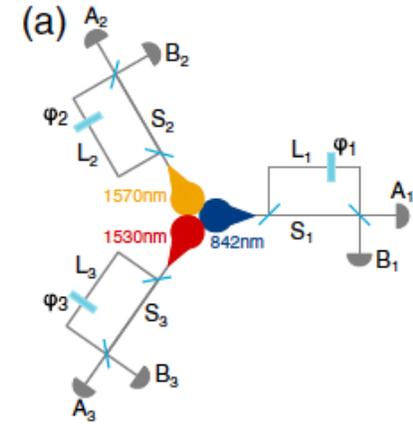
Coincident detection probability at the detectors  $D_1$  and  $D_2$

$$\frac{1}{4} (1 + \cos\phi_1)(1 + \cos\phi_2)$$

$$\frac{1}{2} (1 + \cos(\phi_1 + \phi_2))$$

# Agne *et al* from Innsbruck

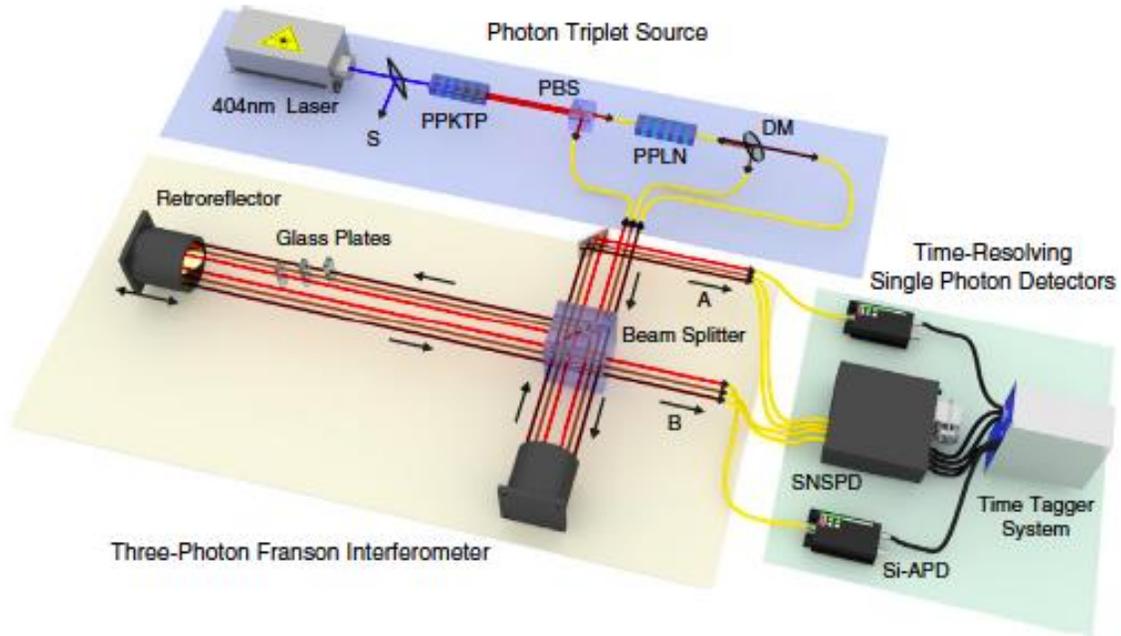
PRL 118, 153602 (2017)



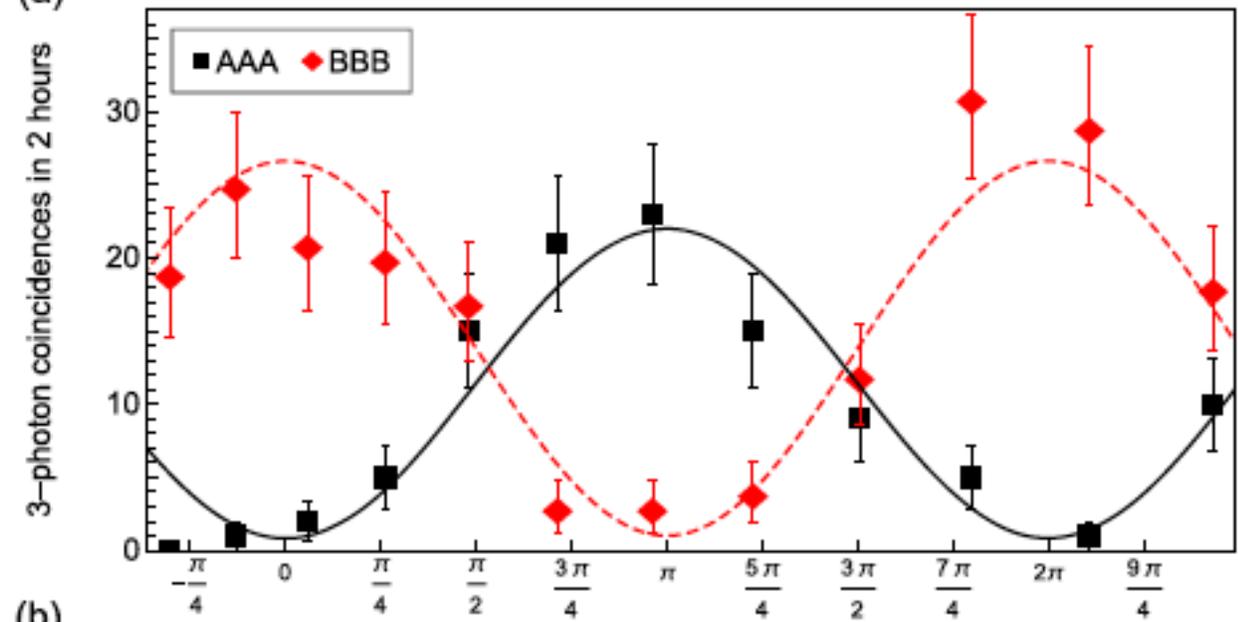
- Extended the Franson interferometer to three-correlations

S. Agne *et al.* PRL 118, 153602 (17)

Red={A1A2A3, A1B2B3, B1A2B3, B1B2A3}



(a)

