NonClassical Correlations in Open Quantum Dynamics

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IISER THIRUVANANTHAPURAM

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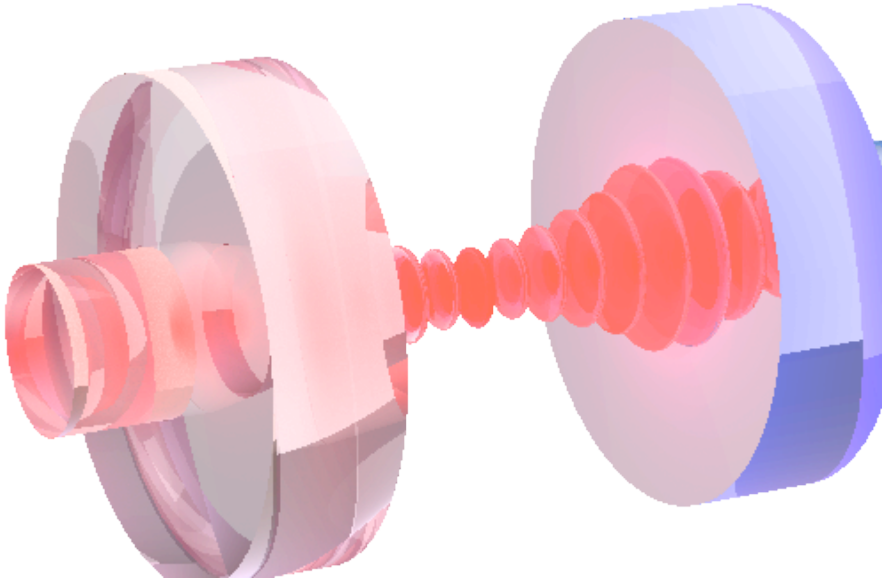

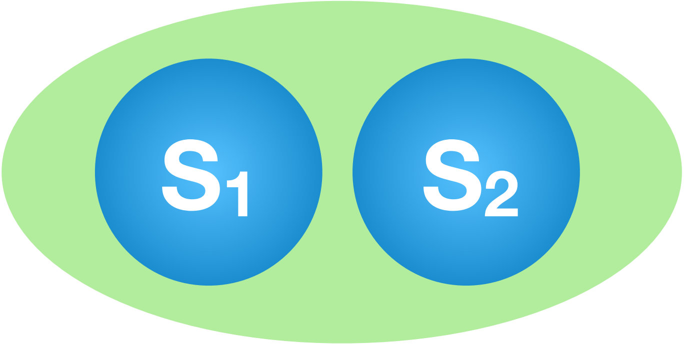
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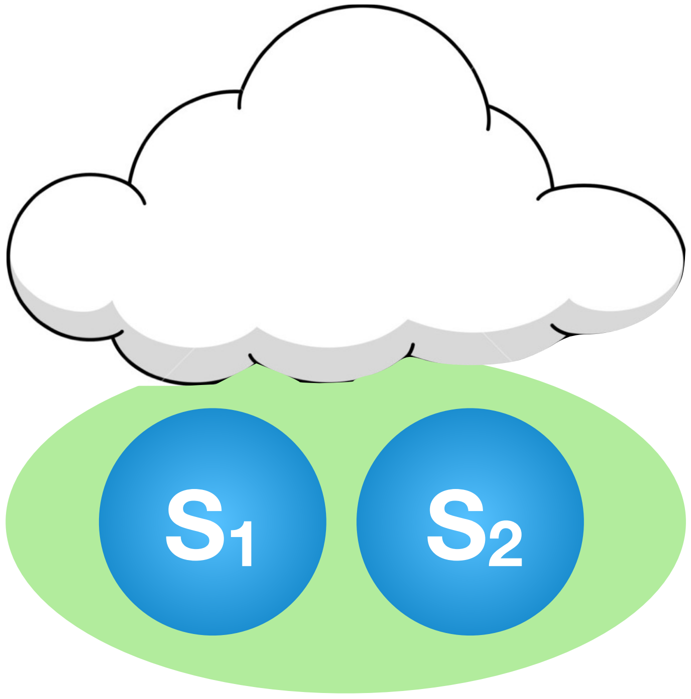
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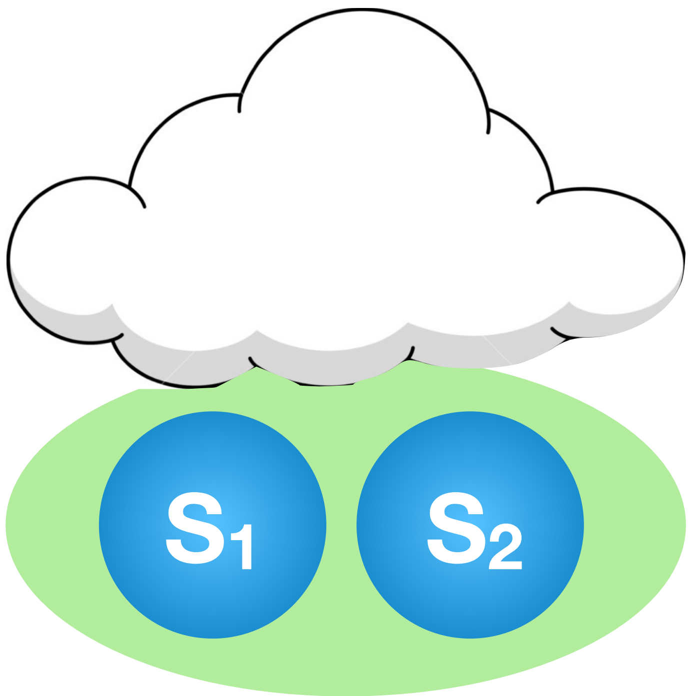
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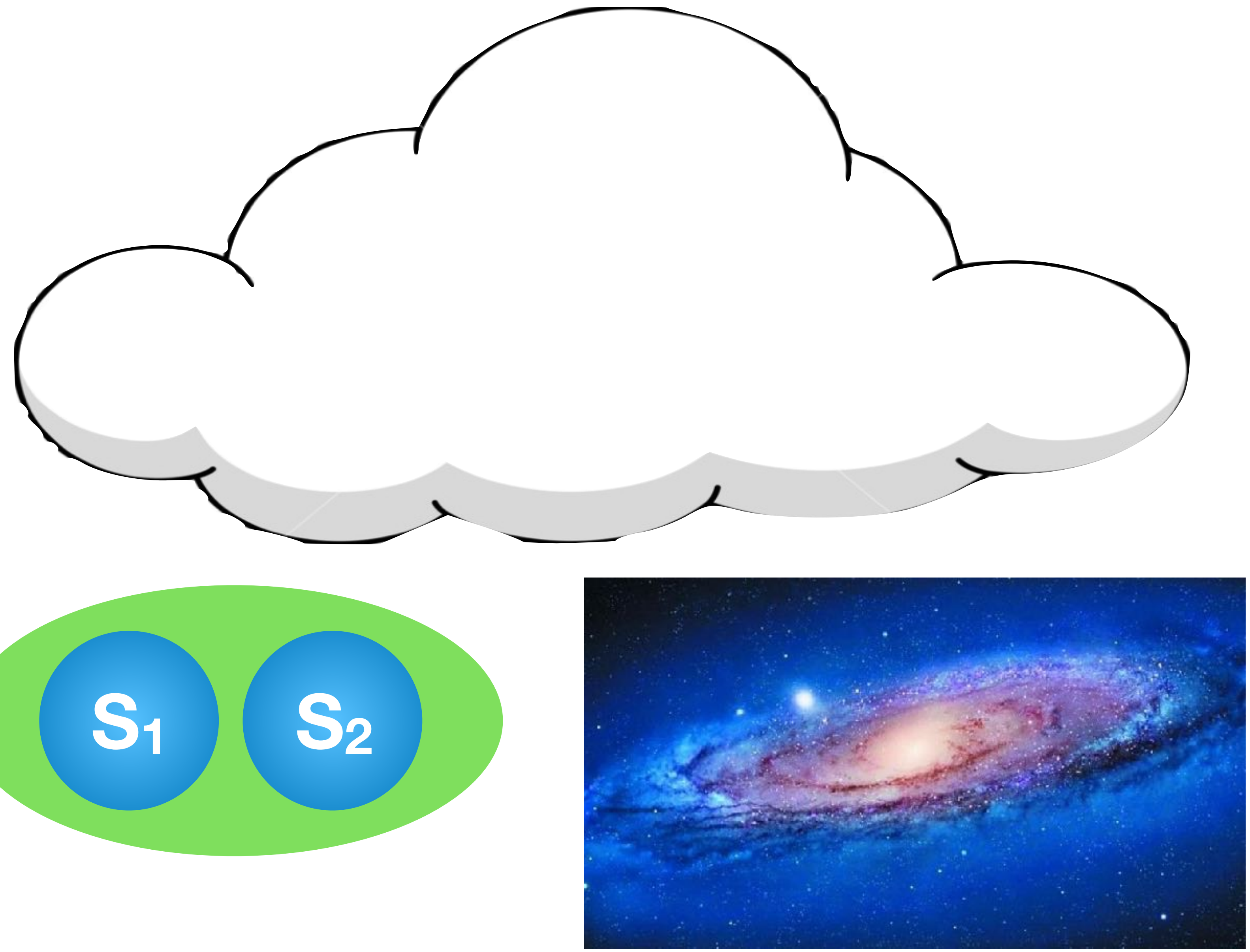
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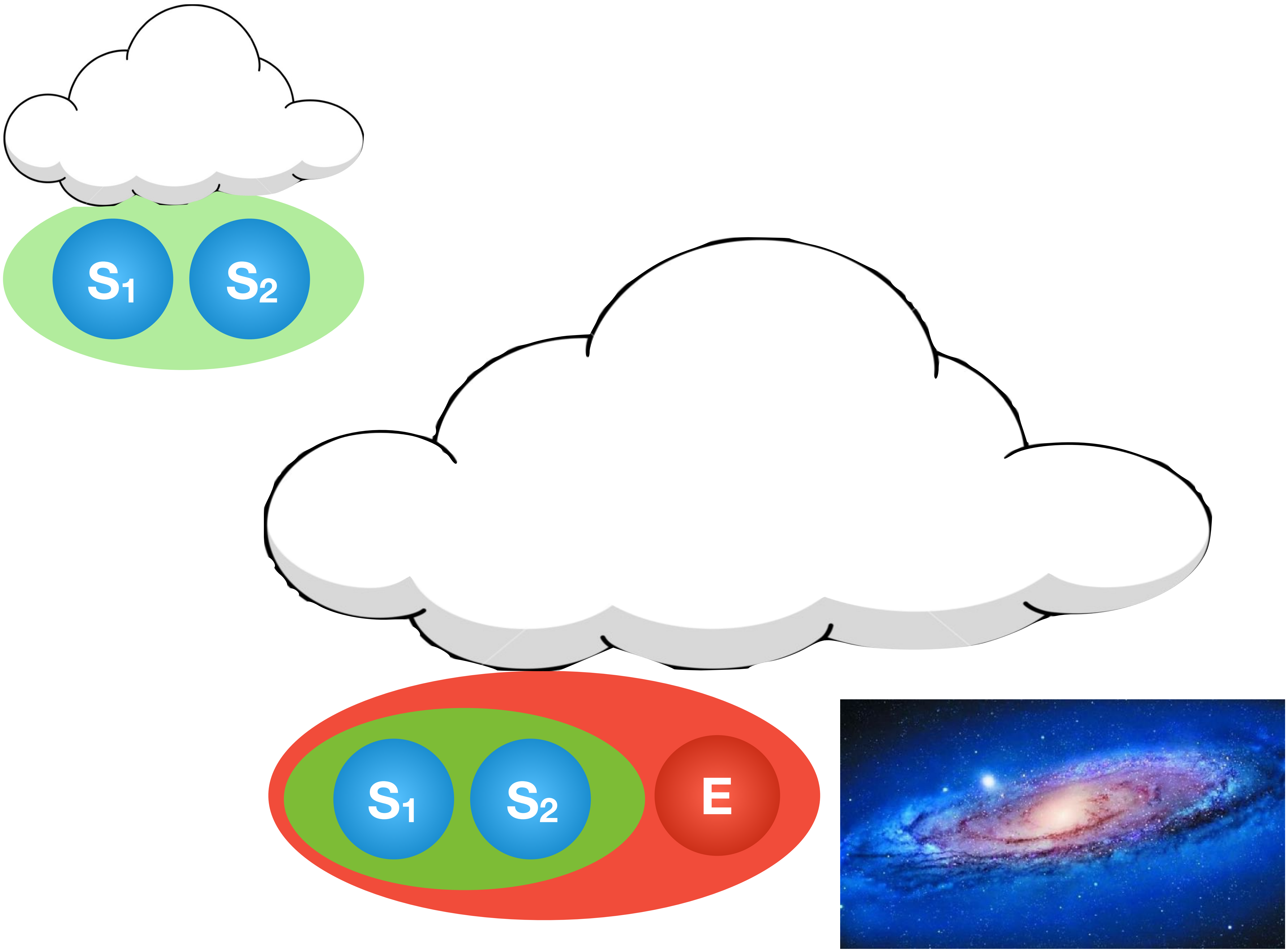
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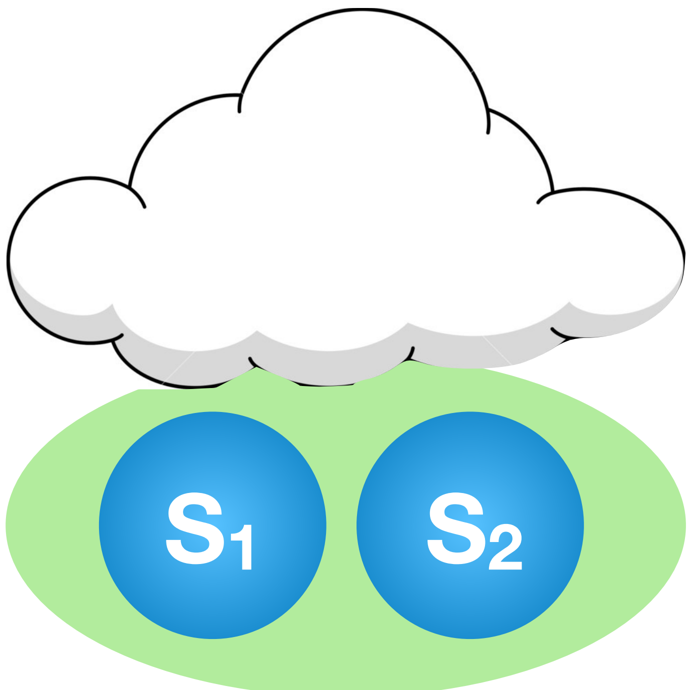
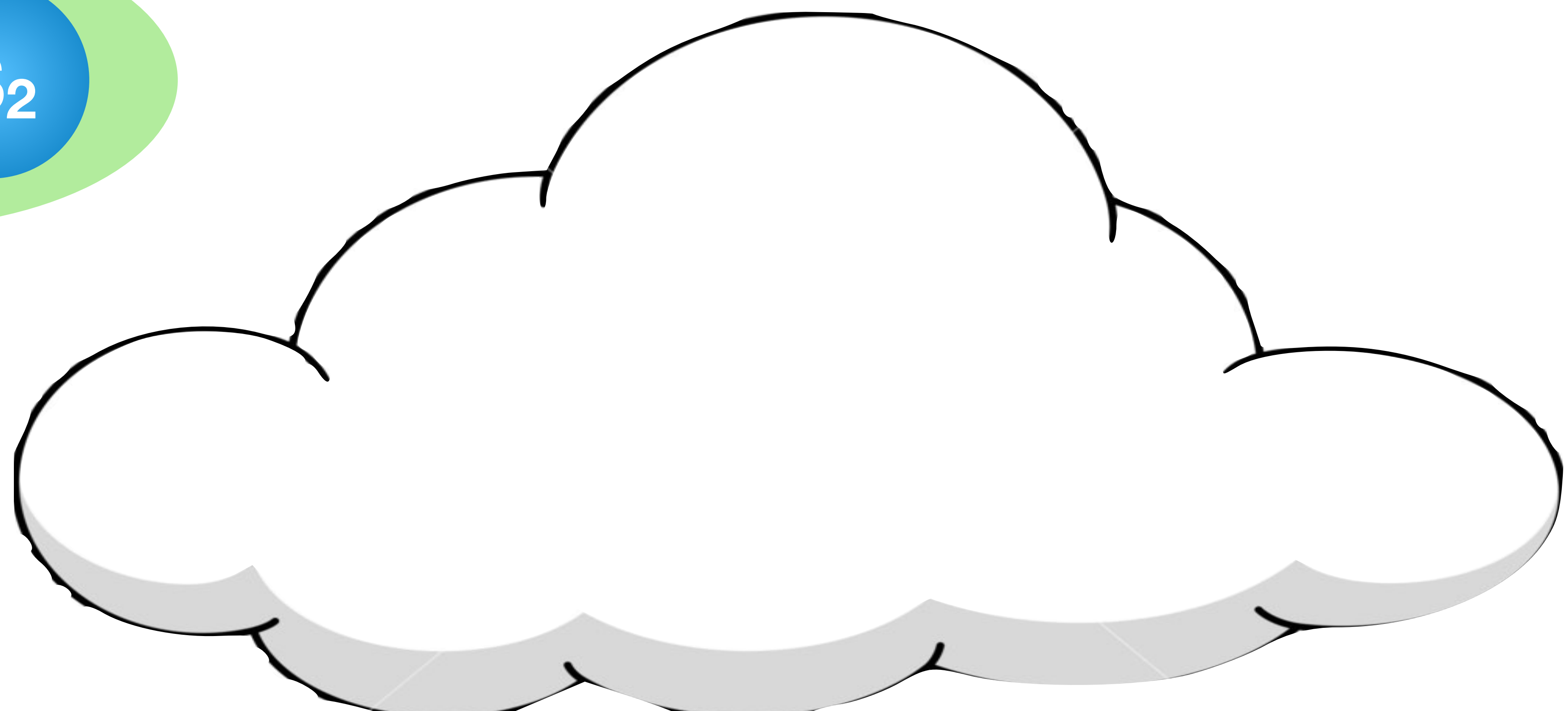
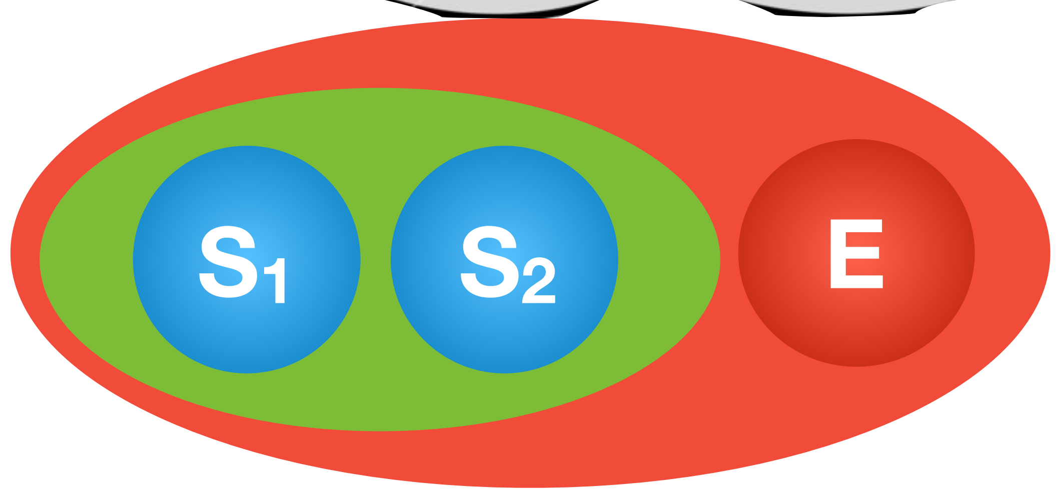
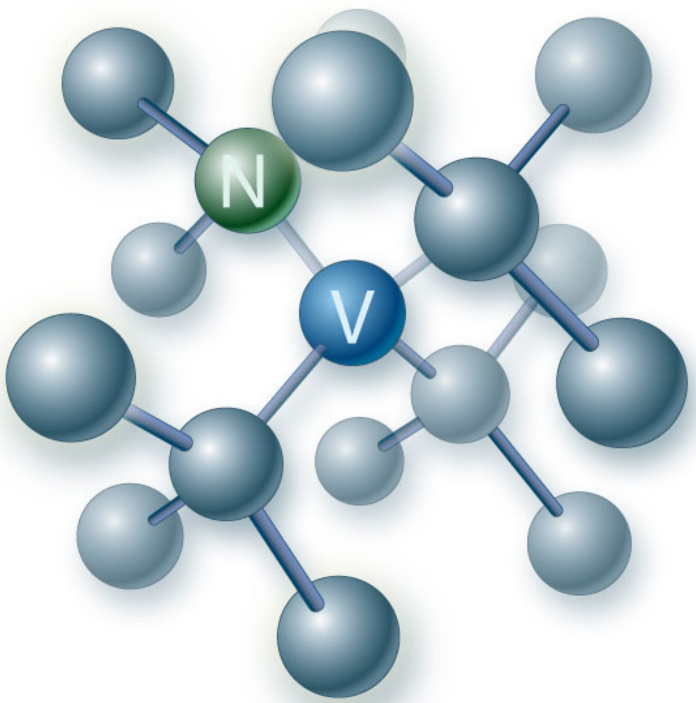


