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MIQC2 OPTICAL METROLOGY FOR QUANTUM-ENHANCED SECURE TELECOMMUNICATION

Violation of a Leggett-Garg inequality exploiting anomalous weak values

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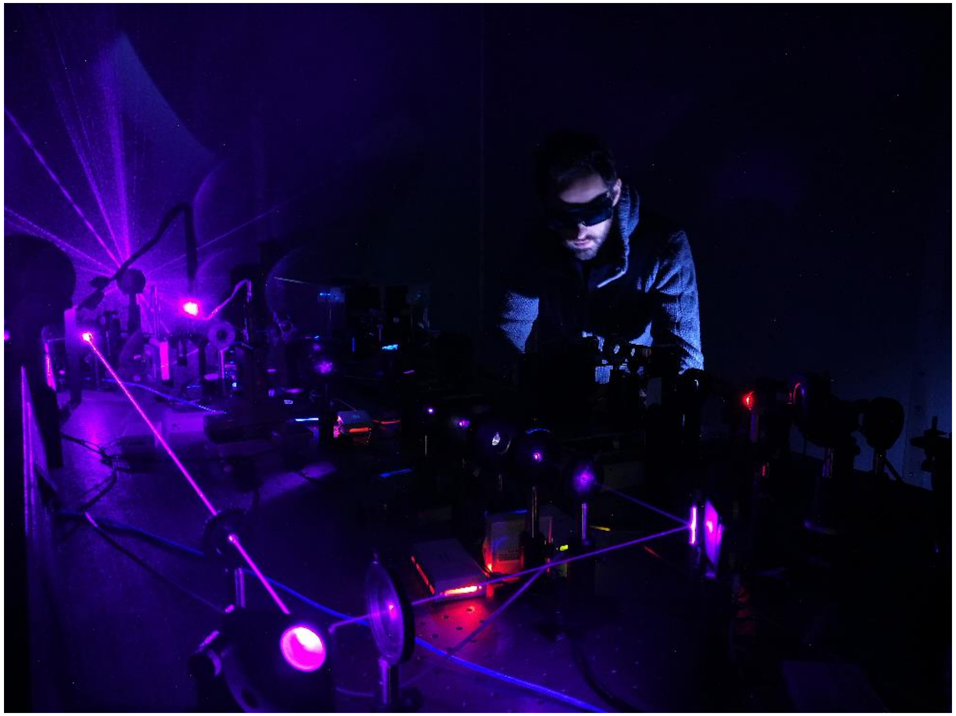


International Symposium on New Frontiers in Quantum Correlations (ISNFQC18)
A tribute to Professor Satyendra Nath Bose on his 125th birth anniversary
January 29th to February 2nd, 2018
S. N. Bose National Centre for Basic Sciences, Kolkata, India

Introduction

Leggett and Garg inequality test

A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985).

Behavior of macroscopic systems when subject to subsequent measurements.
For these objects, it is natural to assume that they will be found in a definite, realistic macrostate (macroscopic realism) and that a measurement, especially when carried out by a microscopic probe, cannot perturb such a macrostate (non-invasive measurability).

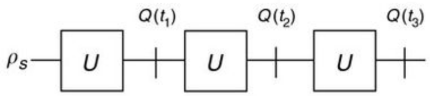
The original observation by Leggett and Garg has led to a fecund production of theoretical and experimental work focusing on the inadequacy of such a macrorealistic view;

A comprehensive review of recent results.

C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. **77**, 016001 (2014)

Leggett and Garg inequality test

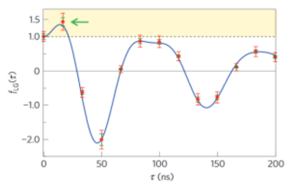
In its simplest form, Leggett and Garg's arrangement considers a macroscopic body undergoing three two-outcome measurements at different times, with the first serving as a preparation.



The correlation among the outcomes can be shown not to be in accordance with macrorealistic prescriptions.

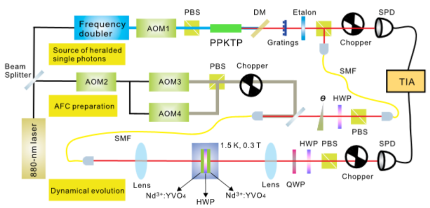
To date, the violation of the Leggett-Garg inequality has been reported on macroscopic objects:

Transmon qubits [15]



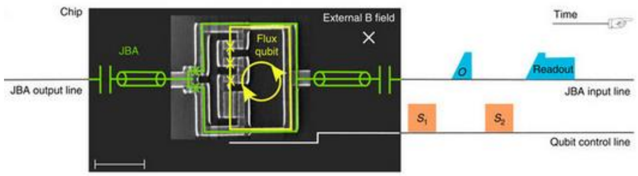
A. Palacios-Laloy et al.
Nat. Phys. **6**, 442 (2010)

Crystals [21]



Z.-Q. Zhou et al.
Phys. Rev. Lett. 115, 113002 (2015)

Superconducting flux qubits [22].

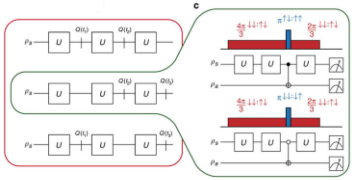


G. C. Knee et al.
Nat. Commun. 7, 13253 (2016).

The testis also suited to highlight the inadequacy of a realistic view to the description of simpler quantum objects:

Phosphorus impurities [19]

G. C. Knee et al.
Nat. Commun. **3**, 606 (2012).



In this case, the focus is on the assessment of the quantum character of the system, being a very significant tool for quantum technologies, rather than on its fundamental value.

Leggett and Garg inequality test

The canonic three-measurement arrangement can be generalized in several directions:

The simplest extension considers longer sequences. Increasing the number of measurements leads to stronger departures from the macrorealistic predictions,

Tested with photons:

J.-S. Tang et al., Chin. Phys. Lett. 28, 060304 (2011)

and nuclear spins:

J.-S. Tang et al., Chin. Phys. Lett. 28, 060304 (2011)

A different take considers substituting the measurement in the middle with a **weak measurement** imparting limited back-action:

Y. Aharonov and L. Vaidman, Phys. Rev. A 41, 11 (1990)

Leggett and Garg inequality test

Shot by shot the measurement delivers only partial information on the observable, it still provides the correct expectation value on a large ensemble.

This concept has been introduced

Leggett-Garg Inequality with a Kicked Quantum Pump
A. N. Jordan, A. N. Korotkov, and M. Buttiker, Phys. Rev. Lett. 97, 026805 (2006).