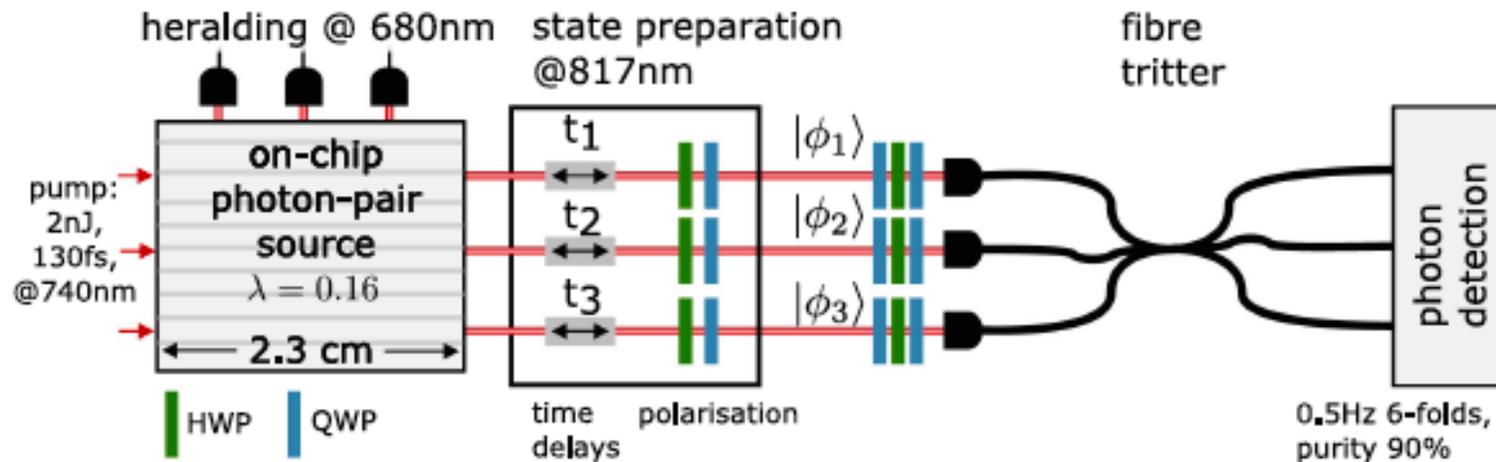


(b)

$$P = \frac{1}{2} [1 \mp \cos(\phi_1 + \phi_2 + \phi_3)]$$

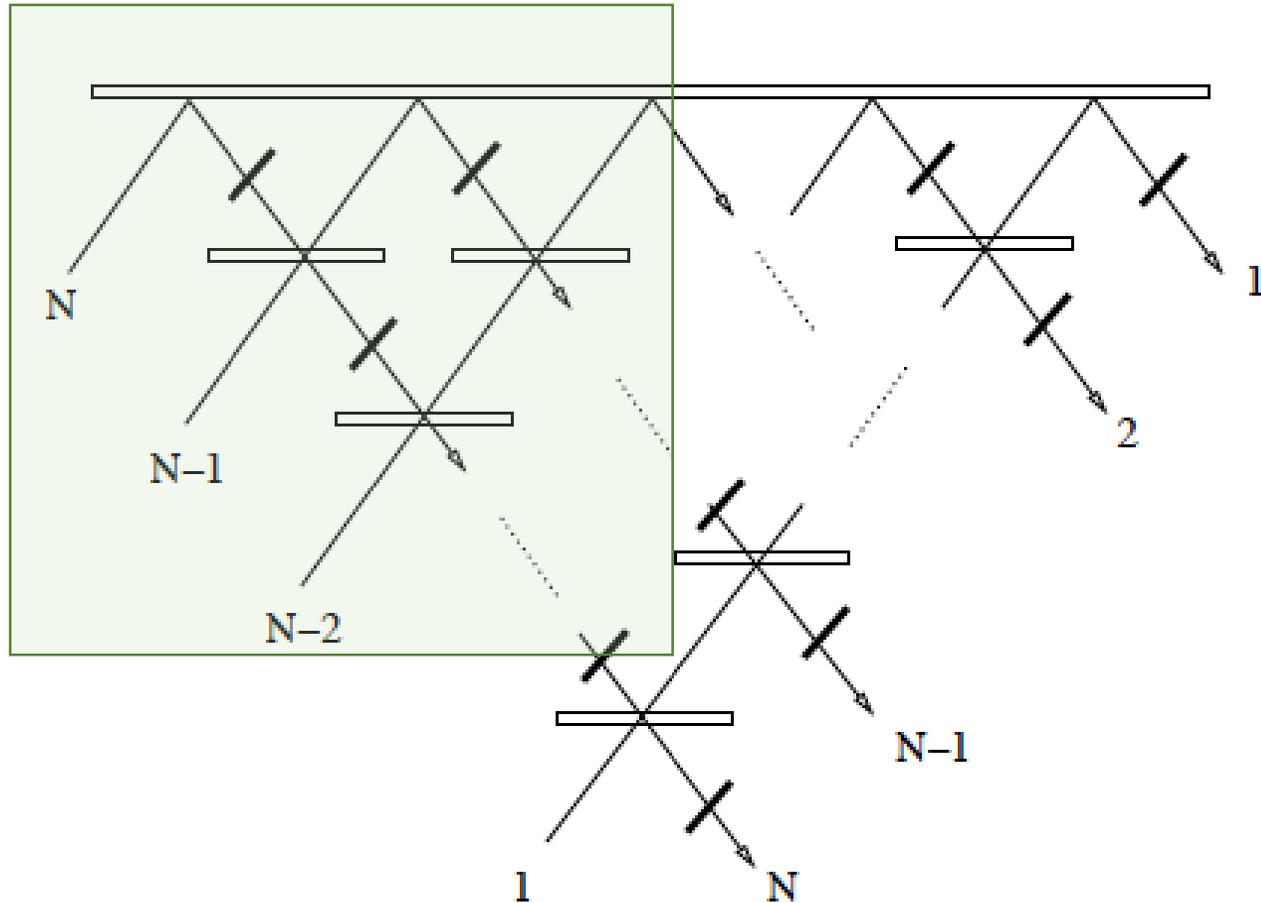
# Experiment 2: Menssen *et al*

PRL 118, 153603 (2017)

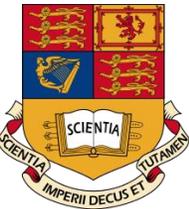


$$\hat{U}_{\text{tritter}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{4\pi i/3} & e^{2\pi i/3} \\ 1 & e^{2\pi i/3} & e^{4\pi i/3} \end{pmatrix}$$

# Decomposition of a beam splitter



Generalised Hong-Ou-Mandel experiments  
with bosons and fermions  
Lim and Beige, New J. Phys 7, 155 (2005)