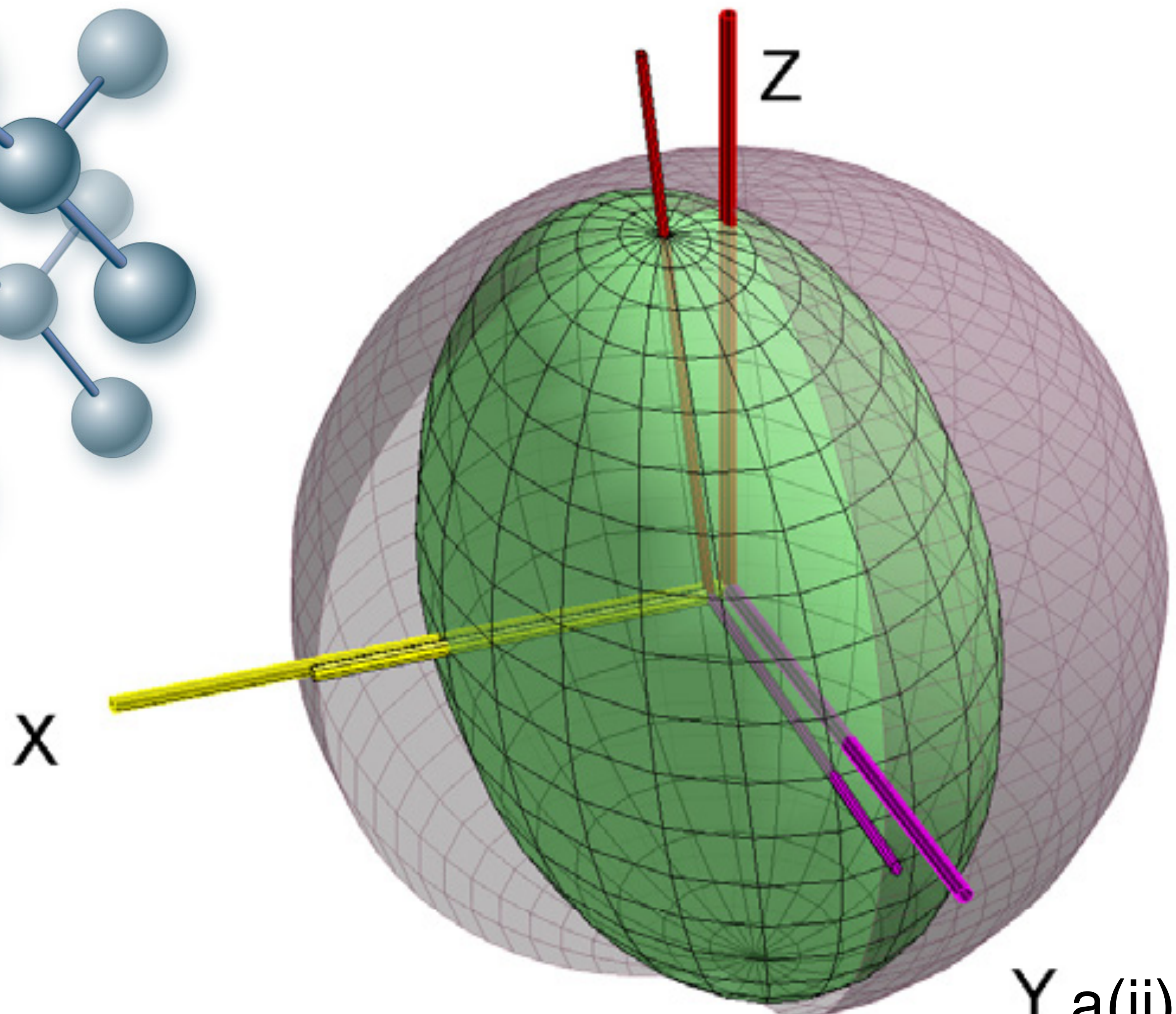
Nonclassical correlations between parts of a system as well as those between the system, among its parts, to its immediate environment and with the rest of the universe.





Quantum process tomography





a(ii)

Howard, M., Twamley, J., Wittmann, C., Gaebel, T., Jelezko, F., & Wrachtrup, J. (2006). Quantum process tomography and Linblad estimation of a solid-state qubit. *New Journal of Physics*, ***8***(3), 33–33.

Open quantum dynamics

Dynamical maps take density matrices to density matrices

- Linear and trace preserving
- Preserves Hermiticity of ρ
- Maps positive matrices to positive matrices

$$\rho_{rs} \longrightarrow A_{rs;r's'}\rho_{r's'} = (A\rho)_{rs}$$

$$A_{rs;r's'}(t) \leftrightarrow B_{rr';ss'}(t)$$

The 'B' Matrix form is often more convenient to use

The B-Matrix is in itself Hermitian

The operator sum form

Since B is hermitian, it can be written in terms of its eigenvalues and eigenvectors.

$$\rho_{rs} = \sum_\alpha \lambda_\alpha \zeta_{rr'}(\alpha) \rho_{r's'} \zeta^\dagger_{s's}(\alpha)$$

If all $\lambda_\alpha \geq 0$ then define $C(\alpha) \equiv \sqrt{\lambda_\alpha}\zeta(\alpha)$

$$\rho \longrightarrow \sum_\alpha C(\alpha) \rho C(\alpha)^\dagger$$

with

$$\sum_\alpha C(\alpha)^\dagger C(\alpha) = \mathbf{1}$$

B is then a **completely positive** map

Too restrictive?

Is complete positivity too restrictive a condition on allowable varieties or open quantum dynamics?

> *One may reasonably doubt this argument. It is very powerful magic: W sits apart from $S+R$ and does absolutely nothing; by doing so, it forces the motion of S to be completely positive with dramatic physical consequences such as $T_2 \leq 2T_1$ for exponential two-state relaxation.*
>
> – *P. Pechukas, Phys. Rev. Lett.* **73**, *1060 (1994)*

There is indeed a rich story to be told going beyond the confines of completely positive maps

Maps as contractions

Dynamical maps may be viewed as contractions of the unitary evolution of an extended system:

$$B_t \rho_S = \mathrm{tr}_E [U_t \mathscr{R}_{SE}(0) U_t^\dagger]$$

where $\mathscr{R}_{SE}(0)$ is the density matrix representing the combined initial state of the system (S) and the environment (E)

- The environment needs to be at most N^2 dimensional to get any map
- If $\mathscr{R}_{SE}(0) = \rho_S \otimes \eta_E$ then B_t is completely positive
- What if $\mathscr{R}_{SE}(0)$ is entangled?
- What if $\mathscr{R}_{SE}(0)$ is a generic separable state?

Making Coffee

Making Coffee

Making Coffee

Making Coffee

Making Coffee

When will the coffee be cool enough to drink for different temperature/pressure and other settings of the preparation device.

Making Coffee

Making Coffee

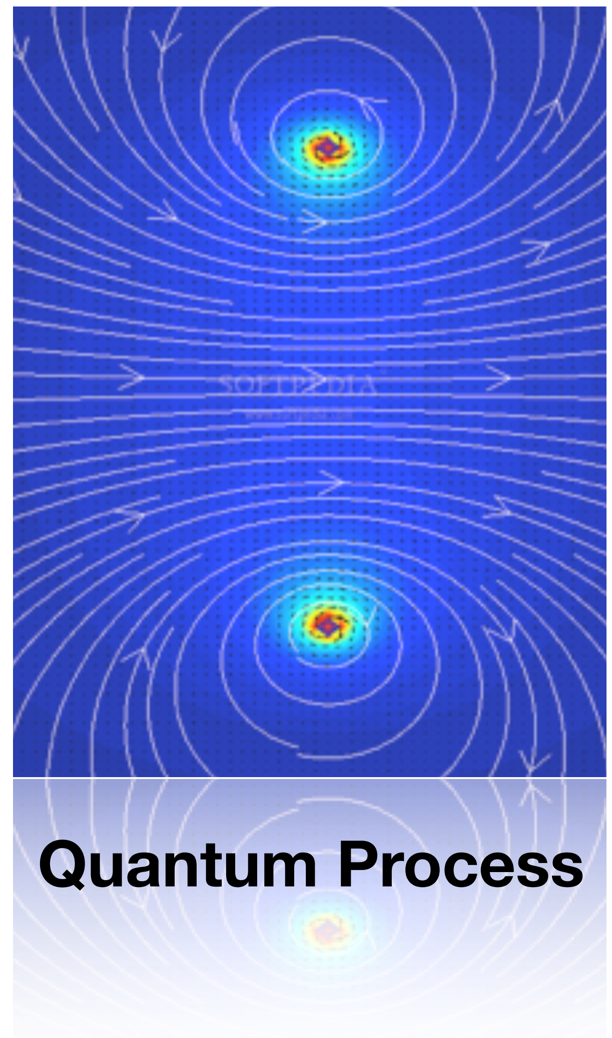
Making Coffee

Making Coffee

Making Coffee

The dogma of CP maps asserts that one should not be asking questions like this where the preparation device influences the environment of the open system as well.