Weak Values and the Leggett-Garg Inequality in Solid-State Qubits
N. S. Williams and A. N. Jordan, Phys. Rev. Lett. 100, 026804 (2008).

has been tested on single photons

Violation of the Leggett–Garg inequality with weak measurements of photons
M. E. Goggin, et al., Proc. Natl. Acad. Sci. USA 108, 1256 (2011).

and on transmons in

Partial-Measurement Backaction and Nonclassical Weak Values in a Superconducting Circuit
J. P. Groen et al., Phys. Rev. Lett. 111, 090506 (2013).

Our goals

We present an experiment encompassing these two generalizations at the same time, by demonstrating a sequential multiple-measurement setting carried out in the weak regime.

We perform a LGT on the polarization of single photons,
estimating non-commuting observables via “weakaverages”,
and draw an explicit link to the emergence of anomalous values.

Our experiment confirms the intimate connection between the observation of anomalies in the postselected statistics of quantum measurement as a genuine manifestation of the quantum character of the observed object.

These anomalies can then be used as practical witnesses of non-classicality for application in quantum technology:

Random number generators

Quantum Key Distribution

Weak Measurements

Weak measurements [Aharonov et al., PRL 60 (1988)]:
little information is extracted from a single measurement, but the state does NOT collapse.

Weak value: $\langle \widehat{A} \rangle_w = \frac{\langle \psi_f | \widehat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$

Pre-selected state: $|\psi_i\rangle$

Post-selected state: $|\psi_f\rangle$

Von Newman coupling between an observable $\widehat{A}$ and a pointer observable: $\widehat{P}$

$$\widehat{U} = \exp(-ig\widehat{A} \otimes \widehat{P})$$

Projective measurement (post-selection on $|\psi_f\rangle$)

$$\widehat{\Pi}_f = |\psi_f\rangle\langle\psi_f|$$

$$|\phi_{out}\rangle = \widehat{\Pi}_f \widehat{U} |\phi_{in}\rangle = \widehat{\Pi}_f \widehat{U} |\psi_i\rangle \otimes |f_i\rangle$$

$\widehat{X}$ and $\widehat{P}$ canonically conjugated

In the weak interaction regime approximation:

$$\langle \widehat{X} \rangle = \frac{\langle \phi_{out} | \widehat{X} | \phi_{out} \rangle}{\langle \psi_i | \widehat{\Pi}_f | \psi_i \rangle} = g \ \mathrm{Re}[\langle \widehat{A} \rangle_w]$$

Weak Measurements

$$\langle \widehat{A} \rangle_w = \frac{\langle \psi_f | \widehat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

$$\widehat{U} = \exp(-ig\widehat{A} \otimes \widehat{P})$$

$$\widehat{\Pi}_f = |\psi_f\rangle\langle\psi_f|$$

Some interesting properties:

- $\langle \widehat{A} \rangle_w$ is a complex number
- $\mathrm{Re}[\langle \widehat{A} \rangle_w]$ is unbounded!

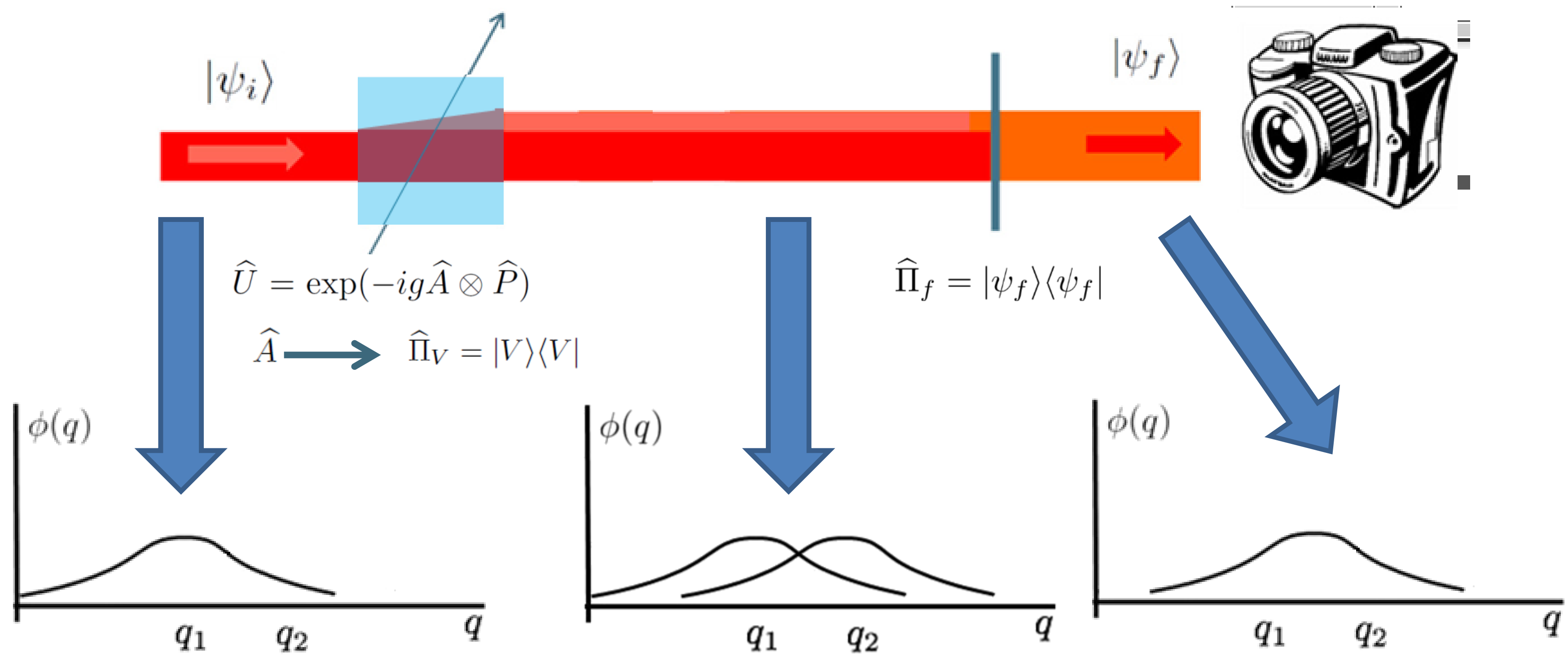
❑ Metrology:

- Amplification of measurement of coupling strength, without amplifying unrelated noise [Boyd et al.]:
  - Light beam displacement [Kwiat et al.]
  - Angular deflection [Dixon et al.]