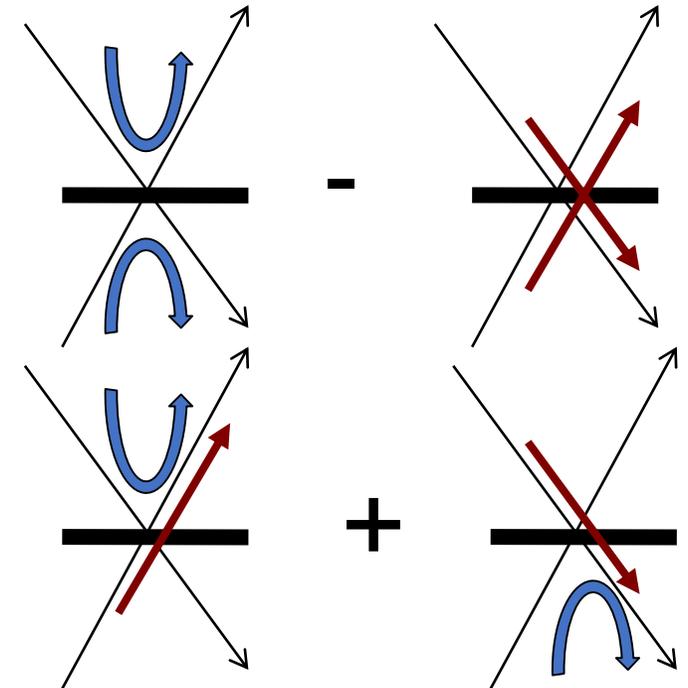
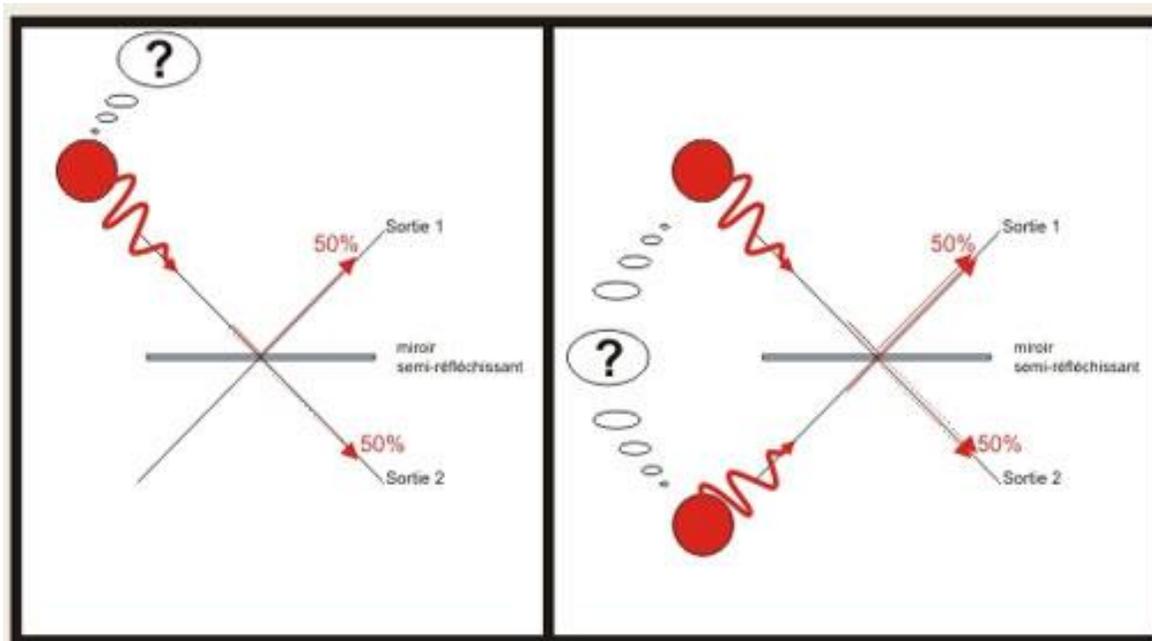


# Nonclassical properties

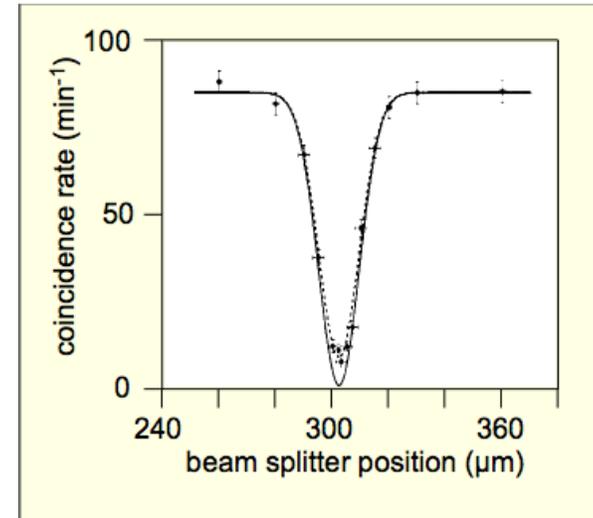
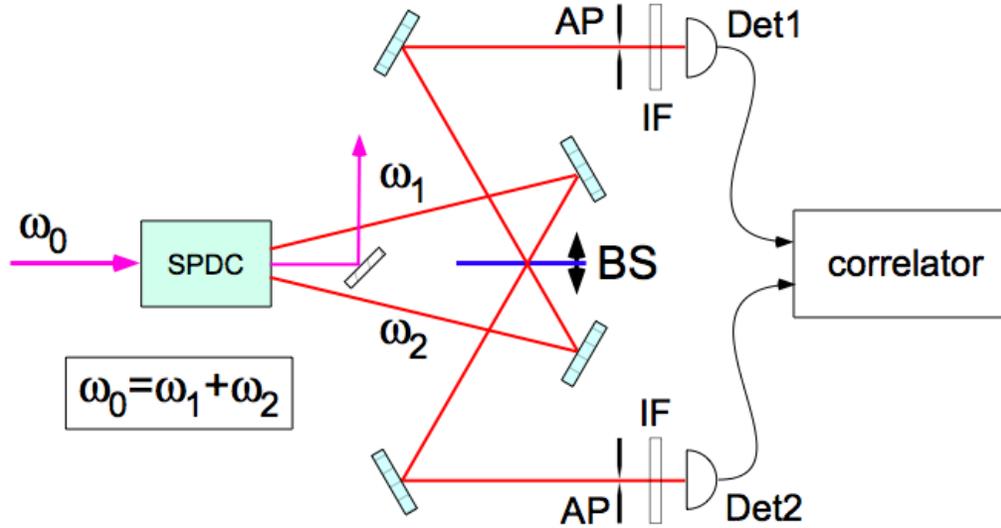
- Q: How do we detect input not classical by measuring output intensities after their interference?
- Let us consider the Hong-Ou-Mandel interferometer