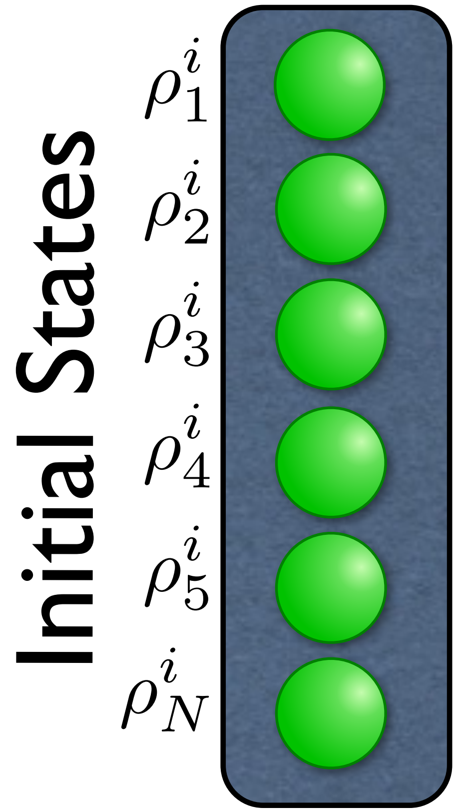
Quantum Process Tomography

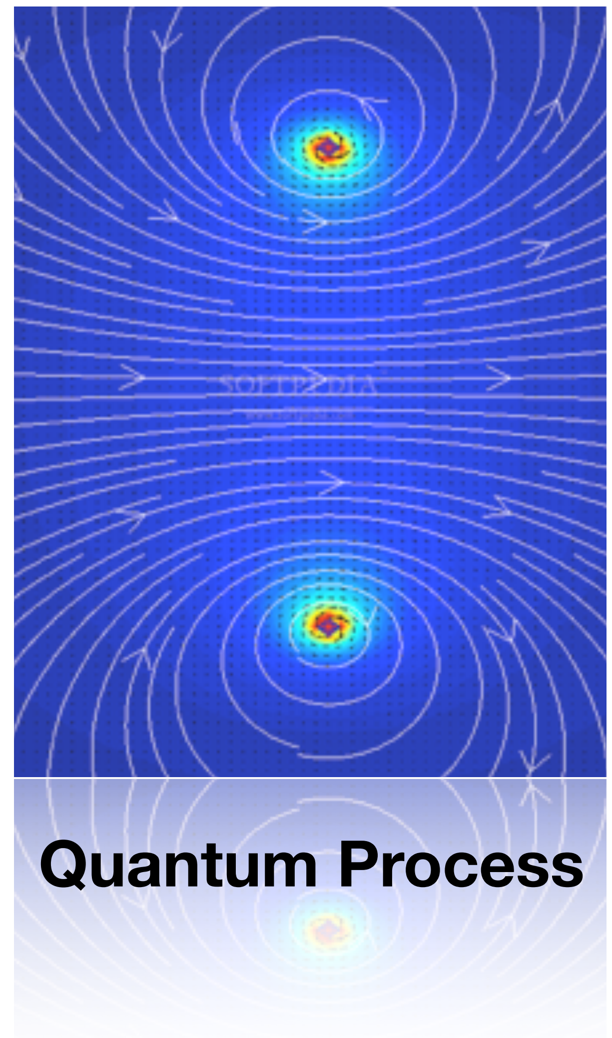


$$\rho_f = A\rho_i$$

The d^4 x d^4 matrix A represents the process to be reconstructed

Quantum Process Tomography





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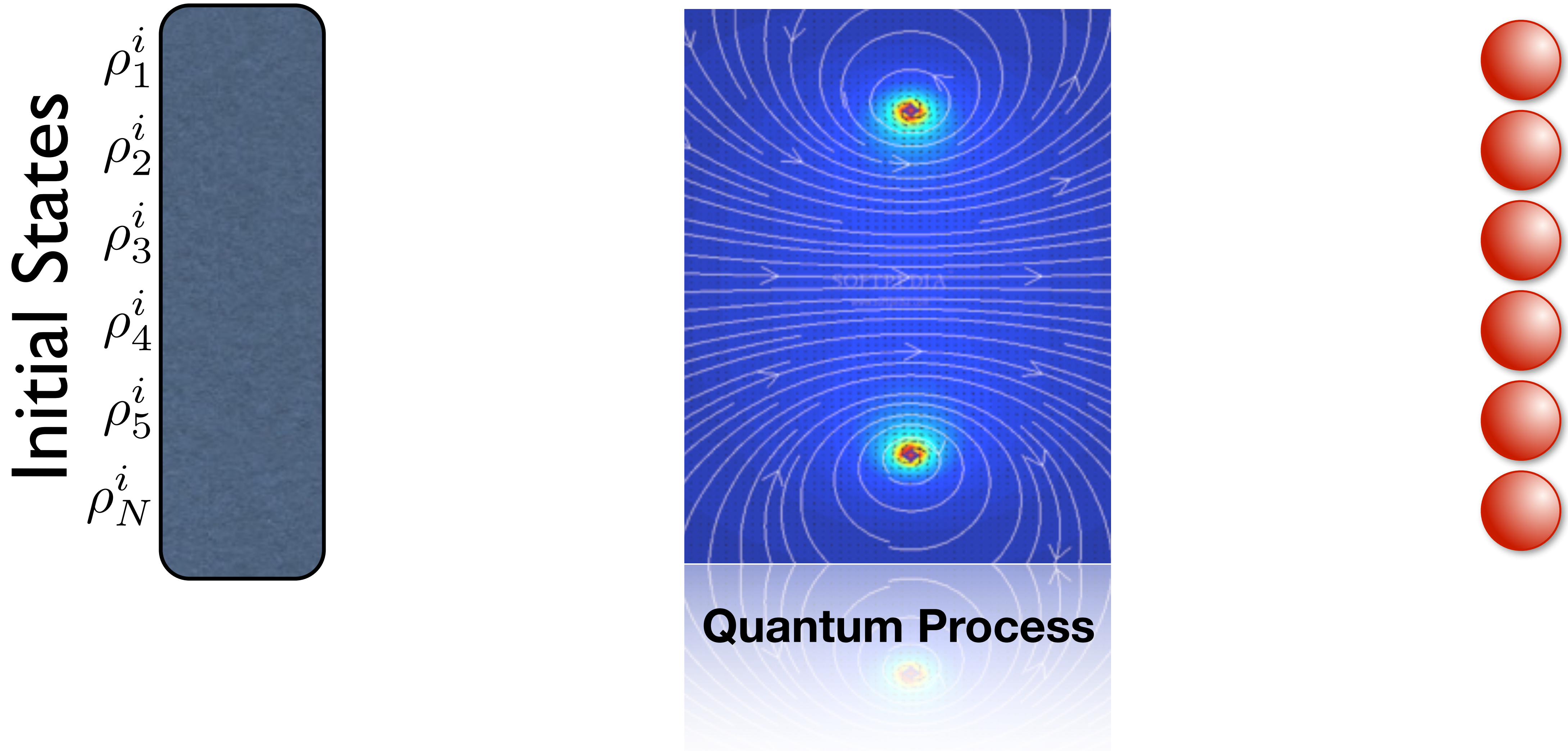

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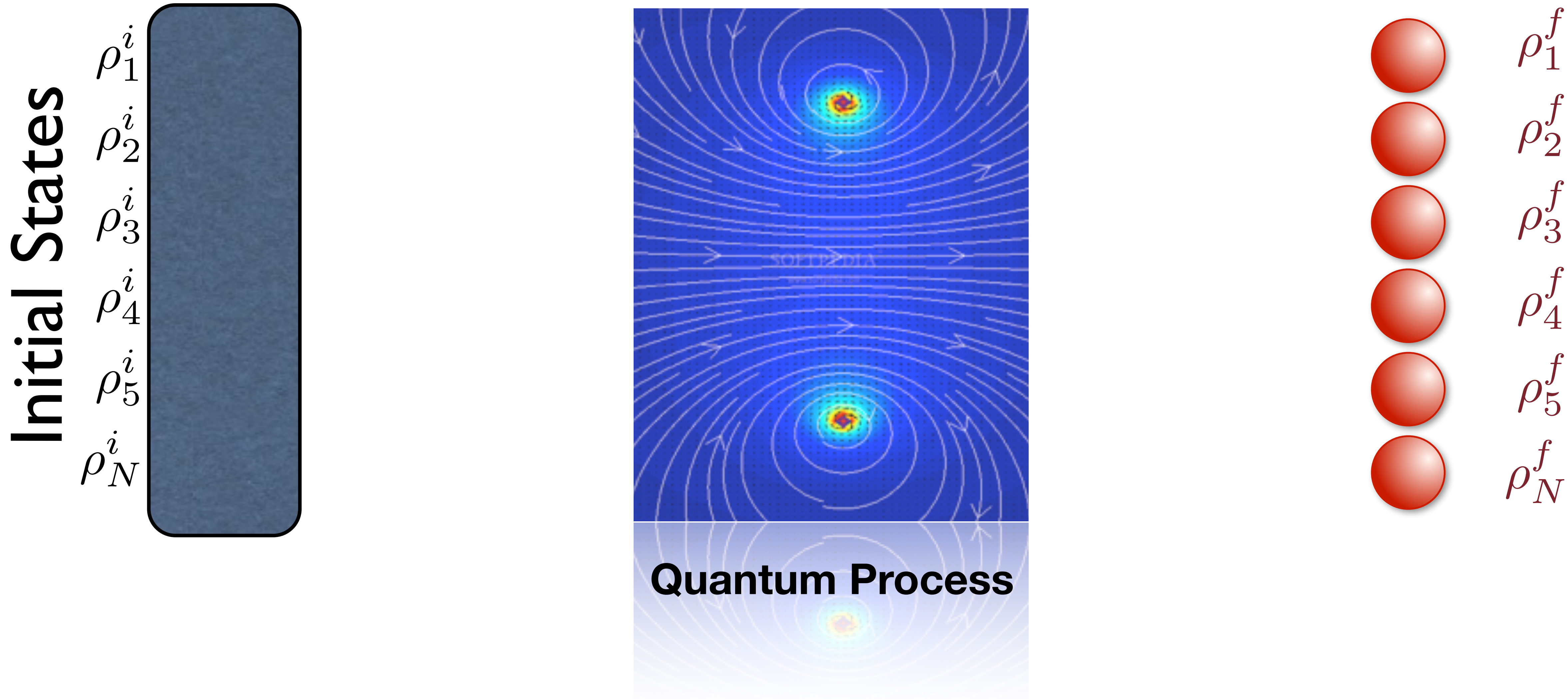

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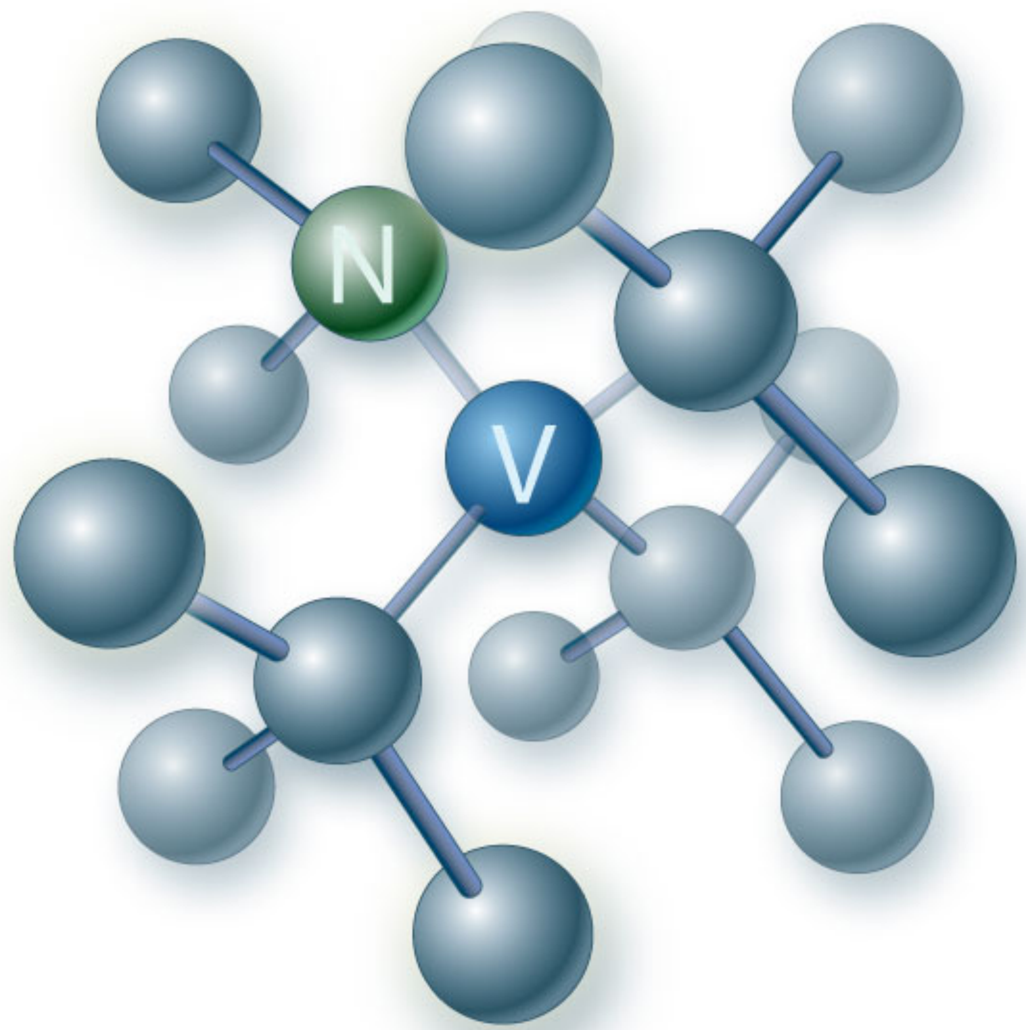