❑ Foundations of Quantum Mechanics:

- Measurement of incompatible observables on the same particle [Mitchinson et al.]
- Tests of Quantum Contextuality [Pusey]
- Hints on Quantum Mechanics interpretations [TSVF, Aharonov et al., …]

Weak Measurements



We measure the position observable $\widehat{X}$, canonically coniugated to the pointer observable $\widehat{P}$

$$\langle\widehat{X}\rangle = g\mathrm{Re}[\langle\widehat{\Pi}_V\rangle_w]$$

Sequential Weak Measurements

«challenge one of the canonical dicta of QM: that non commuting observables cannot be simultaneously measured»
«the fact that one hardly disturbs the systems in making WM means that one can in principle measure different variables in succession»
[Mitchison, Jozsa and Popescu, PRA 76 (2007)]

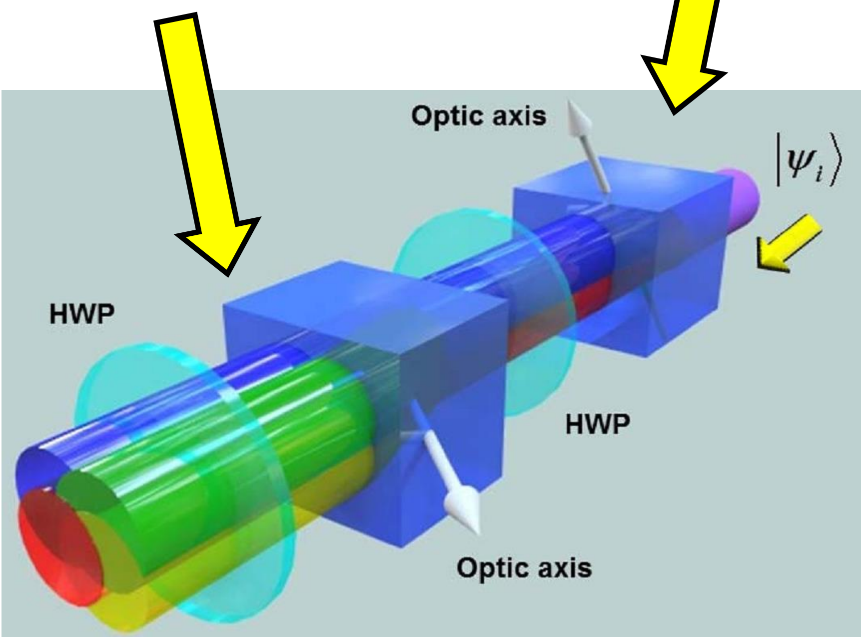
$$\widehat{A} \longrightarrow \widehat{\Pi}_V = |V\rangle\langle V|$$

$$\widehat{B} \longrightarrow \widehat{\Pi}_\psi = |\psi\rangle\langle\psi|$$

$$|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$$

$$\widehat{U}_y = e^{-ig_y \widehat{\Pi}_V \otimes \widehat{P}_y}$$

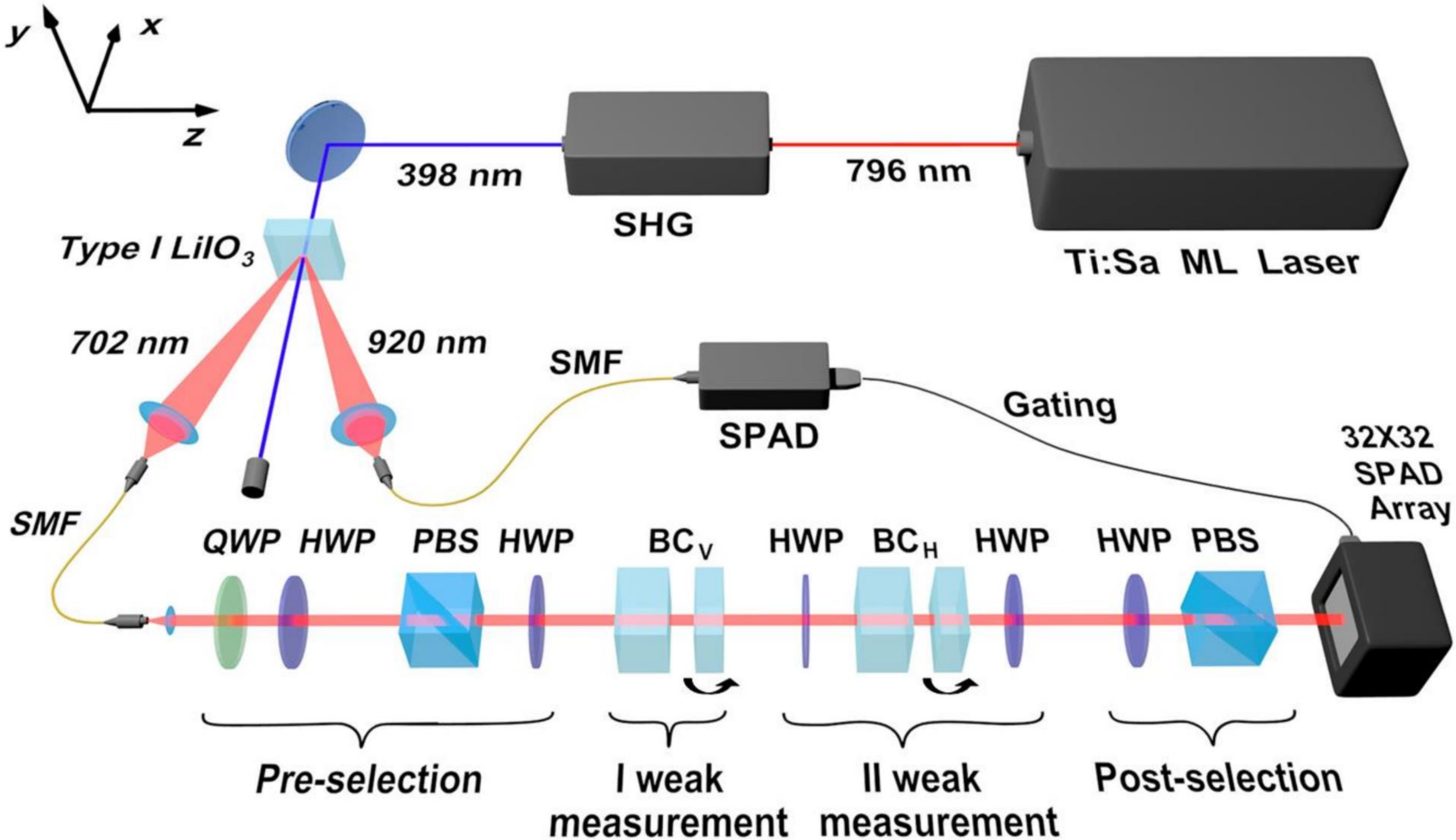
$$\widehat{U}_x = e^{-ig_x \widehat{\Pi}_\psi \otimes \widehat{P}_x}$$