# Two photons do interfere



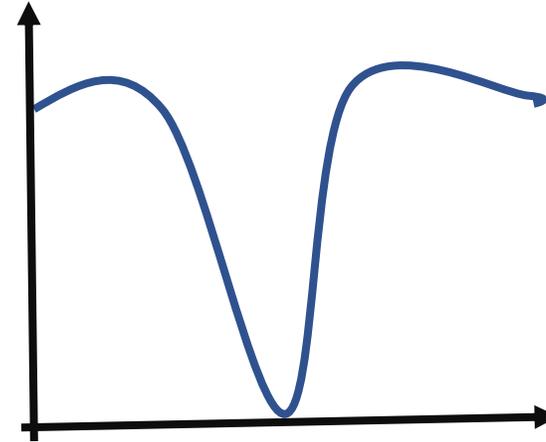
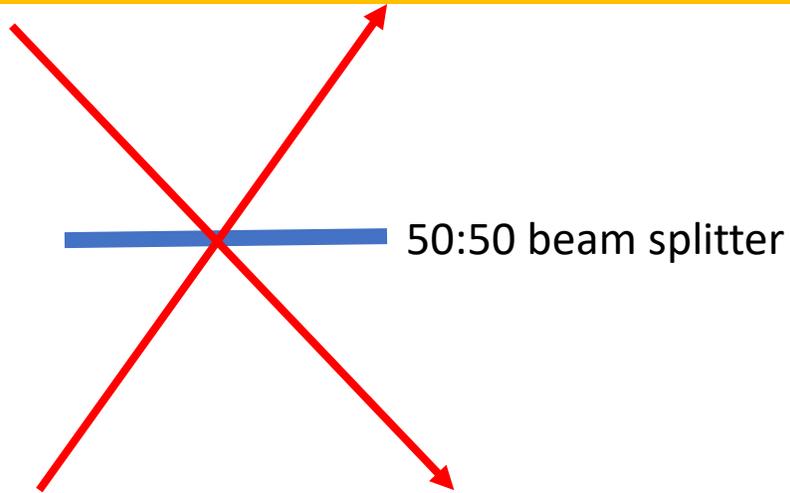
# Hong-Ou-Mandel experiment



Hong et al., PRL

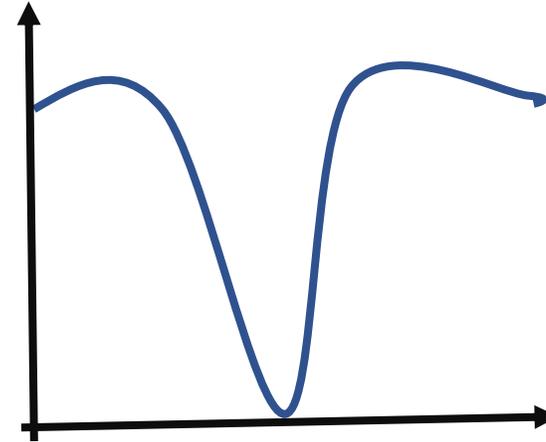
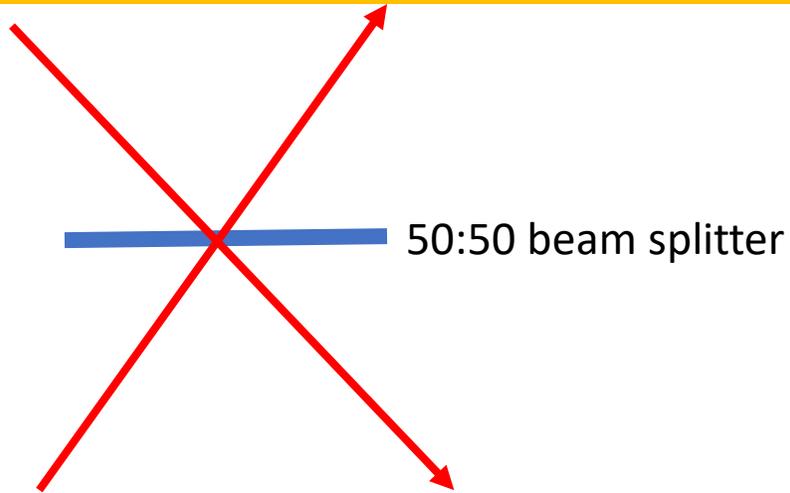
- Zero coincidence detection – Nonclassical effect
- 50% visibility is the classical limit
  - Where is this limit from?

# Hong-Ou-Mandel Interferometer



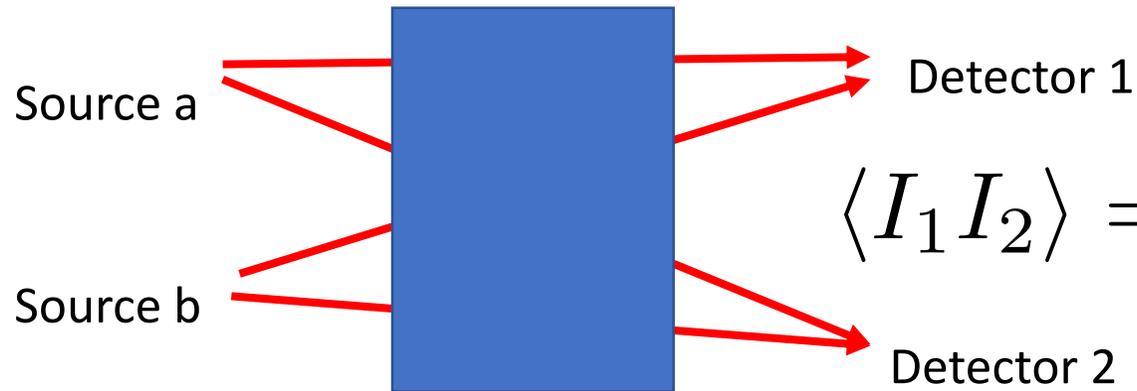
- Two coherent states of the same amplitudes interfere
- One output port will be empty
- Coincident detection rate is zero