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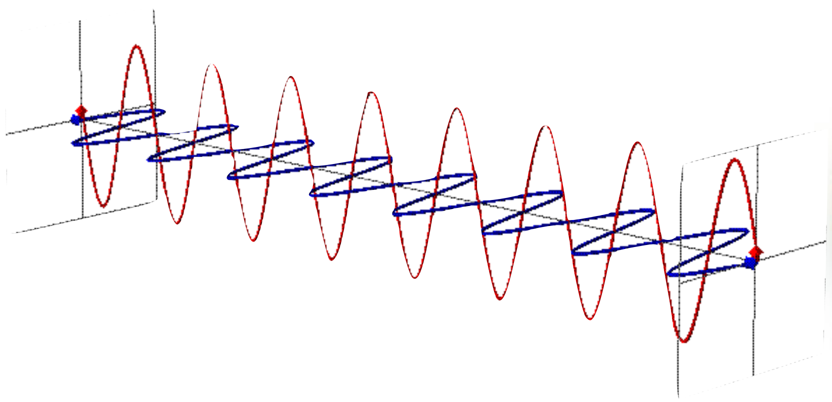
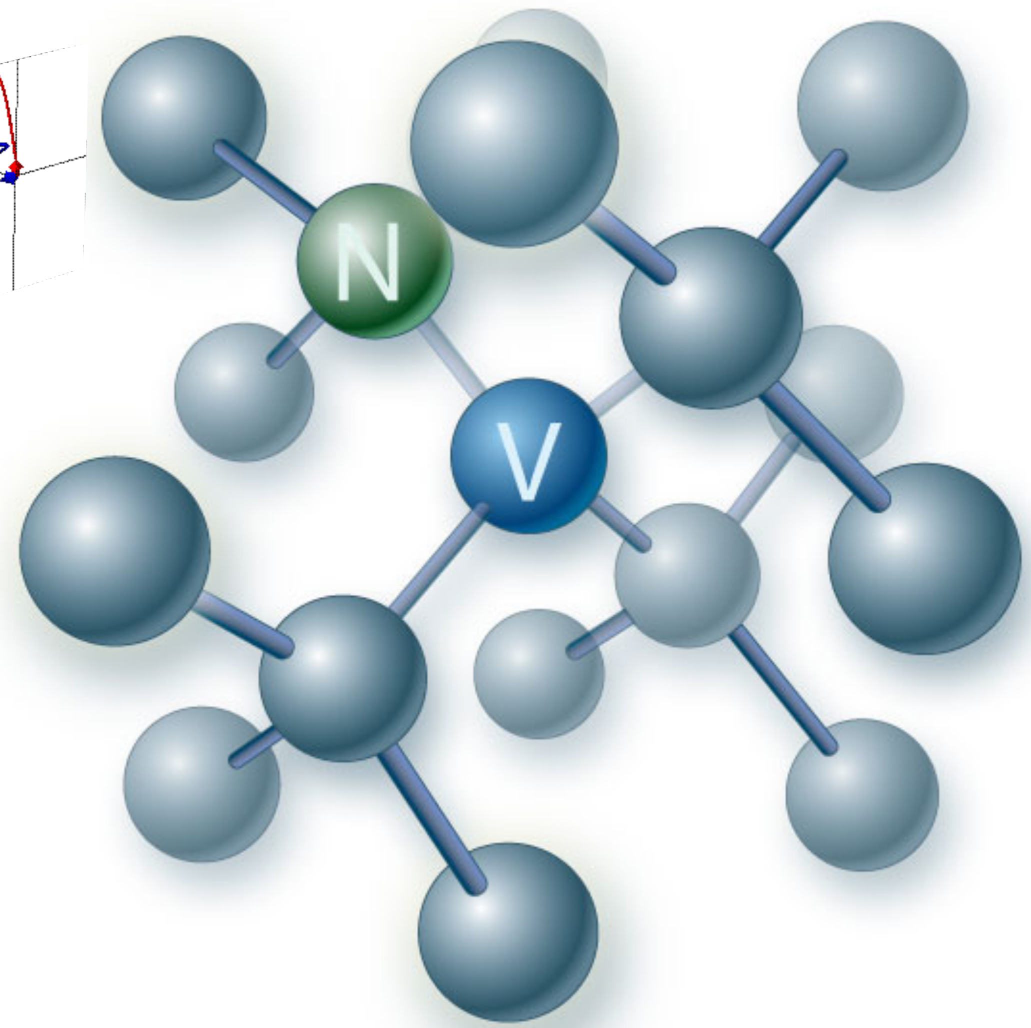
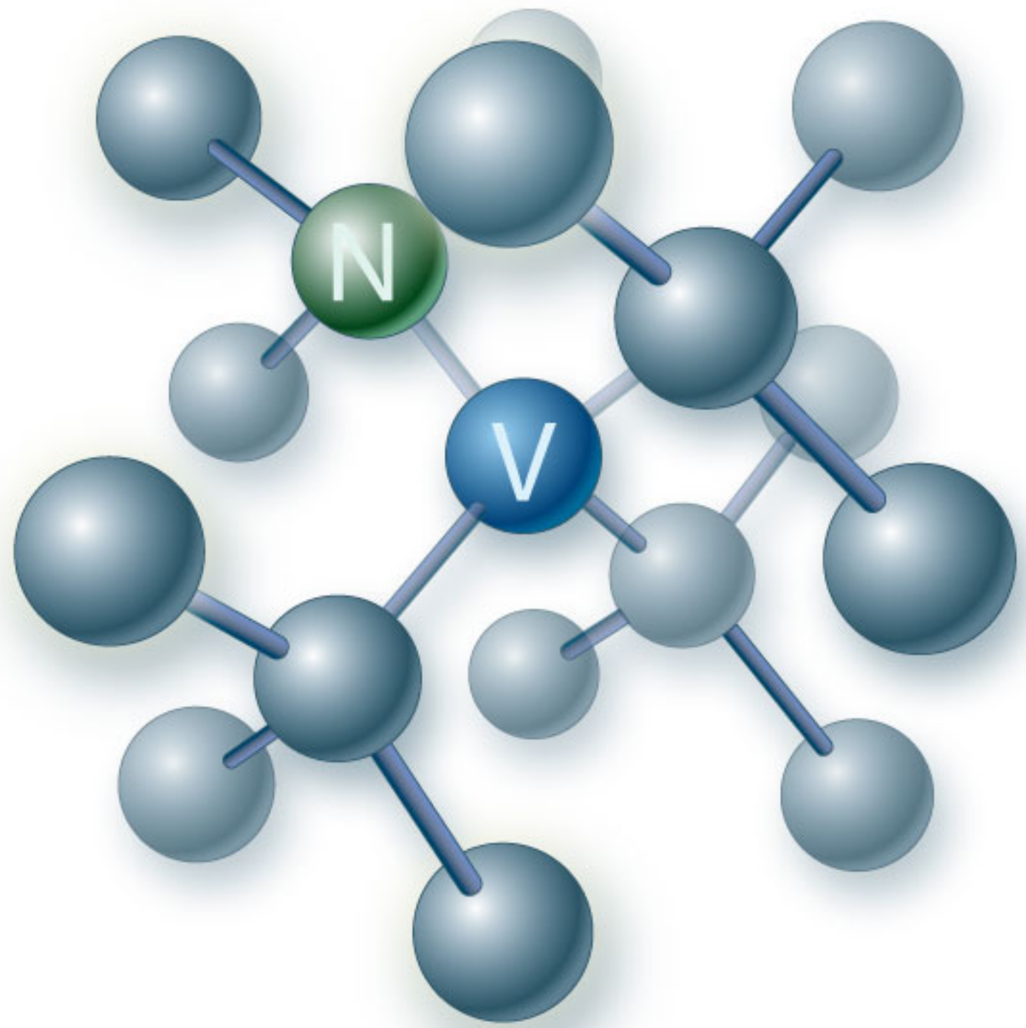


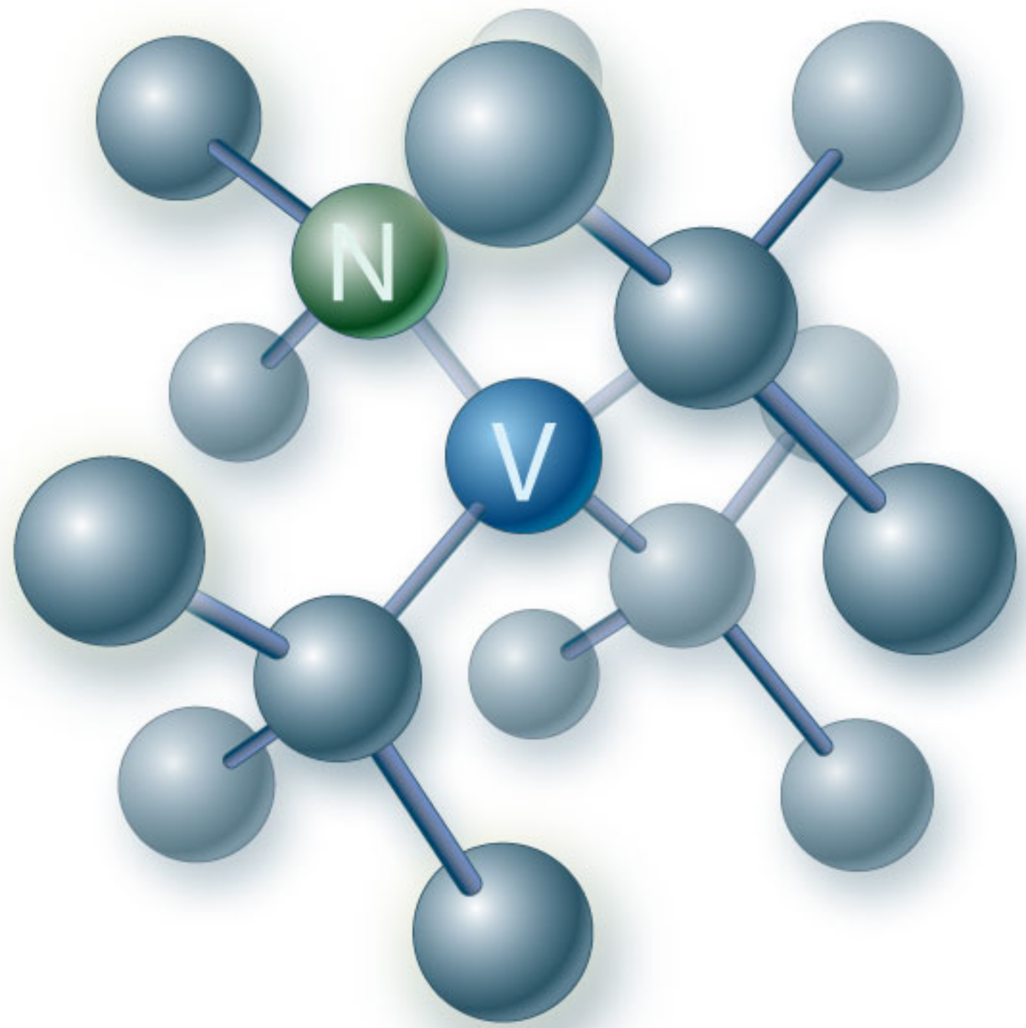


Quantum Process Tomography

1. Prepare



Quantum Process Tomography



1. Prepare
2. Evolve

Quantum Process Tomography

1. Prepare
2. Evolve
3. Read-Out

Quantum Process Tomography

1. Prepare
2. Evolve
3. Read-Out
4. Reconstruct

The preparation and CP

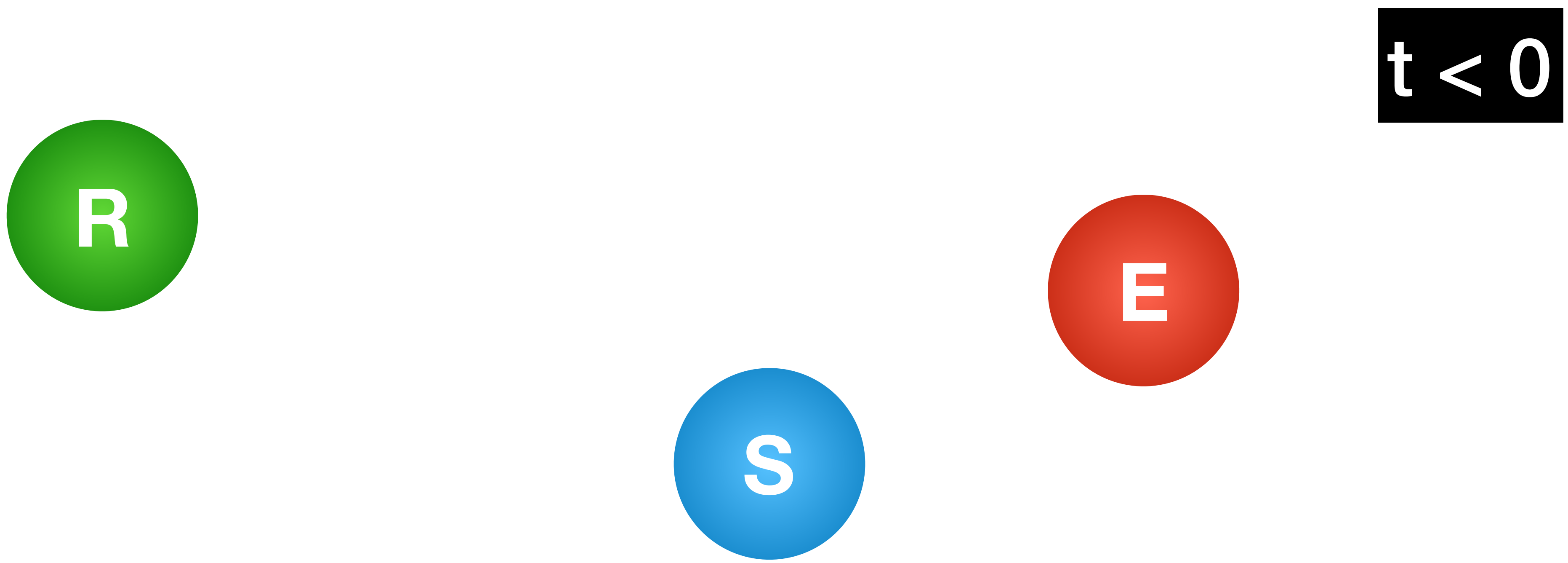
- For CP reduced dynamics, the initial joint state of the system (S) and its environment (E) should be

$$\rho_{SE} = \rho_S \otimes \eta_E$$

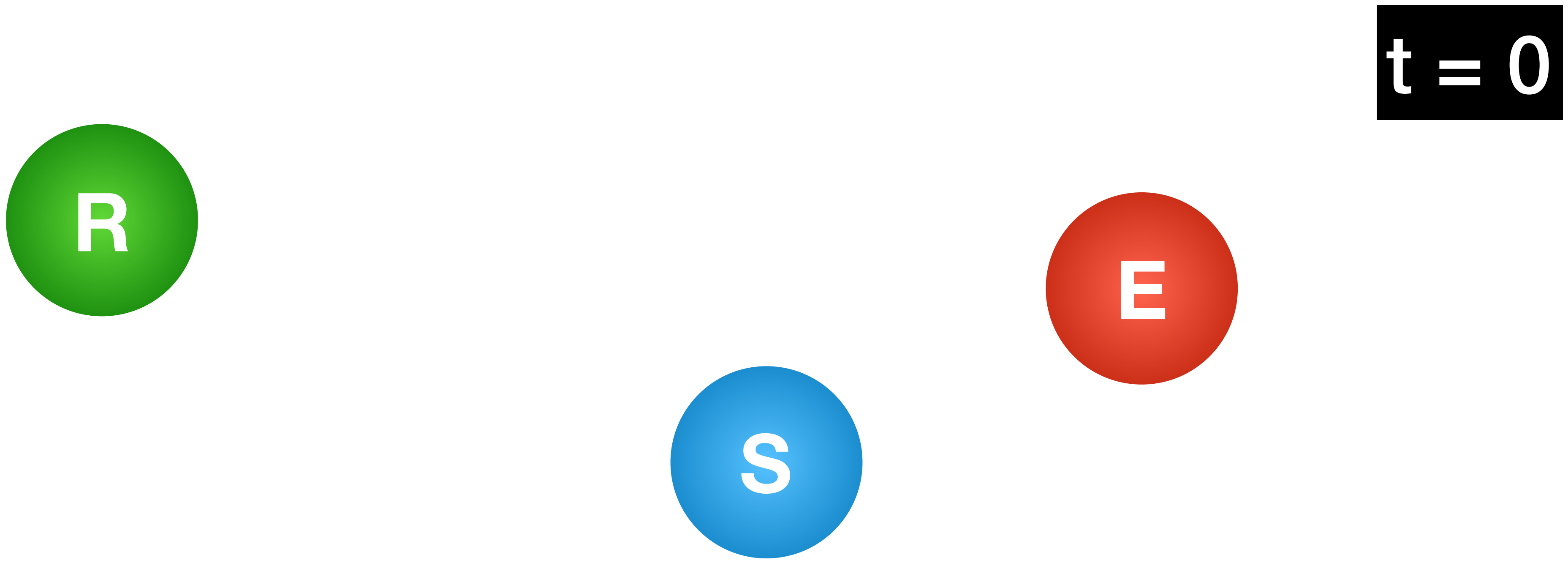
- Here every state of S is associated with fixed state of E
- The preparation pulse in our example can very well affect E.
- So each state of S that is prepared can potentially be associated with distinct states of E initially (The two could even be entangled!)
- Quantum process tomography is then unlikely to yield CP reduced dynamics