Linearly polarized pre- and post-selection states $|\psi_i\rangle, \ |\psi_f\rangle$

$$\begin{cases} \langle\widehat{X}\rangle = g_x \langle\widehat{\Pi}_\psi\rangle_w \\ \langle\widehat{Y}\rangle = g_y \langle\widehat{\Pi}_V\rangle_w \\ \langle\widehat{X}\widehat{Y}\rangle = \frac{1}{2} g_x g_y \left(\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w + \langle\widehat{\Pi}_\psi\rangle_w \langle\widehat{\Pi}_V\rangle_w\right) \end{cases}$$

Sequential Weak Measurements



SPAD lab

32x32 SPAD+TDC camera

Features

- Multi-modality: photon-counting, 2D imaging 3D time-of-flight ranging, TCSPC (time-correlated single-photon counting)
- Image dimension: 32x32 (1024) pixels
- In-pixel counter: 6 bit (photon-counting)
- In-pixel TDC: 10 bit (photon-timing)
- Max frame rate: 100,000 fps (burst) and 10,000 fps (continuous)
- Timing resolution: 312 ps – 0.9 ns
- Full scale range: 320 ns – 0.92 μs
- Hardware interface: USB 2.0
- Software interface: Matlab

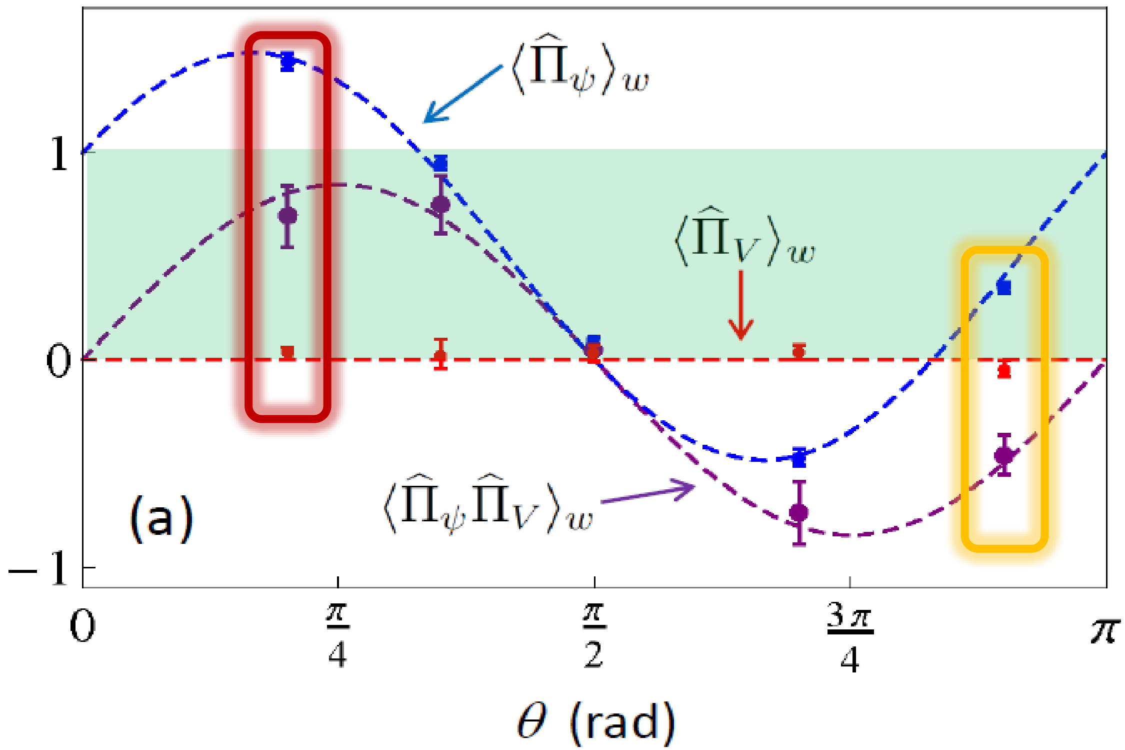
Fig. 1: SPAD camera for 2D imaging, 3D ranging and TCSPC photon-counting.

Sequential Weak Measurements

Measured weak values (data points) compared with the theoretical predictions

$$\widehat{\Pi}_V = |V\rangle\langle V| \qquad \widehat{\Pi}_\psi = |\psi\rangle\langle\psi| \qquad (|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

$$|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle \qquad |\psi_f\rangle = |H\rangle$$



$$\langle\widehat{\Pi}_V\rangle_w = 0.03(3)$$
$$\langle\widehat{\Pi}_\psi\rangle_w = 1.44(4)$$
$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = 0.69(15)$$

$$\langle\widehat{\Pi}_V\rangle_w = 0.04(3)$$
$$\langle\widehat{\Pi}_\psi\rangle_w = 0.35(4)$$
$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = -0.46(10)$$

"Measuring Incompatible Observables by Exploiting Sequential Weak Values" **PRL 117, 170402 (2016)**

Sequential Weak Measurements

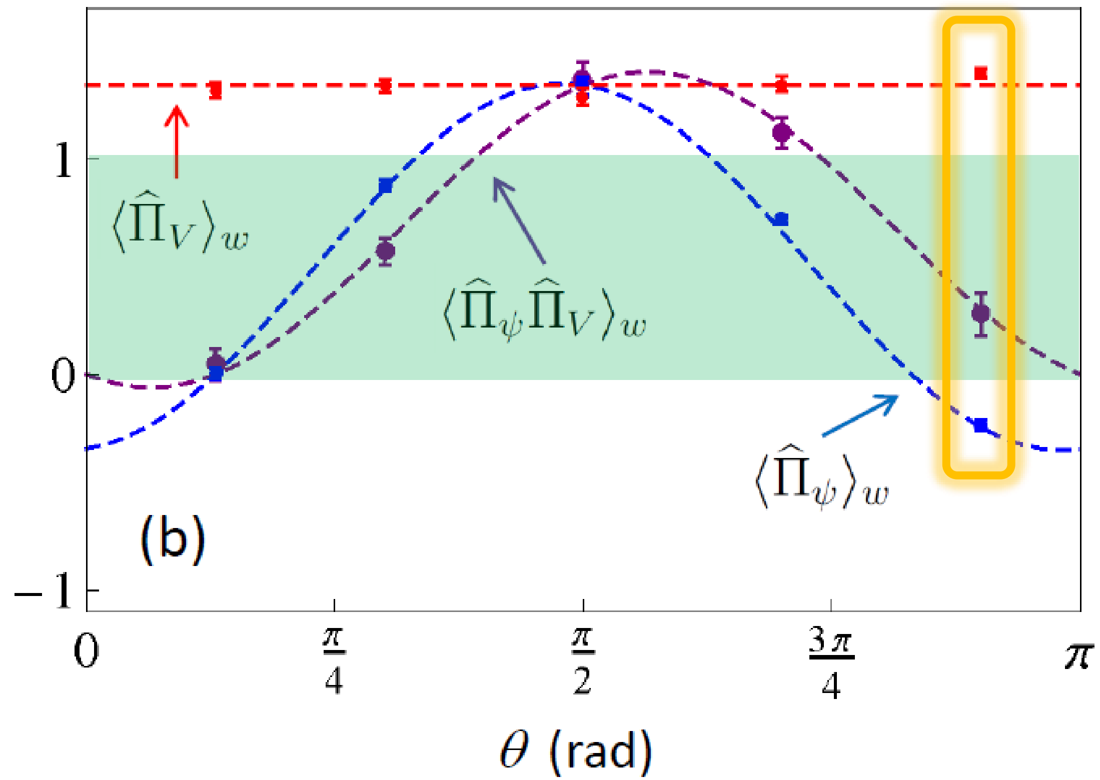
Measured weak values (data points) compared with the theoretical predictions