# Hong-Ou-Mandel Interferometer



- Assumption 1: Two input fields have random phases  $\langle E \rangle = 0$
- How about each input is from  $\{E, -E\}$  then  $\langle E \rangle = 0$ : Coincident detection rate is zero
- Assumption 2:  $\langle E_1 E_2 \rangle = 0$

# Hanbury Brown and Twiss Interferometer



$$\langle I_1 I_2 \rangle = \langle (I_a + I_b)^2 \rangle + 2 \langle I_a I_b \rangle \cos \phi$$

Visibility

$$\langle : \hat{I}_a^2 : \rangle \rightarrow \langle \hat{n}_a (\hat{n}_a - 1) \rangle$$

$$\frac{2 \langle I_a I_b \rangle}{\langle (I_a + I_b)^2 \rangle} \leq \frac{1}{2}$$

Not subject to Cauchy inequality

~~$$\frac{2 \langle I_a I_b \rangle}{\langle (I_a + I_b)^2 \rangle} \leq \frac{1}{2}$$~~

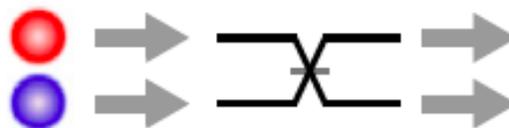
$$\hat{\rho} = \int d^2 \alpha P(\alpha) |\alpha\rangle \langle \alpha| \rightarrow \langle I^n \rangle = \langle \hat{I}^n \rangle$$

in QM



# partially identical photons

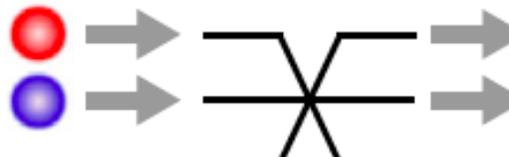
(a)  $\langle \text{blue} | \text{red} \rangle = \text{overlap} e^{i\varphi_{\text{br}}}$

(b)   $P_{11} = \frac{1}{2} (1 - \text{overlap}^2)$

(c)   $P_{111} = \frac{1}{9} [2 + 4 \text{overlap}_{\text{br}} \text{overlap}_{\text{rg}} \text{overlap}_{\text{gb}} \cos(\varphi) - \text{overlap}_{\text{br}}^2 - \text{overlap}_{\text{rg}}^2 - \text{overlap}_{\text{gb}}^2]$

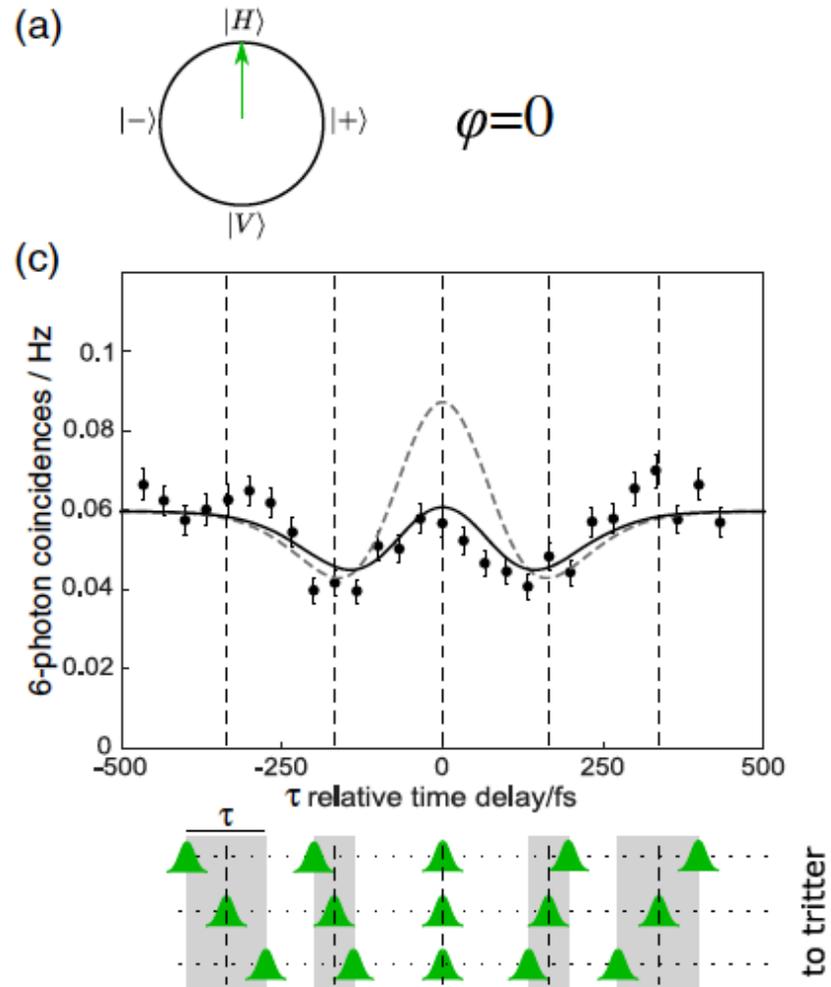
Overlap phase information becomes relevant