The reference system

The reference system



The reference system



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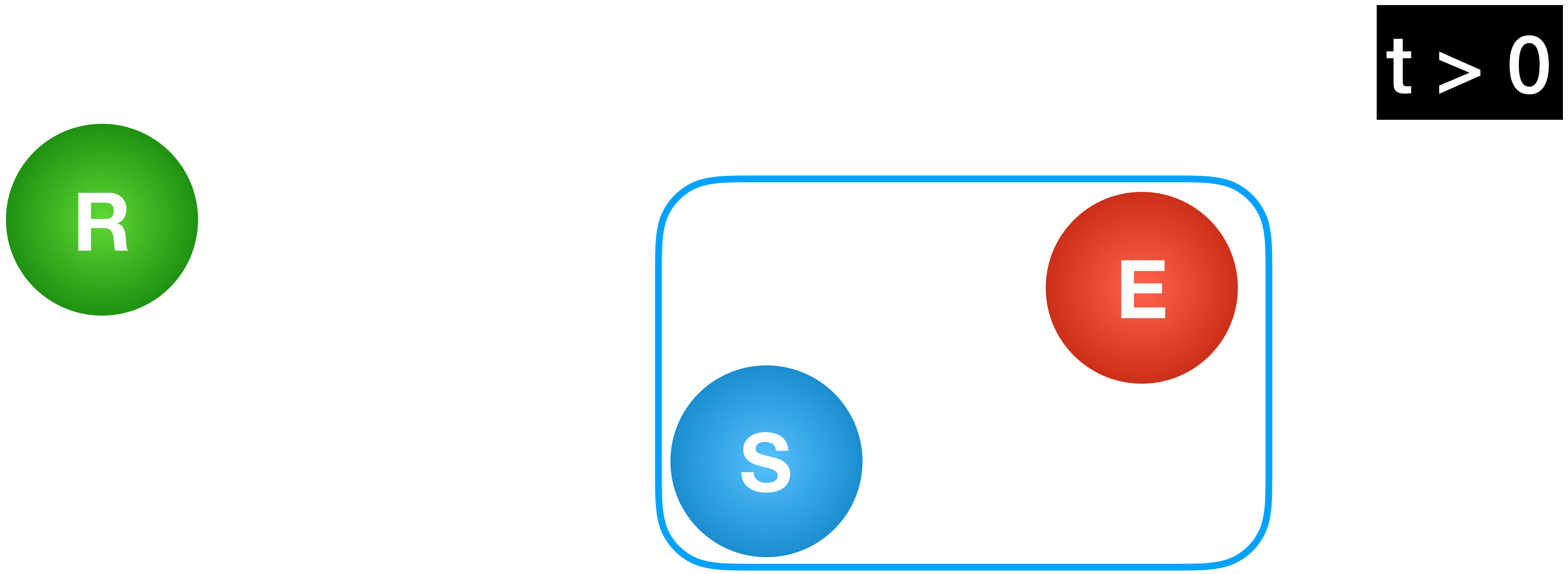

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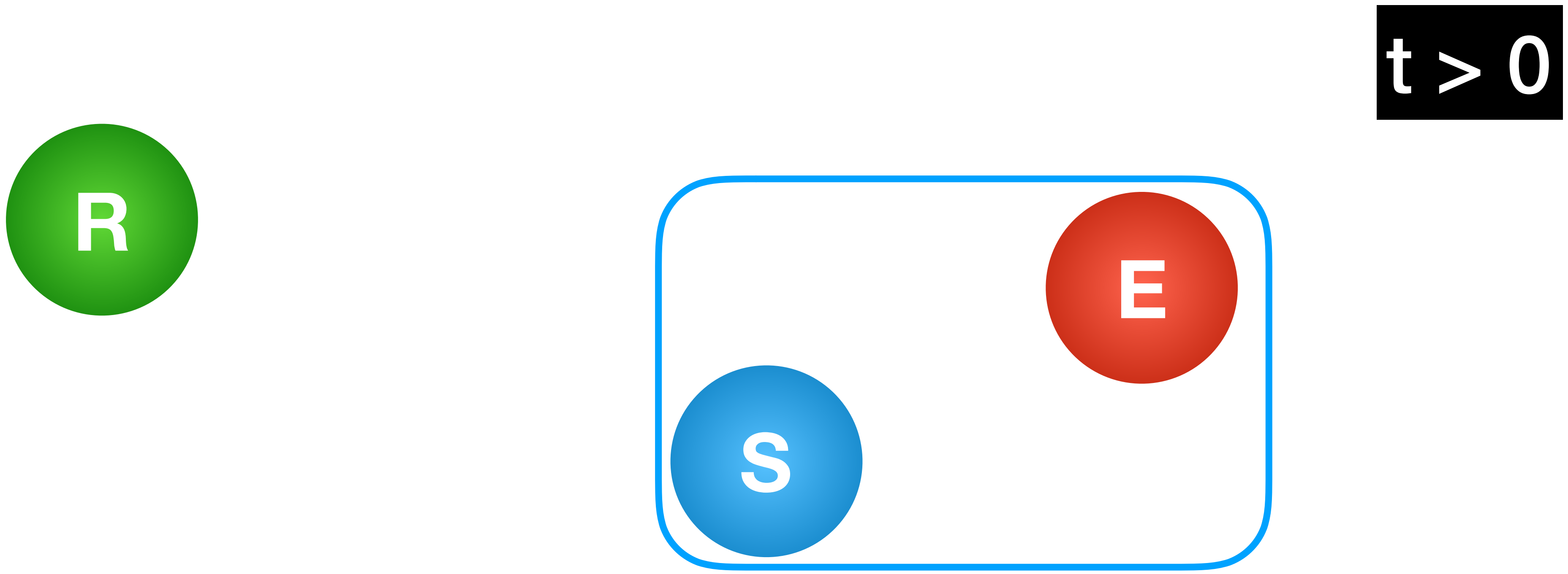
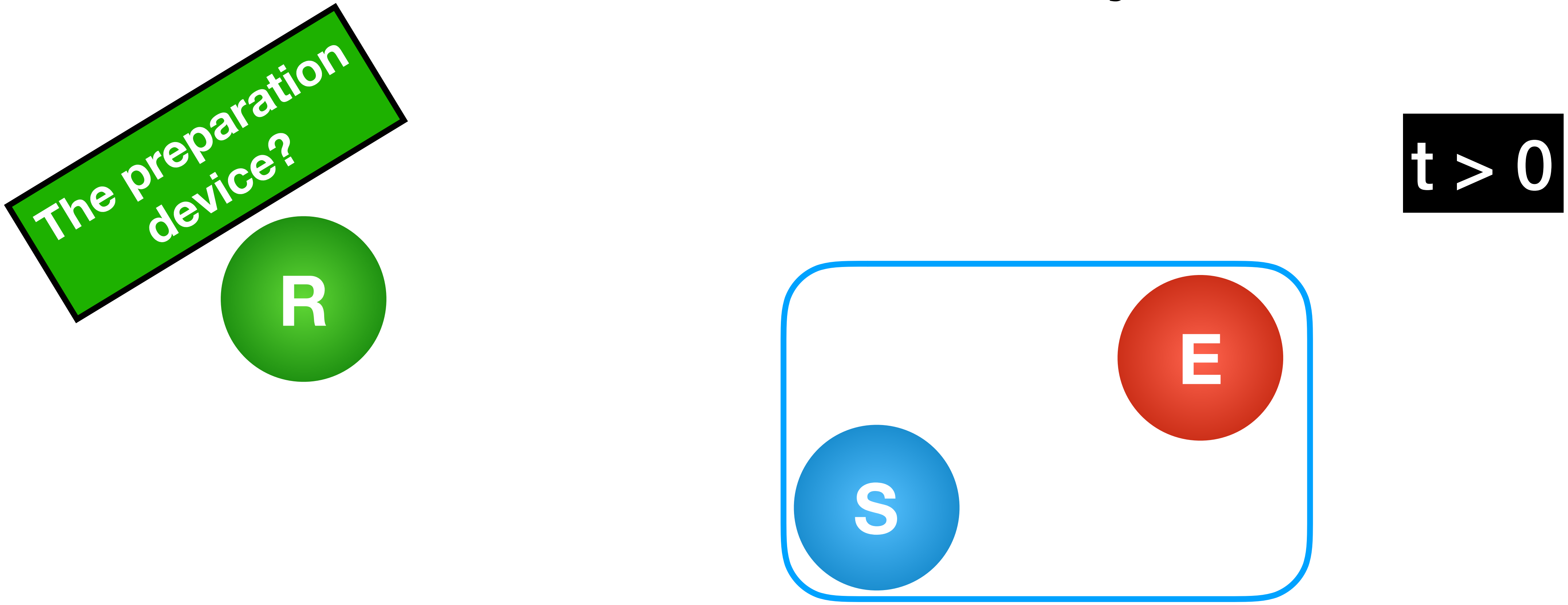


- The reference system can share prior entanglement with S and E or both.
- S and E can be entangled as well
- Is there a well defined map in this case?