$$\widehat{\Pi}_V = |V\rangle\langle V| \qquad \widehat{\Pi}_\psi = |\psi\rangle\langle\psi| \quad (|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

$$|\psi_i\rangle = 0.509|H\rangle + 0.861|V\rangle$$

$$|\psi_f\rangle = -0.397|H\rangle + 0.918|V\rangle$$

$$\langle\widehat{\Pi}_V\rangle_w = 1.40(4)$$
$$\langle\widehat{\Pi}_\psi\rangle_w = -0.24(3)$$
$$\langle\widehat{\Pi}_\psi\widehat{\Pi}_V\rangle_w = 0.28(10)$$



"Measuring Incompatible Observables by Exploiting Sequential Weak Values" **PRL 117, 170402 (2016)**

Theoretical background

The simplest LGT one can design involves three measurements, which we label as I_A, I_B, and I_C; these are two-outcome observables which can take either the value $+1$ or the value -1.

The inequality can be written as follows:

$$-3 \leqslant \mathcal{B}_3 = \langle I_A I_B \rangle + \langle I_B I_C \rangle - \langle I_A I_C \rangle \leqslant 1$$

The measurement of I_A can be taken to coincide with the initial preparation in the state $|\psi_A\rangle$; hence one can assign the fixed value $+1$ for I_A:

$$-3 \leqslant \mathcal{B}_3 = \langle I_B \rangle + \langle I_B I_C \rangle - \langle I_C \rangle \leqslant 1$$

The connection with anomalous postselected values of I_B is established by considering the two instances $I_C = 1$ and $I_C = -1$ separately, each with the respective occurrence probabilities $p_C(1)$ and $p_C(-1)$:

$$\mathcal{B}_3 = \langle I_B \rangle + [{}_{+}\langle I_B \rangle - 1] p_C(1) - [{}_{-}\langle I_B \rangle - 1] p_C(-1)$$

Theoretical background

$$\mathcal{B}_3 = \langle I_B \rangle + [{}_+\langle I_B \rangle - 1] p_C(1) - [{}_-\langle I_B \rangle - 1] p_C(-1)$$

$$\boxed{\langle I_B \rangle = {}_+\langle I_B \rangle p_C(1) + {}_-\langle I_B \rangle p_C(-1)}$$

$$\mathcal{B}_3 = 1 + 2 p_C(1)({}_+\langle I_B \rangle - 1)$$

This connection can be extended to the multiple measurement LGT that considers four measurements, including state preparation I_A:

$$|\mathcal{B}_4| = |\langle I_B \rangle + \langle I_B I_C \rangle + \langle I_C I_D \rangle - \langle I_D \rangle| \leqslant 2$$

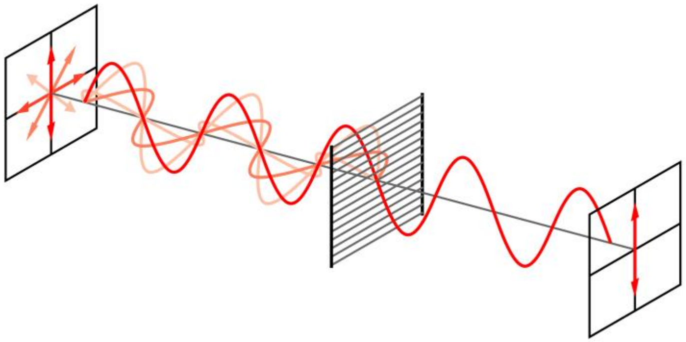
$$|\mathcal{B}_4| = |\langle I_B \rangle + \langle I_B I_C \rangle + p_D(1)[{}_+\langle I_C \rangle - 1] - p_D(-1)[{}_-\langle I_C \rangle - 1]|$$

In our case, violation of the bound for demands a minimal value:

$${}_-\langle I_C \rangle \geqslant \frac{3 - M}{2 p_D(-1)} \qquad M = \langle I_B \rangle + \langle I_B I_C \rangle + \langle I_C \rangle$$

Leggett and Garg inequality test based on weak values

$$|\mathcal{B}_4| = |\langle I_A I_B \rangle + \langle I_B I_C \rangle + \langle I_C I_D \rangle - \langle I_A I_D \rangle| \leq 2$$