(d)  $\varphi = \varphi_{\text{br}} + \varphi_{\text{rg}} + \varphi_{\text{gb}}$

(e)   $P_{110} = \frac{1}{9} (2 - \text{overlap}^2)$

# Three photon coincidences

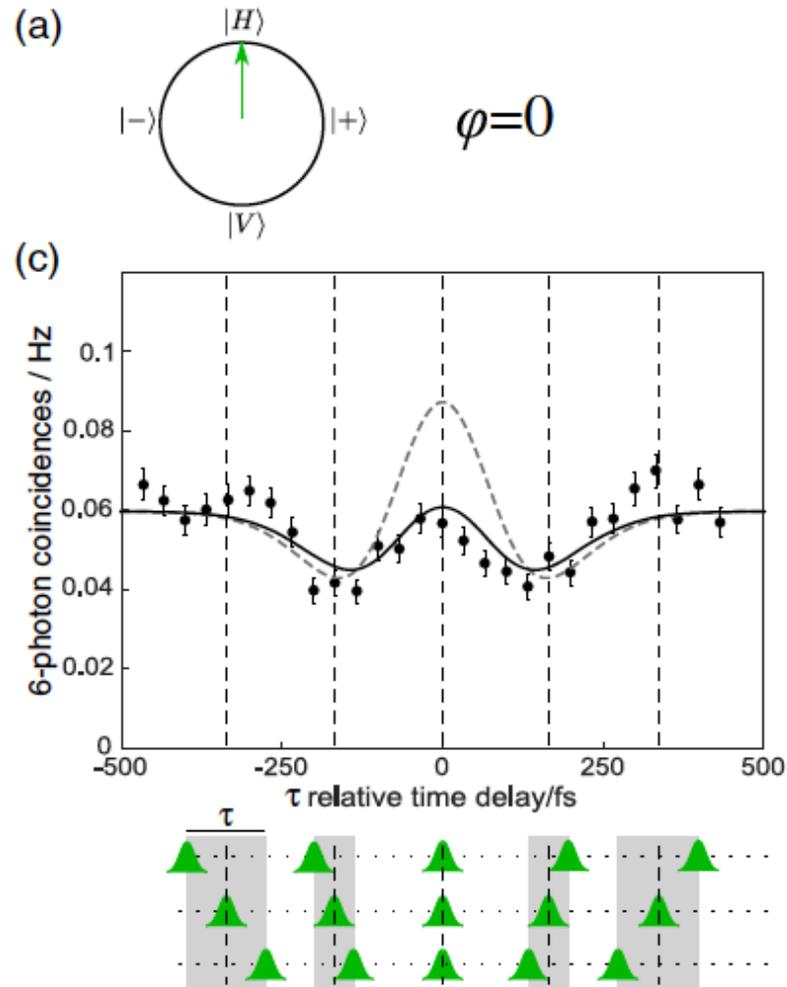
$$|\phi_1\rangle = |t_1\rangle \otimes |H\rangle$$



No bunching effects

$$|\phi_1\rangle = |t_1\rangle \otimes |H\rangle$$

# Three photon coincidences



No bunching effects

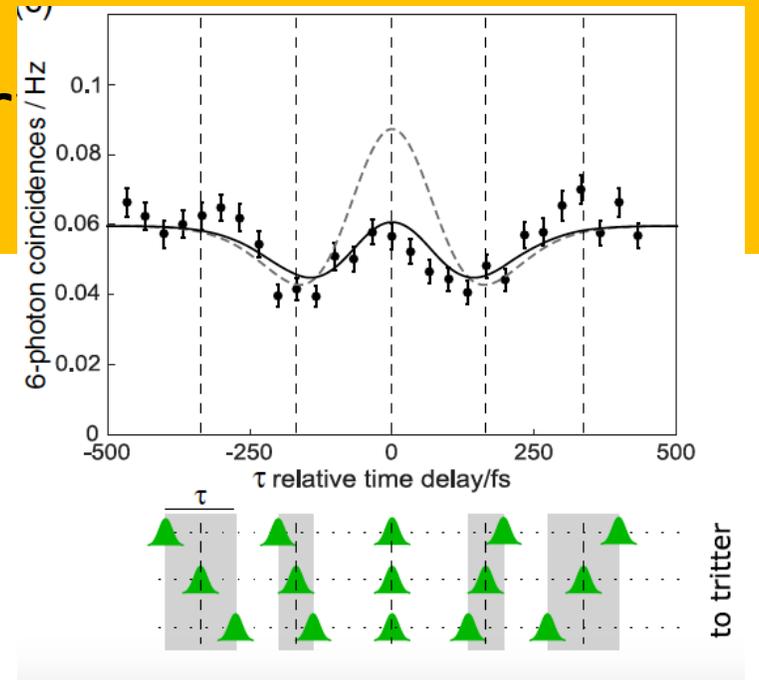
# Classical theory

$$g_C^{(3)} = \frac{\langle I_1 I_2 I_3 \rangle}{\langle I_1 \rangle \langle I_2 \rangle \langle I_3 \rangle}$$

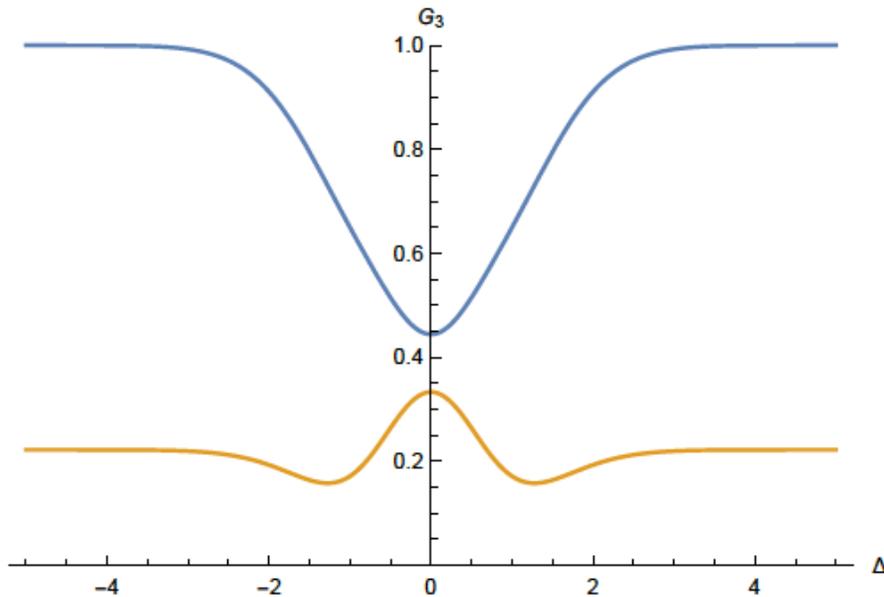
- Third-order correlation function

$$g_C^{(3)} = 1 + \frac{4}{9} r_{12} r_{23} r_{31} \cos \varphi - \frac{3}{9} (r_{12}^2 + r_{23}^2 + r_{31}^2)$$

$$g_Q^{(3)} = \frac{2}{9} + \frac{4}{9} r_{12} r_{23} r_{31} \cos \varphi - \frac{1}{9} (r_{12}^2 + r_{23}^2 + r_{31}^2)$$