The reference system



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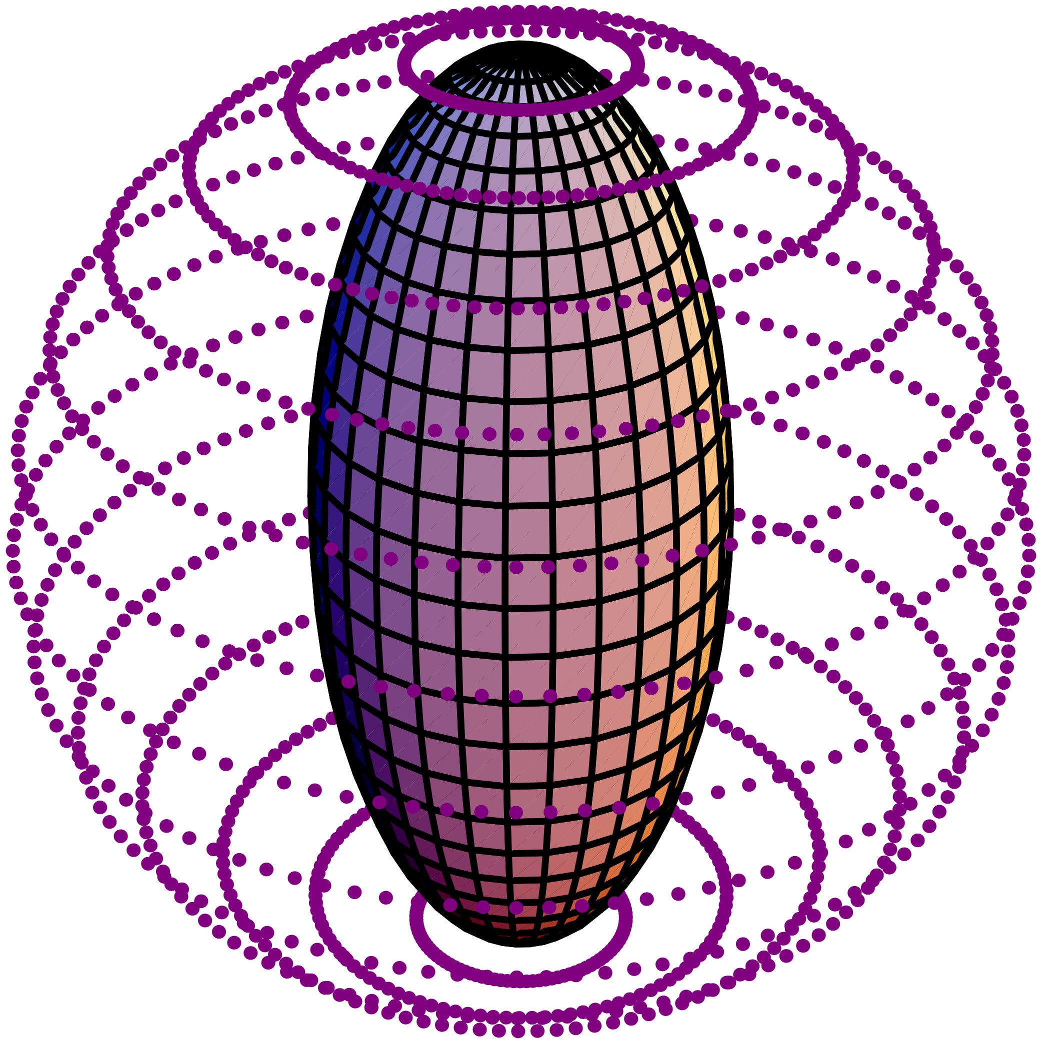
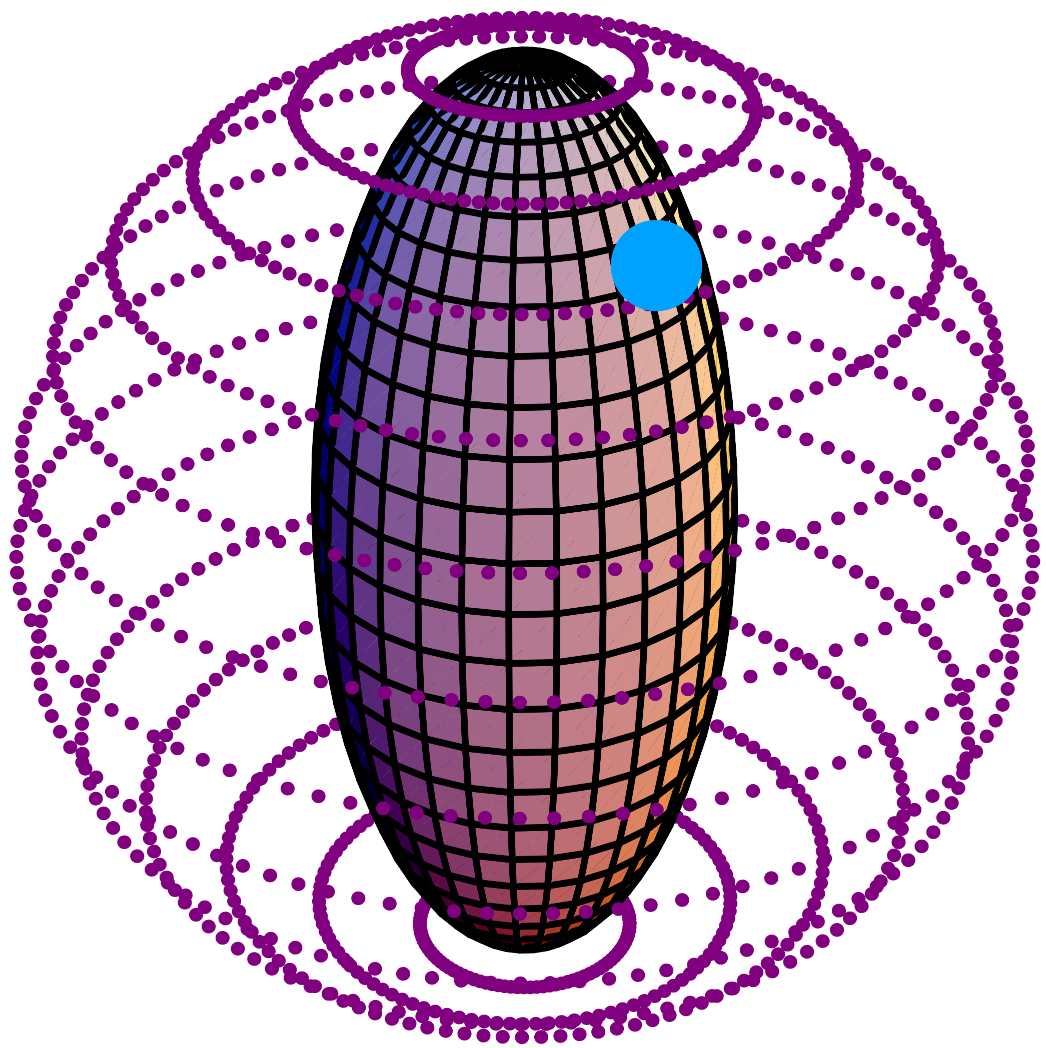
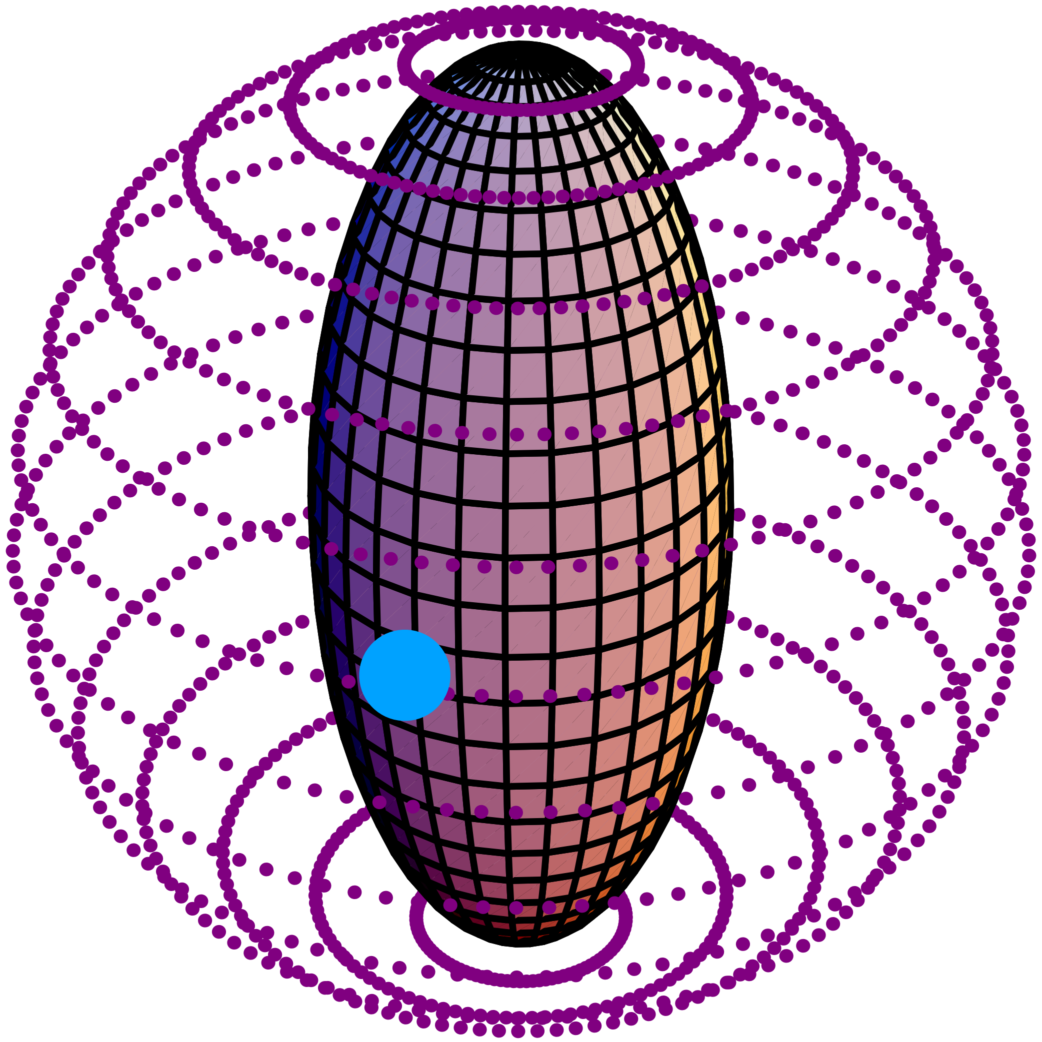
Point transformations and Maps