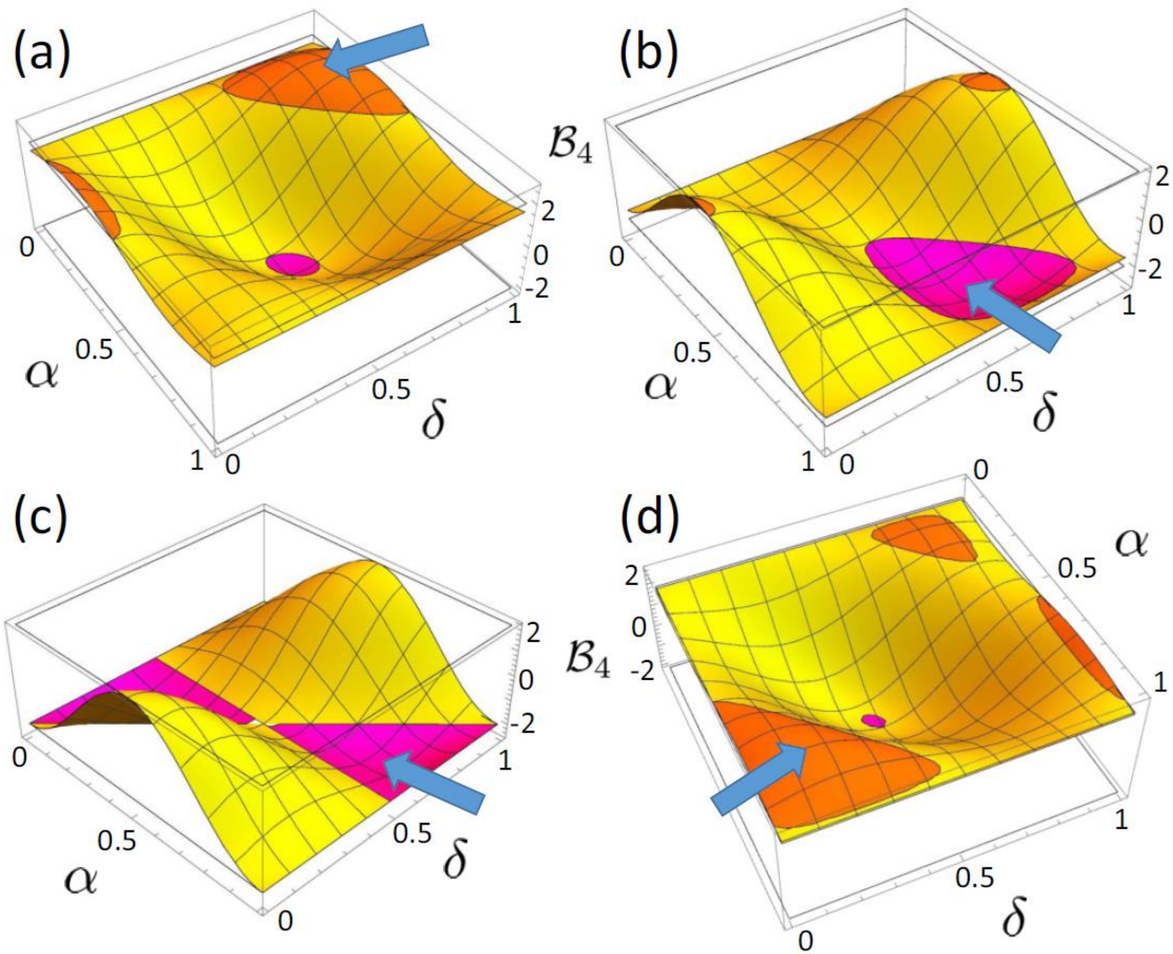
$$I_A = |\psi_A\rangle\langle\psi_A| - |\psi_A^\perp\rangle\langle\psi_A^\perp| \qquad |\psi_A\rangle = \cos\alpha|H\rangle + \sin\alpha|V\rangle$$ (preparation)

$$I_B = |\psi_\gamma\rangle\langle\psi_\gamma| - |\psi_\gamma^\perp\rangle\langle\psi_\gamma^\perp| \qquad |\psi_\gamma\rangle = \cos\gamma|H\rangle + \sin\gamma|V\rangle$$

$$I_C = |H\rangle\langle H| - |V\rangle\langle V|$$

$$I_D = |\psi_D\rangle\langle\psi_D| - |\psi_D^\perp\rangle\langle\psi_D^\perp| \qquad |\psi_D\rangle = \cos\delta|H\rangle + \sin\delta|V\rangle$$ (post-selection)

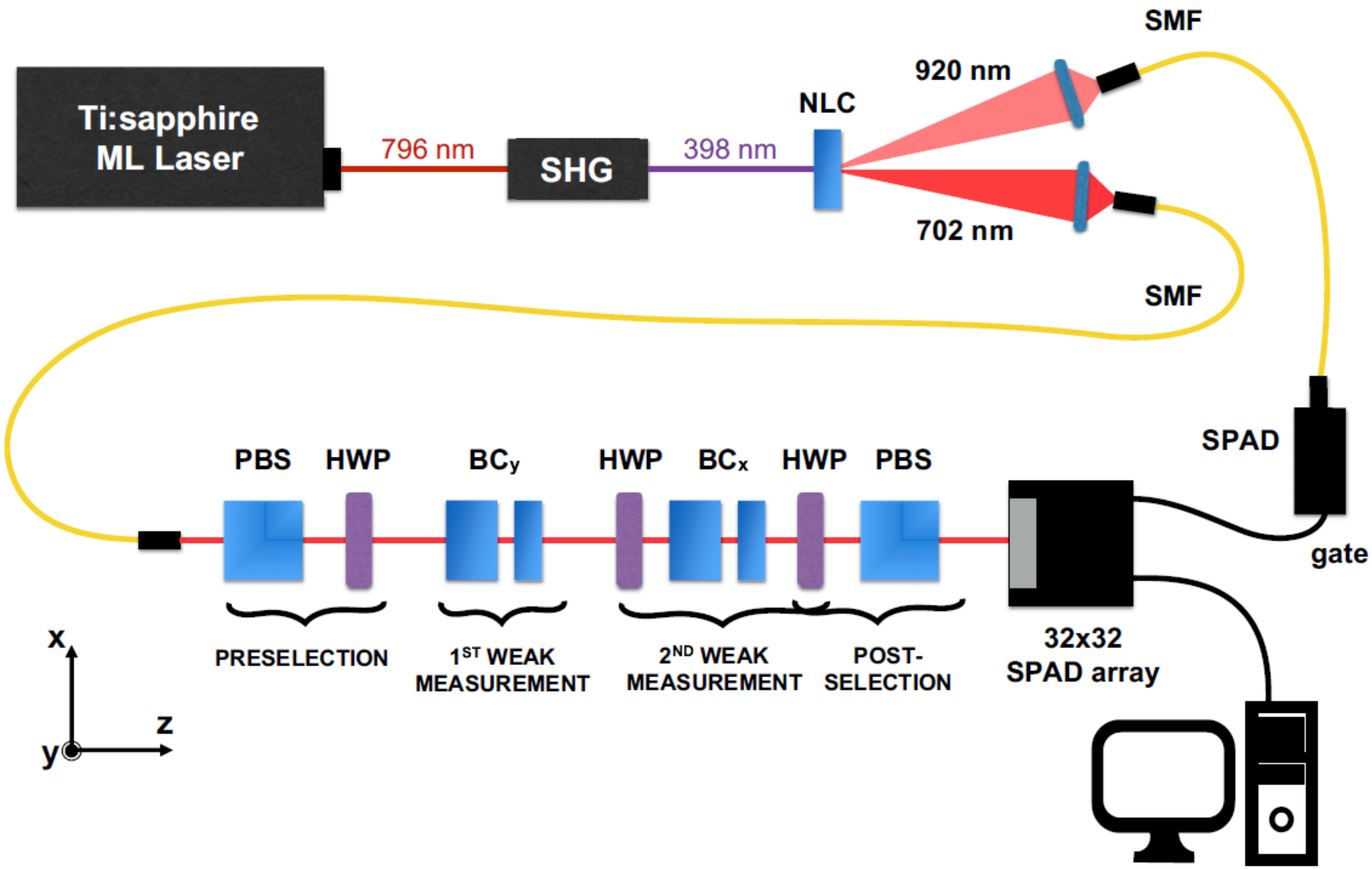
Results and conclusions



$$|\psi_A\rangle = \cos\alpha|H\rangle + \sin\alpha|V\rangle \qquad \text{(preparation)}$$

$$|\psi_D\rangle = \cos\delta|H\rangle + \sin\delta|V\rangle \qquad \text{(post-selection)}$$