# Q vs C

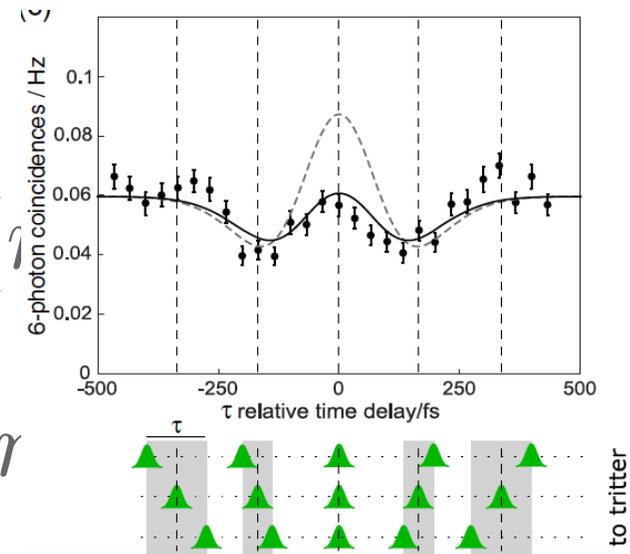


Revival of high coincidence counting is a quantum signature

Valid for input fields having identical intensities

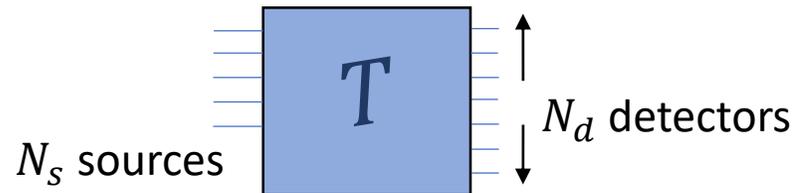
$$g_Q^{(3)} = \frac{2}{9} + \frac{4}{9} r_{12} r_{23} r_{31} \cos \varphi - \frac{1}{9} \left( \dots \right)$$

$$g_C^{(3)} = 1 + \frac{4}{9} r_{12} r_{23} r_{31} \cos \varphi - \frac{3}{9} \left( \dots \right)$$



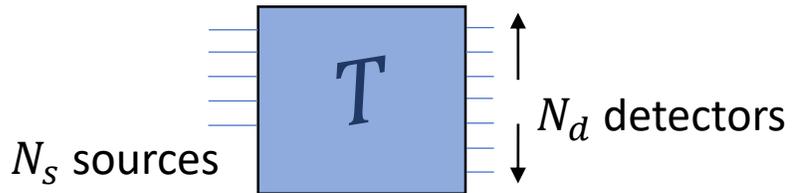
# Multimode Extension

Hypotheses: Independent sources emit light pulses with random phases



# Multimode Extension

Hypotheses: Independent sources emit light pulses with random phases



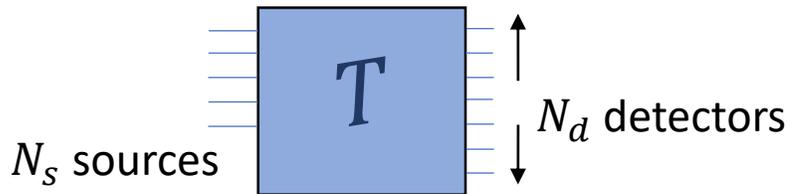
$$\bar{G}^{(cl)} = \frac{1}{\binom{M}{2}} \sum_{i < j}^M \frac{\langle I_i I_j \rangle}{\langle I_i \rangle \langle I_j \rangle} \quad \text{Average correlations among all output pairs}$$

$$\mathbf{E}_\alpha(t) = \sum_\lambda \int d\omega \mathbf{g}_{\omega,\lambda} e^{-i\omega t} \epsilon_{\omega,\lambda}$$

$$\mathbf{O}_i(t) = \sum_{\alpha=1}^N T_{i\alpha} \mathbf{E}_\alpha(t - \tau_{i\alpha})$$

# Multimode Extension

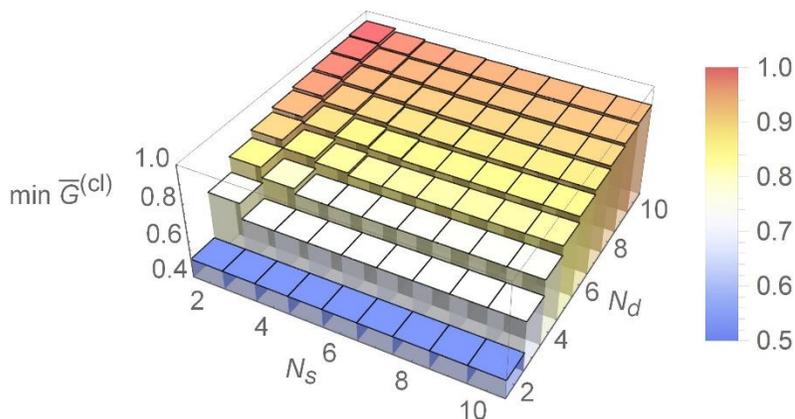
**Hypotheses:** Independent sources emit light pulses with random phases



$$\bar{G}^{(cl)} = \frac{1}{\binom{M}{2}} \sum_{i < j}^M \frac{\langle I_i I_j \rangle}{\langle I_i \rangle \langle I_j \rangle}$$

Average correlations among all output pairs

$\bar{G}^{(cl)}$  has a minimum for every  $N_d, N_s$ !

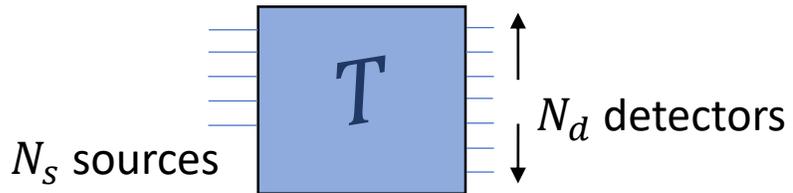


$$\min \bar{G}_{N,M}^{(cl)} = 1 - \frac{N - 1}{N(M - 1)} \text{ if } N \leq M$$

$$= 1 - \frac{1}{M} \text{ if } N \geq M$$

# Multimode Extension

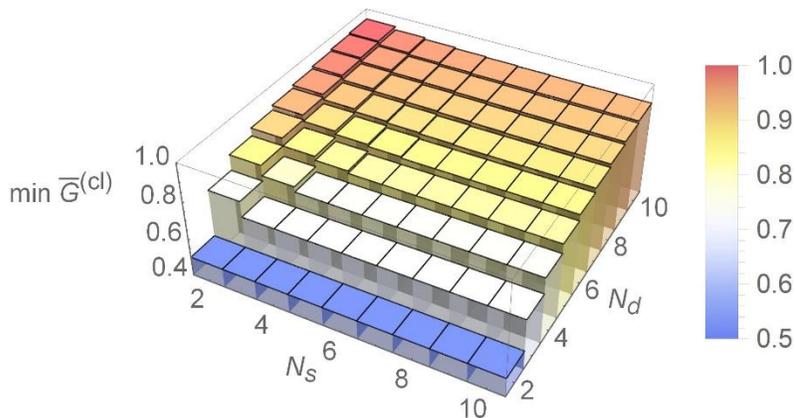
**Hypotheses:** Independent sources emit light pulses with random phases



$$\bar{G}^{(cl)} = \frac{1}{\binom{M}{2}} \sum_{i < j}^M \frac{\langle I_i I_j \rangle}{\langle I_i \rangle \langle I_j \rangle}$$