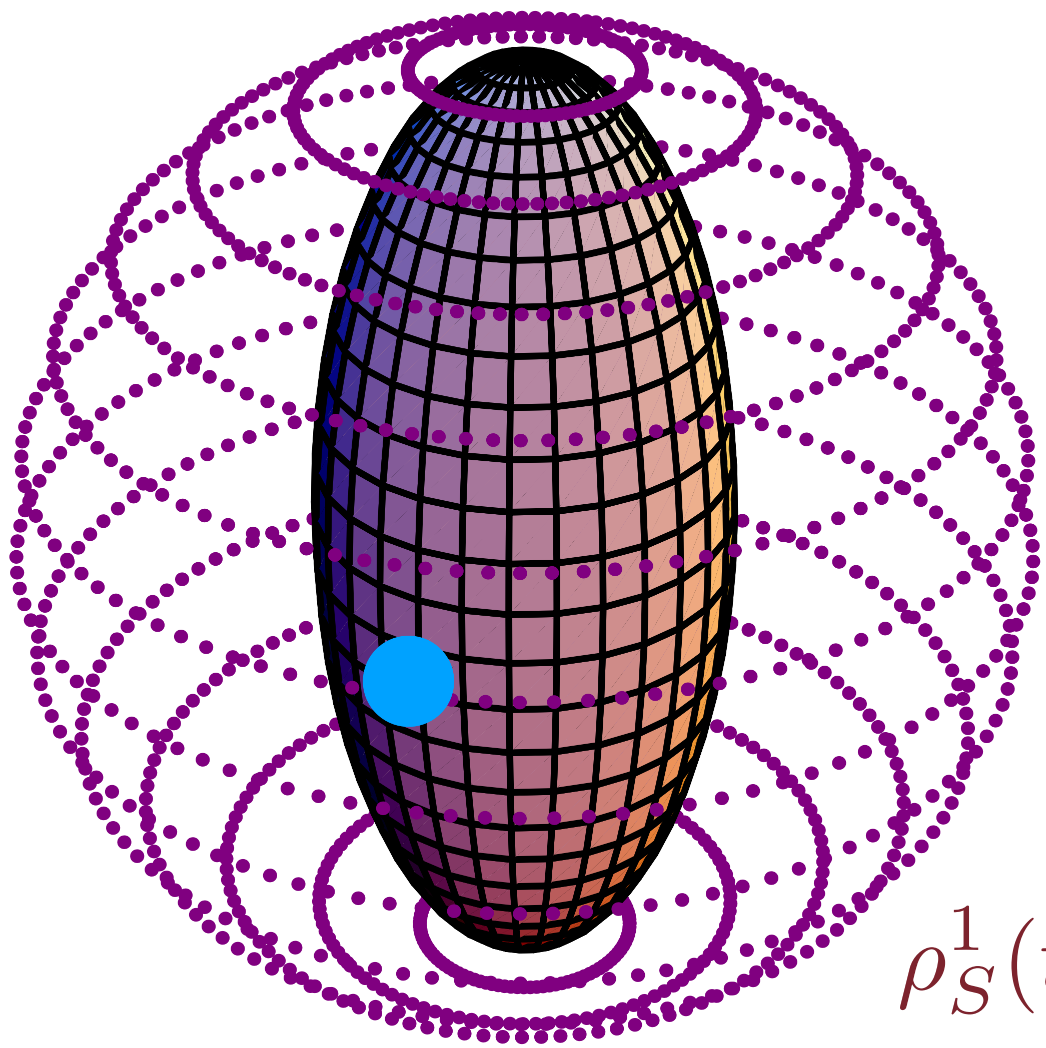
Point transformations and Maps



Point transformations and Maps



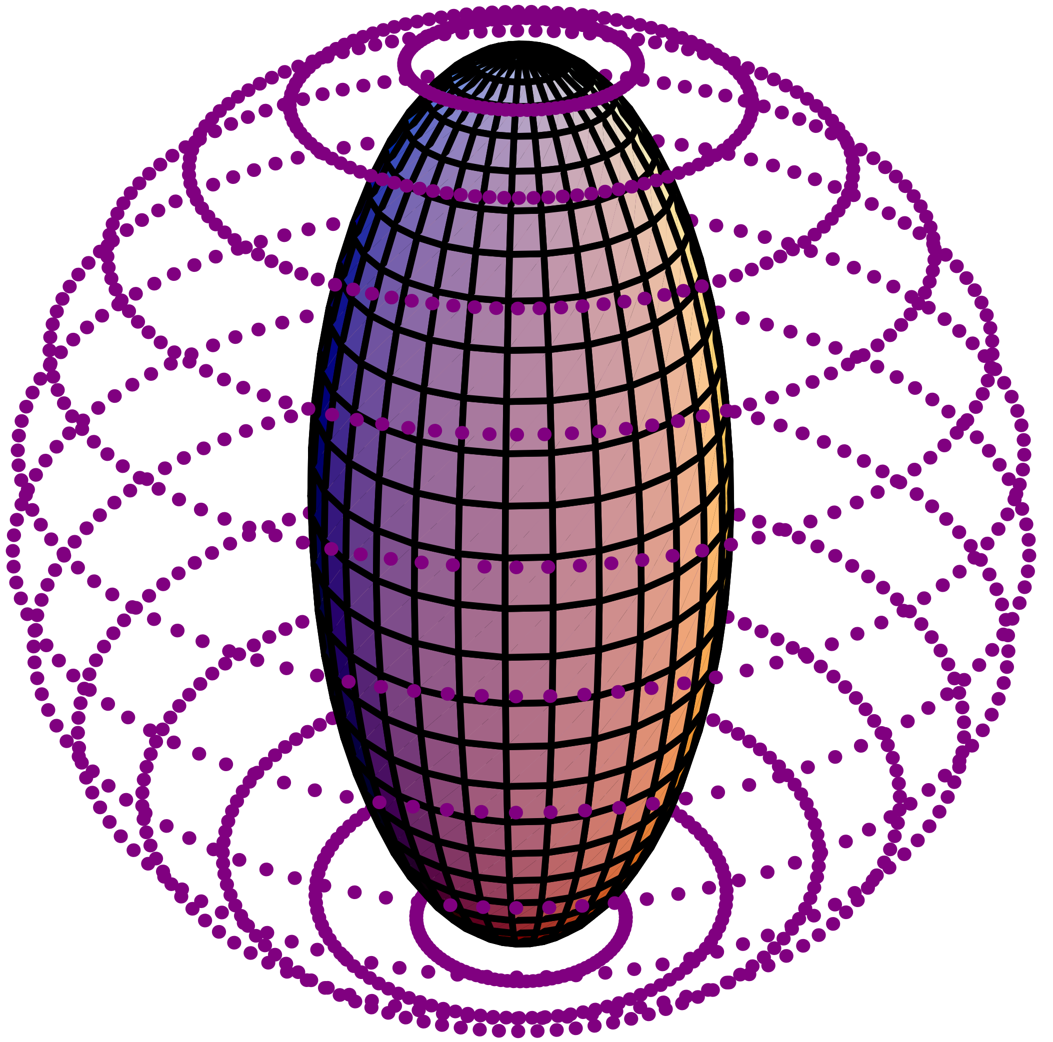
Point transformations and Maps



$$\rho_S^1(t) = \mathrm{tr}[U\rho_S^1 \otimes \rho_E U^\dagger]$$

Point Transformation

Point transformations and Maps



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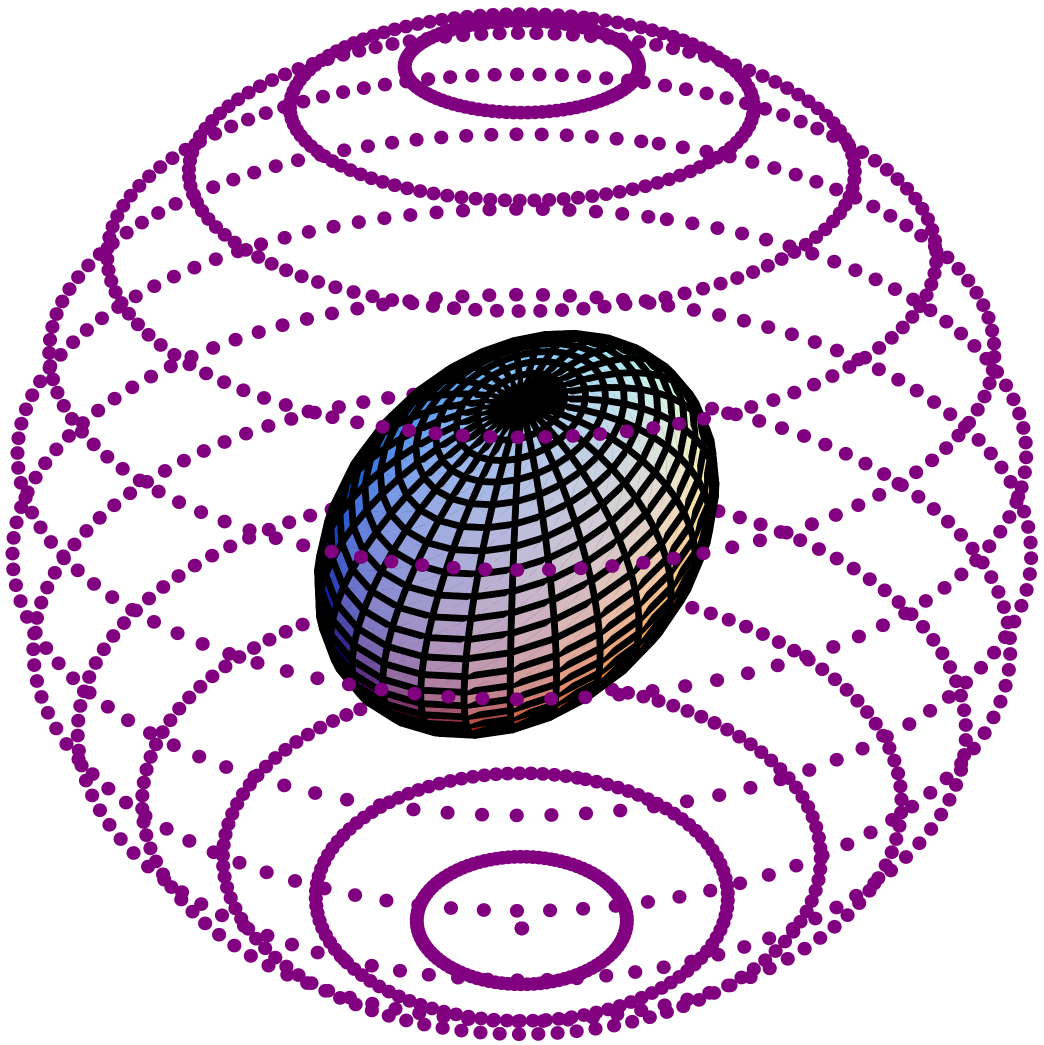
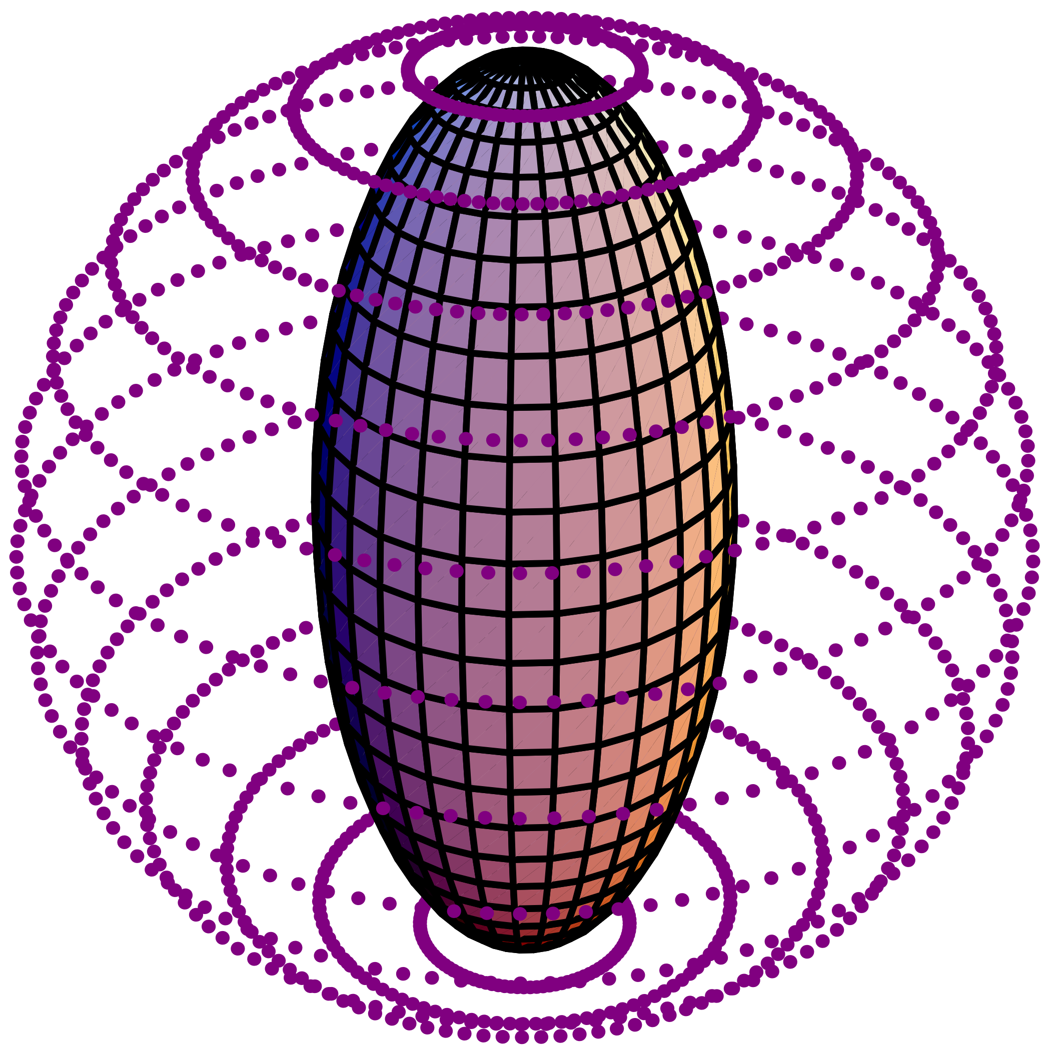


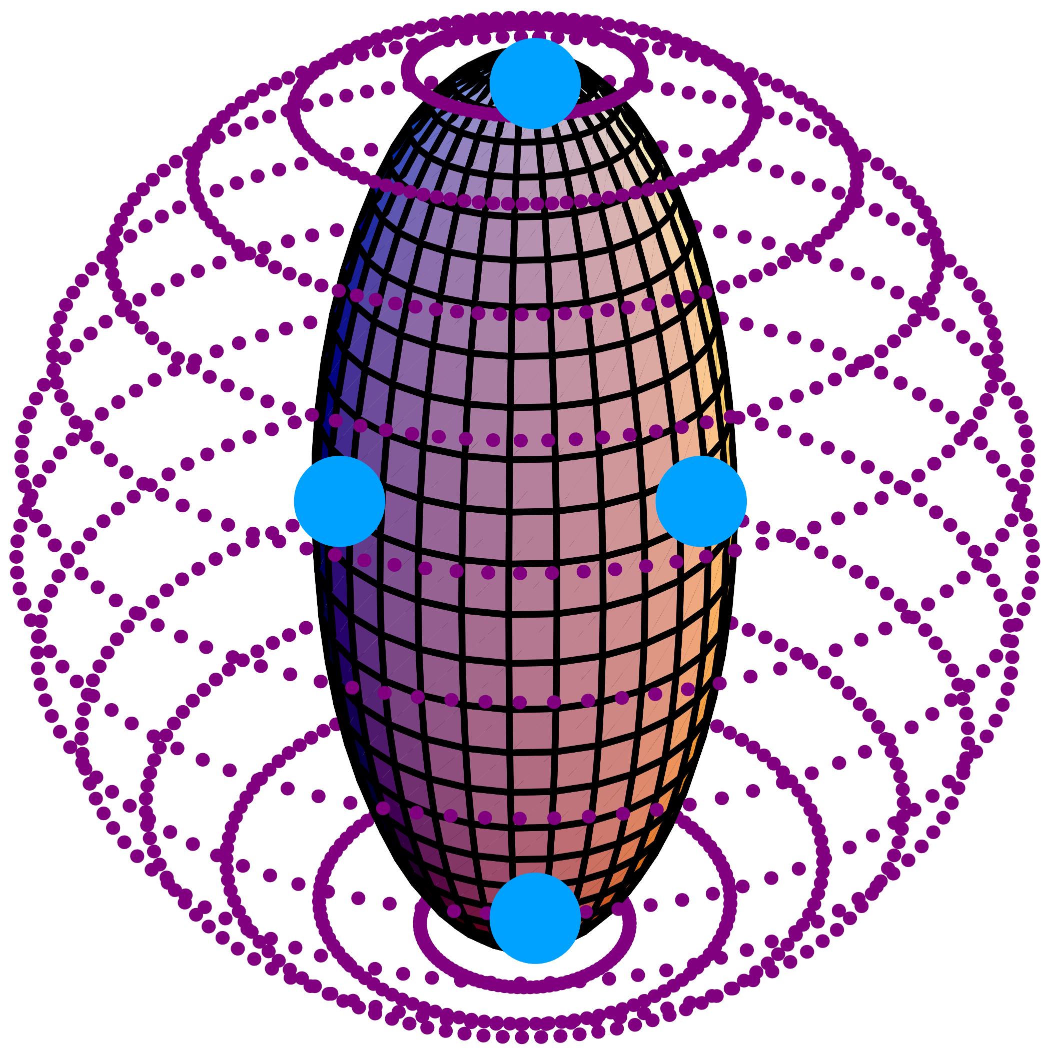
Map

Process Tomography Again



- Sufficient number of linearly independent initial states
- All states in **compact, dense set** undergoing the same transformation

Process Tomography Again



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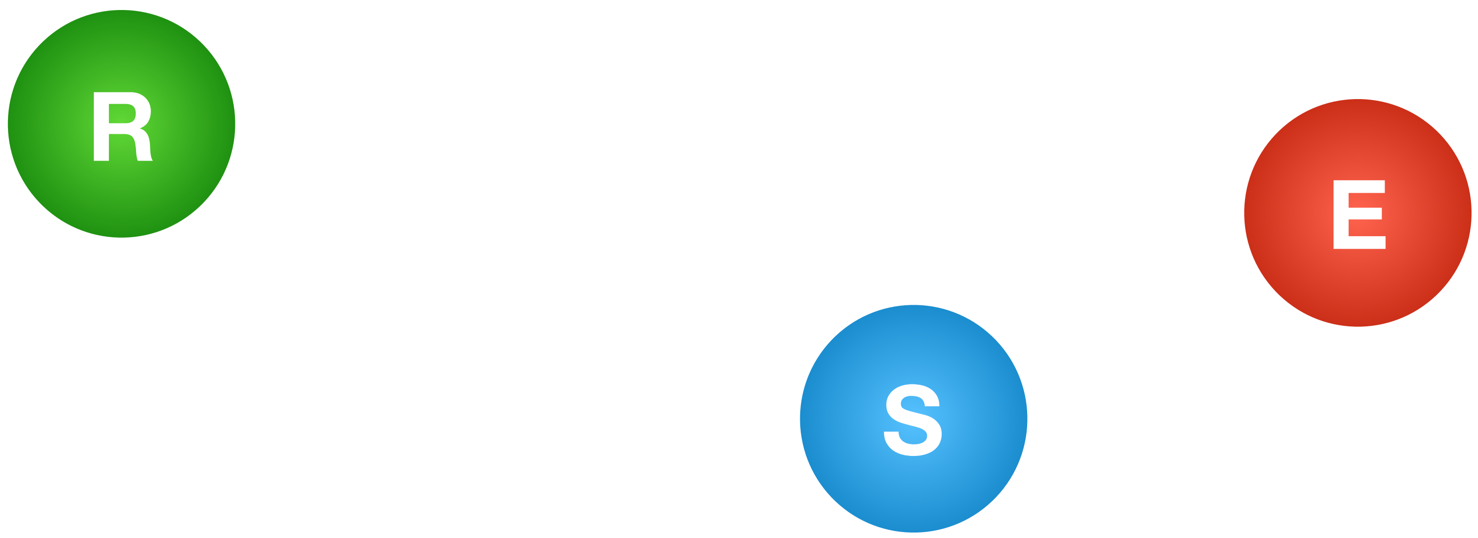
- All states in **compact, dense set** undergoing the same transformation

A natural set to define a map



A natural set to define a map

A natural set to define a map

- The steering set on SE induced by measurements on R

$$\mathcal{S}_{SE}(\rho_{RSE}) := \left\{ \frac{\mathrm{Tr}_{\mathrm{R}}[(\mathcal{P}_{\mathrm{R}} \otimes \mathbf{1}_{\mathrm{S}} \otimes \mathbf{1}_{\mathrm{E}})\rho_{\mathrm{RSE}}]}{\mathrm{Tr}[(\mathcal{P}_{R} \otimes \mathbf{1}_{S} \otimes \mathbf{1}_{E})\rho_{RSE}]} \right\}$$

Short Markov Chains and DPI

- Buscemi showed that if the initial RSE state forms a short Markov chain and if the SE, unitary dynamics satisfy the quantum data processing inequality, then the reduced dynamics on S is CP and defined on the reduced steering set of S.
- The reduced steering set is obtained by tracing out E from the steering set of SE due to R starting from a single state of the RSE tripartite system

F. Buscemi, Physical Review Letters 113, 140502 (2014)

Again restrictive assumptions

- The assumptions that RSE form a short Markov chain and the dynamics satisfy the quantum DPI are not physically mandated or justified.
- The reduced dynamics may be Not Completely Positive but still it is well defined on a dense set.
- The reduced steering set is the domain of action of the map
- Steering gives an aesthetically pleasing and comprehensive solution of the puzzle of how an appropriate state of E gets associated with each set of S so that every state of S in the domain gets acted upon by the same reduced dynamics

Three qubit example

$$
\begin{aligned}
\rho_{RSE} = \frac{1}{8}\big(&\mathbb{I}_R \otimes \mathbb{I}_S \otimes \mathbb{I}_E + a_i \sigma_R^i \otimes \mathbb{I}_S \otimes \mathbb{I}_E \\
&+ e_j \mathbb{I}_R \otimes \sigma_S^j \otimes \mathbb{I}_E + e_{3+k} \mathbb{I}_R \otimes \mathbb{I}_S \otimes \sigma_E^k \\
&+ e_{6+3(j-1)+k} \mathbb{I}_R \otimes \sigma_S^j \otimes \sigma_E^k \\
&+ T_{j,i} \sigma_R^i \otimes \sigma_S^j \otimes \mathbb{I}_E + T_{3+k,i} \sigma_R^i \otimes \mathbb{I}_S \otimes \sigma_E^k \\
&+ T_{6+3(j-1)+k,i} \sigma_R^i \otimes \sigma_S^j \otimes \sigma_E^k \big)
\end{aligned}
$$

The parameters of the three qubit state can be packaged as

$$
\Theta = \begin{pmatrix} 1 & \vec{a}^T \\ \vec{e} & T \end{pmatrix}
$$

Steering set on SE

$$\Theta = \begin{pmatrix} 1 & \vec{a}^T \\ \vec{e} & T \end{pmatrix} \qquad \hat{E} = X_\mu \sigma_R^\mu, \quad \mu = 1, \ldots, 4$$

$$\vec{e}^X = \Theta X$$

$$\rho_{SE}^X = \frac{1}{4}\big(\mathbb{I}_S \otimes \mathbb{I}_E + e_j^X \sigma_S^j \otimes \mathbb{I}_E + e_{3+k}^X \mathbb{I}_S \otimes \sigma_E^k + e_{6+3(j-1)+k}^X \sigma_S^j \otimes \sigma_E^k\big)$$

Reduced steering set of S

$$\rho_S^X = \frac{1}{2}\left[\mathbb{I}_S + \Theta_{j,\mu} X_\mu \sigma_S^j\right]$$

S. Jevtic, M. Pusey, D. Jennings, and T. Rudolph, Phys. Rev. Lett. **113**, 020402 (2014)

Joint unitary evolution of S and E

$$\rho_S^X = \frac{1}{2}\left[\mathbb{I}_S + \Theta_{j,\mu} X_\mu \sigma_S^j\right]$$

$$\rho_S^X \to \widetilde{\rho}_S^X = \frac{1}{2}\left[\mathbb{I}_S + \widetilde{\Theta}_{j,\mu} X_\mu \sigma_S^j\right]$$

$$\widetilde{\Theta}_{j,\mu} = u_{j0}^{l0}\Theta_{l,\mu} + u_{j0}^{0k}\Theta_{3+k,\mu} + u_{j0}^{lk}\Theta_{6+3(l-1)+k,\mu}$$

$$U\sigma_S^\mu \otimes \sigma_E^\nu U^\dagger = u_{\alpha,\beta}^{\mu,\nu}\sigma_S^\alpha \otimes \sigma_E^\beta$$

Linta Joseph and Anil Shaji, arXiv 1702.08718

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$$U\sigma_S^\mu \otimes \sigma_E^\nu U^\dagger = u_{\alpha,\beta}^{\mu,\nu}\sigma_S^\alpha \otimes \sigma_E^\beta$$

- The map is independent of X and so the same on all the states in the reduced steering set of S

Linta Joseph and Anil Shaji, arXiv 1702.08718

Discussion

- Approaching the problem using steering states provides a clear, elegant and aesthetically pleasing resolution to the question whether there can be a consistent mathematical definition of NCP dynamics with an unambiguous physical interpretation.