Experimental implementation



Results and conclusions

Leggett-Garg inequality violation results obtained in our four experimental scenarios:

Parameters	$\mathcal{B}_4^{(\text{th})}$	$\mathcal{B}_4^{(\text{exp})}$	${}_1\langle I_c\rangle$	${}_{-1}\langle I_c\rangle$
$\gamma = 0.1\pi$ $\alpha = 0.233\pi$ $\delta = 0.867\pi$	2.82	2.76 ± 0.17	2.34 ± 0.04	-0.34 ± 0.04
$\gamma = 0.4\pi$ $\alpha = 0.767\pi$ $\delta = 0.633\pi$	-2.82	-2.74 ± 0.18	-0.30 ± 0.04	2.20 ± 0.04
$\gamma = 0.5\pi$ $\alpha = 0.833\pi$ $\delta = 0.667\pi$	-2.50	-2.56 ± 0.16	0.01 ± 0.06	1.86 ± 0.06
$\gamma = 0.95\pi$ $\alpha = 0.8\pi$ $\delta = 0.15\pi$	2.71	2.86 ± 0.19	1.86 ± 0.04	-0.12 ± 0.06

We demonstrated the capability of our setup to address single photons with **negligible disturbance**, certifying it by a **LGT** and, in a complementary way, by the presence of **anomalous weak values** upon postselection.

"Anomalous weak values and the violation of a multiple-measurement Leggett-Garg inequality"

Phys. Rev. A 96, 052123

Conclusions

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"Measuring incompatible observables of a single photon"

PRL 117, 170402 (2016)

"An experiment investigating the connection between weak values and contextuality"

PRL 116, 180401 (2016)

"Determining the Quantum Expectation Value by Measuring a Single Photon"

Nature Physics

doi:10.1038/nphys4223

"Anomalous weak values and the violation of a multiple-measurement Leggett-Garg inequality"

Phys. Rev. A 96, 052123

"Investigating the Effects of the Interaction Intensity in a Weak Measurement"
arXiv:1709.04869

Weak Measurements

Weak regime approximated solution:

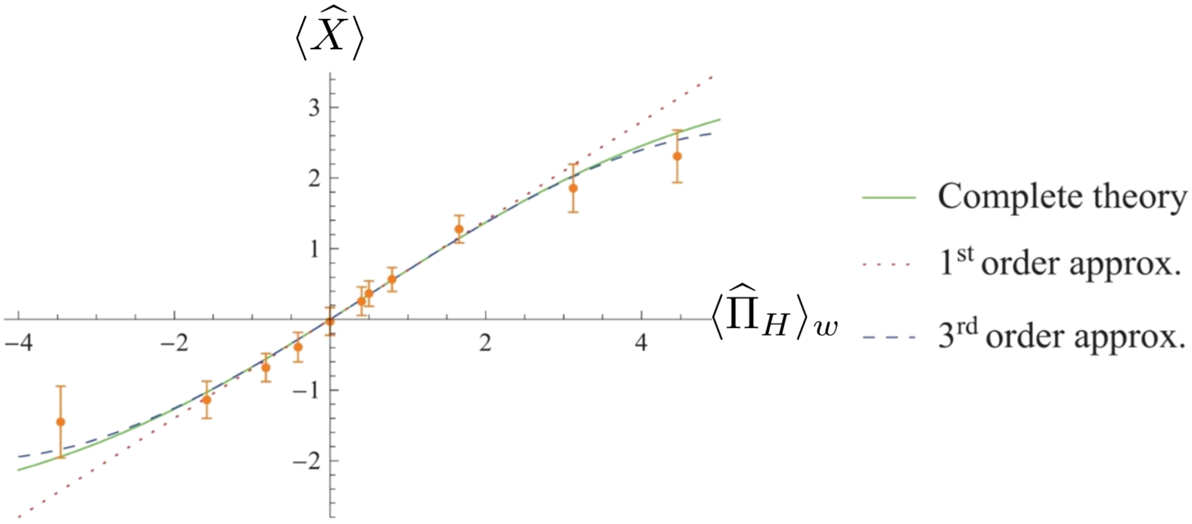
$$\langle\widehat{Y}\rangle = \frac{\mathrm{Tr}[\widehat{\Pi}_f \otimes \widehat{Y}\,\widehat{U}\,\widehat{\rho}_{\psi_i} \otimes \widehat{\rho}_{f_i}\,\widehat{U}^\dagger]}{\mathrm{Tr}[\widehat{\Pi}_f\,\widehat{\rho}_{\psi_i}]} \sim g\,\langle\widehat{\Pi}_V\rangle_w + \mathcal{O}(g^3)$$

Exact solution:

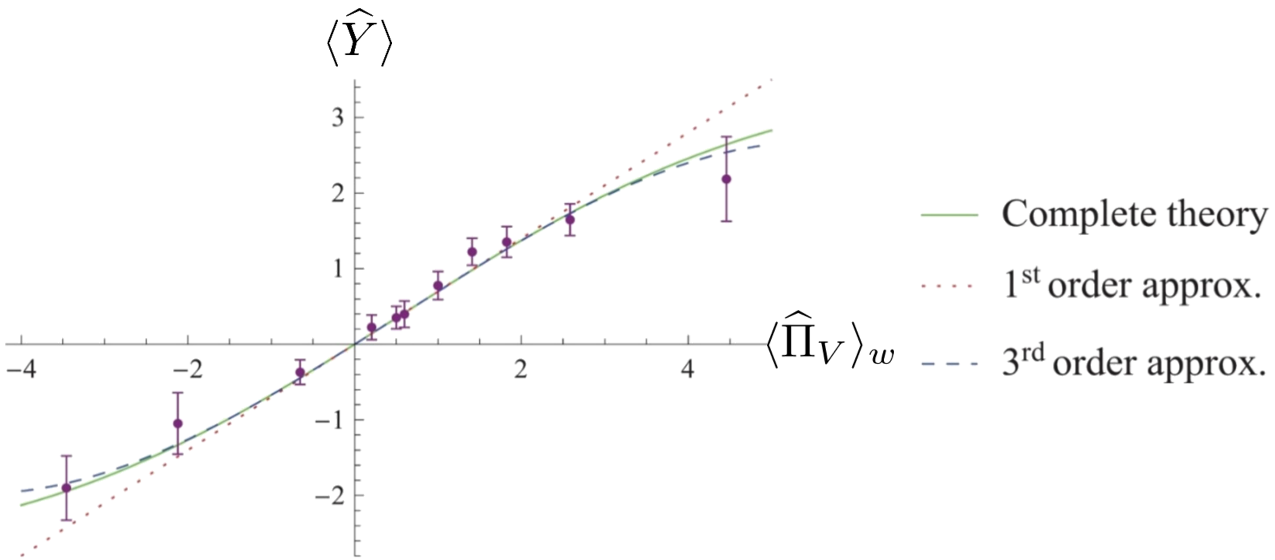
$$\langle\widehat{Y}\rangle = \frac{\mathrm{Tr}[\widehat{\Pi}_f \otimes \widehat{Y}\,\widehat{U}\,\widehat{\rho}_{\psi_i} \otimes \widehat{\rho}_{f_i}\,\widehat{U}^\dagger]}{\mathrm{Tr}[\widehat{\Pi}_f\,\widehat{\rho}_{\psi_i}]} = \frac{g\,\langle\widehat{\Pi}_V\rangle_w\{\langle\widehat{\Pi}_V\rangle_w[e^{(g/2\sigma)^2}-1]-1\}}{e^{(g/2\sigma)^2}+2[e^{(g/2\sigma)^2}-1](\langle\widehat{\Pi}_V\rangle_w+\langle\widehat{\Pi}_V\rangle_w^2)}$$

$$\boxed{\widehat{U} = \exp(-ig\widehat{\Pi}_V \otimes \widehat{P})}$$

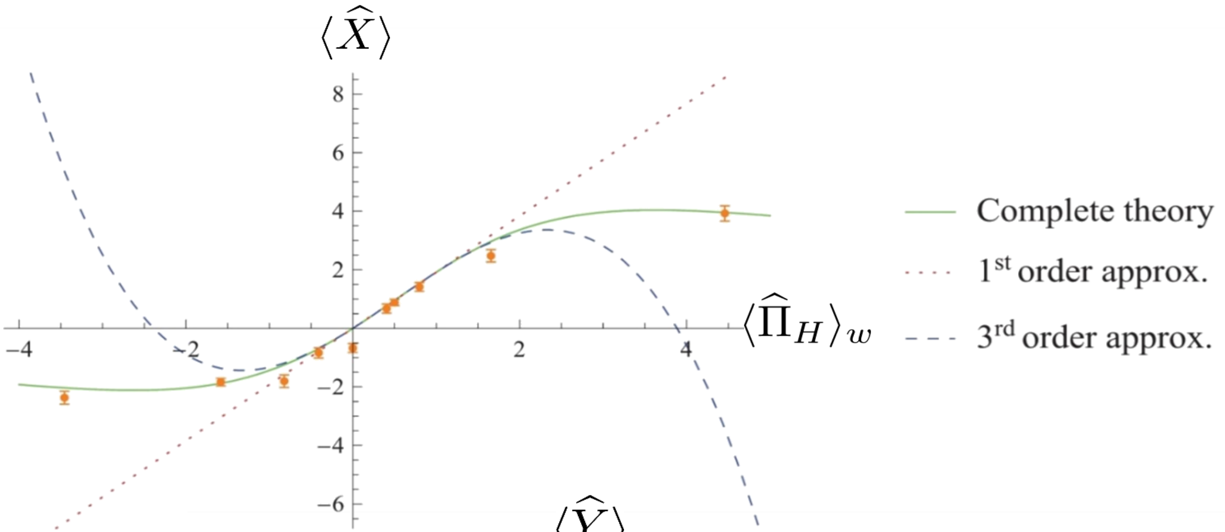
Weak Measurements

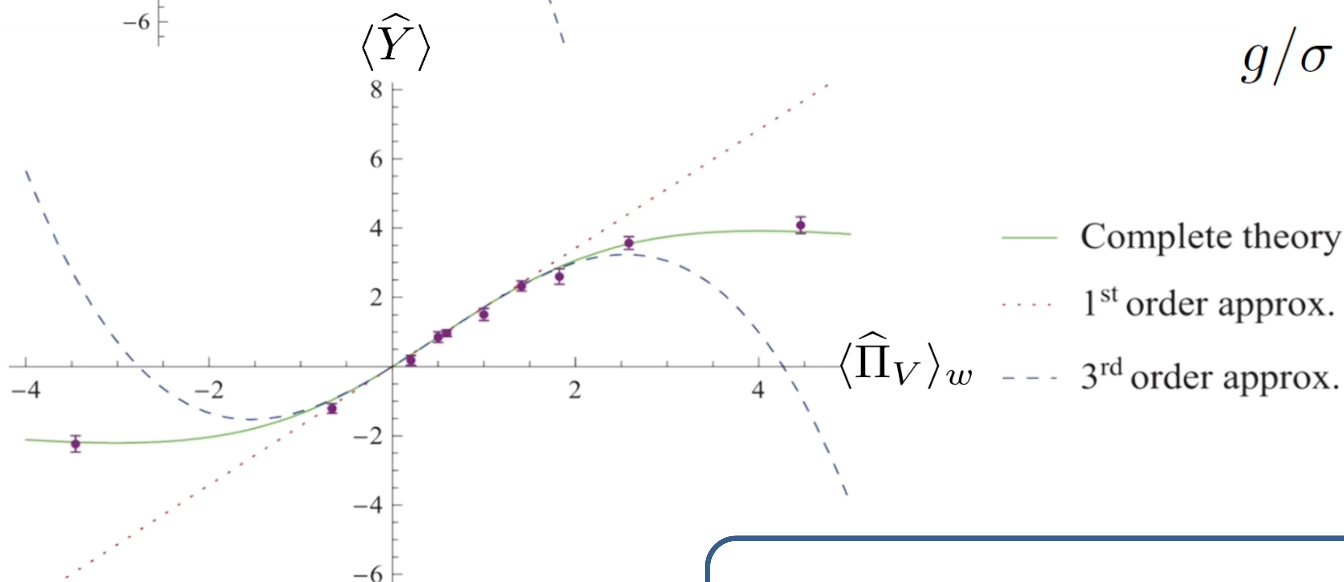


$g/\sigma \sim 0.15$



Weak Measurements





$$g/\sigma \sim 0.32$$

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