Average correlations among all output pairs

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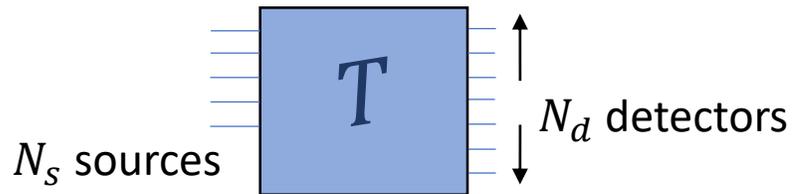


$$\min \bar{G}^{(Q)} = 1 - \frac{1 + \eta}{M} \leq \min \bar{G}^{(cl)}$$

$$0 \leq \eta = - \frac{(\Delta n)^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2} \leq 1$$

# Multimode Extension

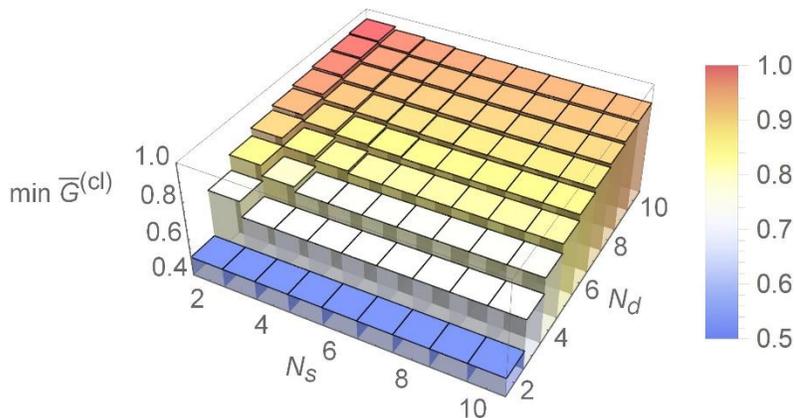
**Hypotheses:** Independent sources emit light pulses with random phases



$$\bar{G}^{(cl)} = \frac{1}{\binom{M}{2}} \sum_{i < j}^M \frac{\langle I_i I_j \rangle}{\langle I_i \rangle \langle I_j \rangle}$$

Average correlations among all output pairs

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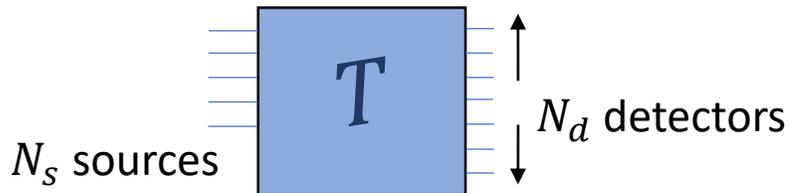


What happens with quantum input states?

- ✓ Classical threshold can be violated
- ✓ Necessity of sub-Poissonian light
- ✓ Optimality of single photons

# Multimode Extension

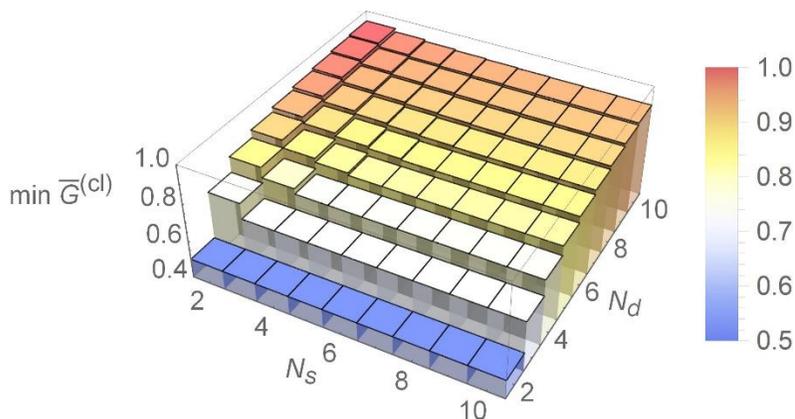
Hypotheses: Independent sources emit light pulses with random phases



$$\bar{G}^{(cl)} = \frac{1}{\binom{M}{2}} \sum_{i < j}^M \frac{\langle I_i I_j \rangle}{\langle I_i \rangle \langle I_j \rangle}$$

Average correlations among all output pairs

$\bar{G}^{(cl)}$  has a minimum for every  $N_d, N_s$ !



What happens with quantum input states?

- ✓ Classical threshold can be violated
- ✓ Necessity of sub-Poissonian light
- ✓ Optimality of single photons
- For identical input states, the gap shrinks with system dimensionality

# Remarks

**Problem**: detecting signatures of nonclassicality in a multiport linear optical interferometer

- Generalization of Hong-Ou-Mandel result
- Usefulness of low-order correlation functions
- Lower bound for classical intensity correlations
- Threshold violated with quantum states of light

Moreover:

Possibility of deducing information on sources/interferometer by the third-order correlation function

Thank you!

Rigovacca *et al.*, arXiv 1712.07259 (2017)

Rigovacca *et al.*, PRL, **117**, 213602 (2016)

# Third-order correlation function

$$g_Q^{(3)} = \frac{\langle \hat{I}_1 \hat{I}_2 \hat{I}_3 \rangle}{\langle \hat{I}_1 \rangle \langle \hat{I}_2 \rangle \langle \hat{I}_3 \rangle}$$

$$g_Q^{(3)} = \frac{2}{9} + \frac{4}{9} r_{12} r_{23} r_{31} \cos \varphi - \frac{1}{9} (r_{12}^2 + r_{23}^2 + r_{31}^2)$$

$$\langle \phi_\alpha | \phi_\beta \rangle = r_{\alpha\beta} e^{i\varphi_{\alpha\beta}}$$