- a given unitary acting on SE steering set will induce the same transformation on all the states in the reduced set of S

- The set of states on which the transformation acts is dense and compact. Operationally this also means that one can reconstruct the map from observing the transformation occurring to a sufficient, finite number of linearly independent initial states of S

Discussion

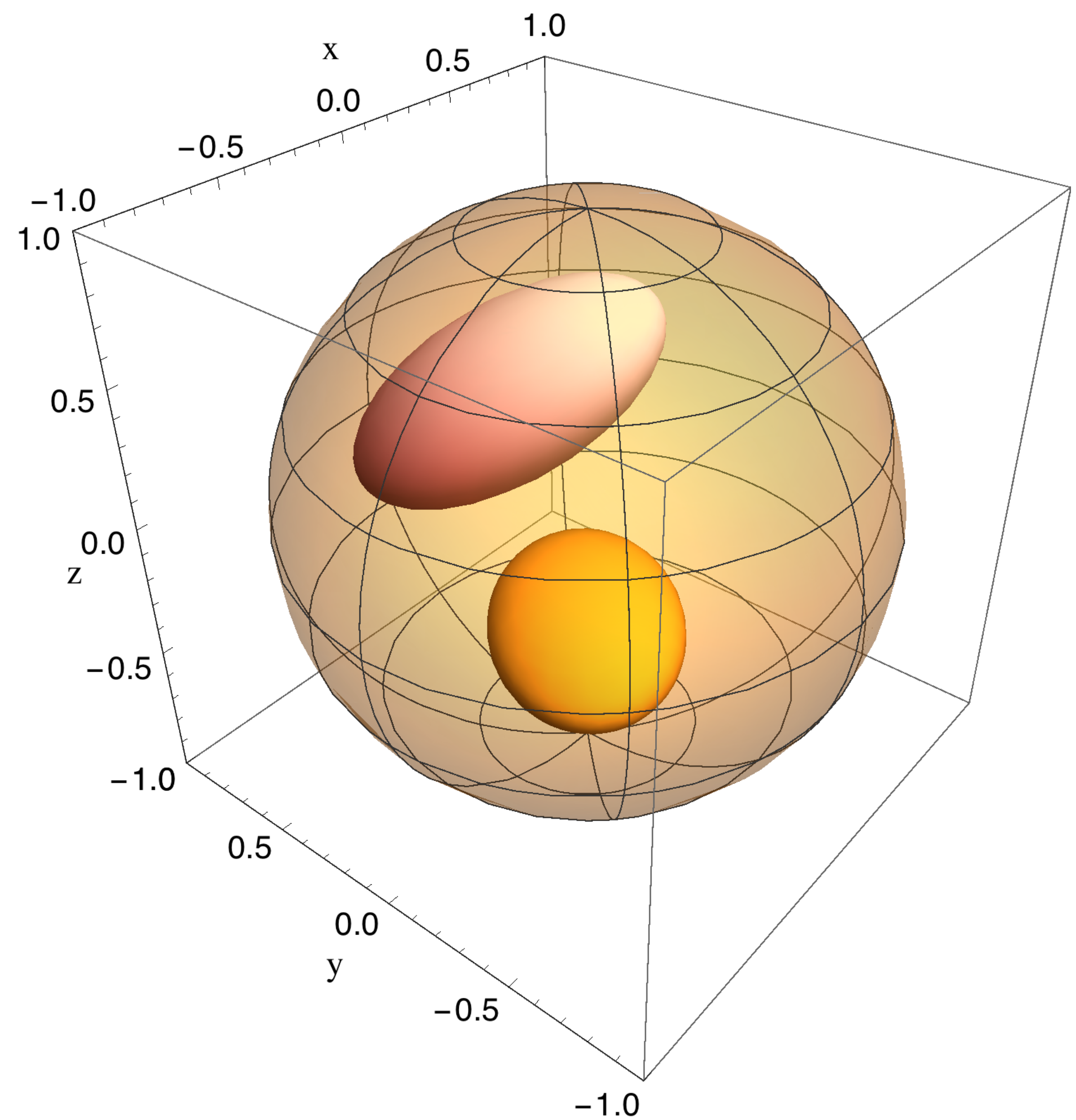
- Since the steering set of SE from which the domain of action of the maps follows is itself derived from a **single** state of RSE, there is no longer any mystery in having the initial state of the environment dependent on the state of S

- When S is a single quantum system and when we are concerned about its open dynamics over short time periods, its immediate environment is more often than not in itself microscopic or mesoscopic. The action of preparing an initial state of E not affecting the state of E in any way is an exceptional scenario in this case and in this sense so is CP reduced dynamics of the system.

Preparing devices

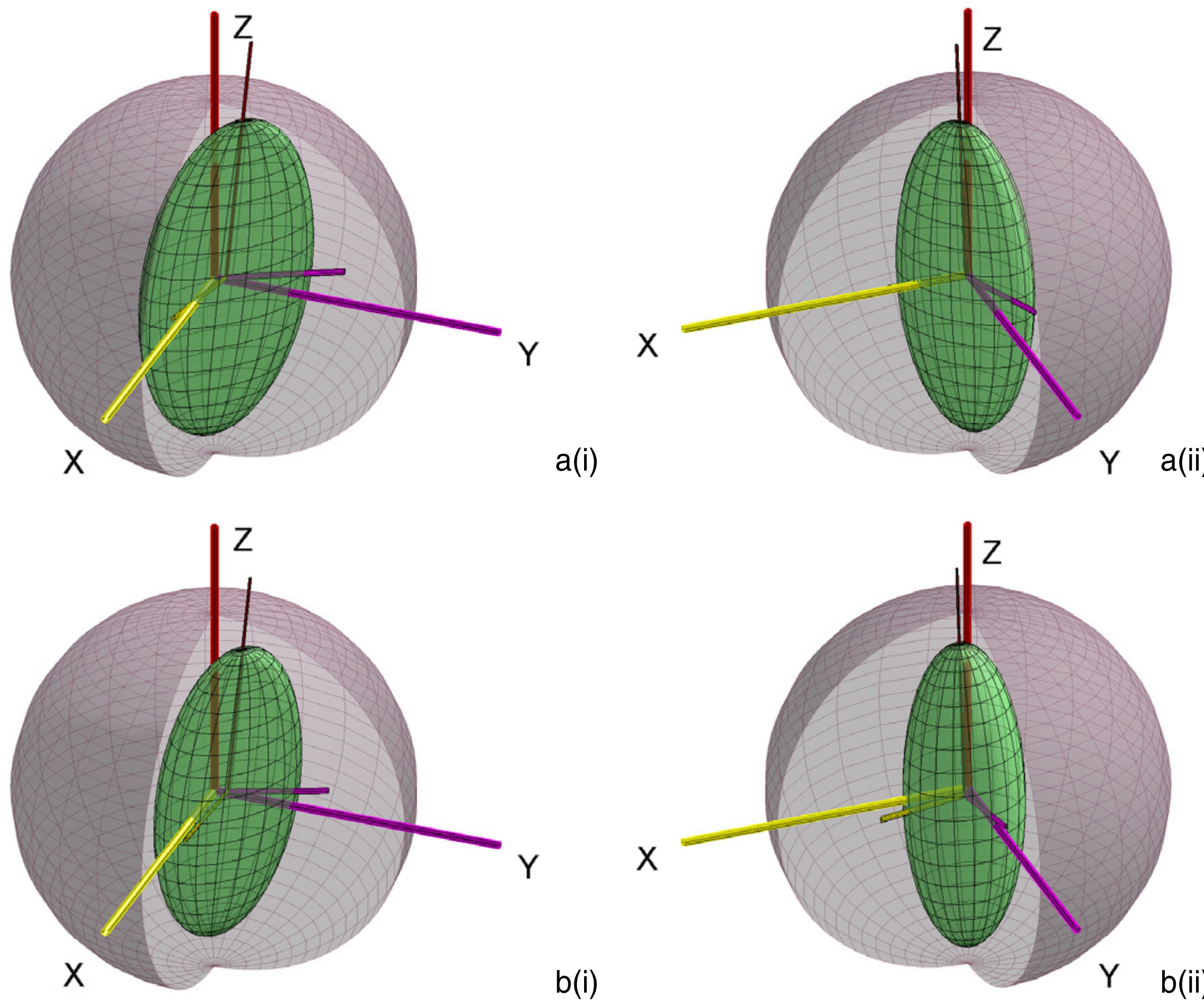
- An ideal preparation device does not interact with E. It can ‘steer’ S to any state in its state space and so there is no SE entanglement or nonClassical correlations. CP dynamics is obtained with the entire state space as its domain.
- Buscemi’s measuring device interacts with S but not with E. However SE can be entangled/correlated. Reduced dynamics is CP on the steering set if the quantum DPI holds.
- In general R can interact with S and E, and even during the process reconstruction, R can continue to interact with E. The reduced dynamics is not CP and the steering set is the domain.

The reduced steering set



The reduced steering set of S obtained from that of SE generated from a tripartite RSE state with RS and RE entanglements

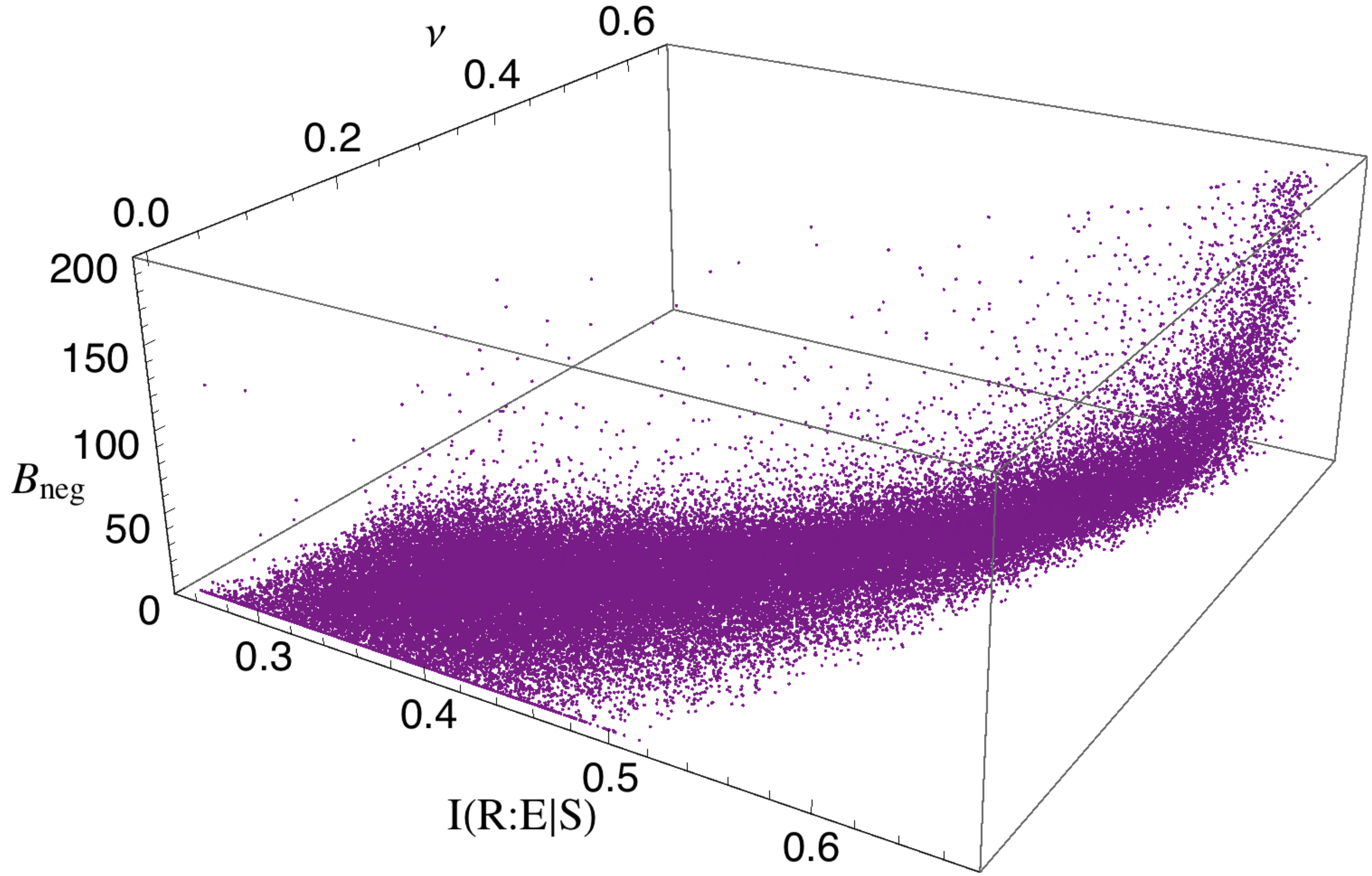
Simpler explanations



It is this property which we use to check the physicality of a process E; if the χ matrix reconstructed from experimental data has negative eigenvalue(s) then this indicates that noise and/or finitely sampled expectation values has caused the output data to infer an unphysical process. To overcome this problem, a physical matrix $\chi^{\sim}$ is found which is as close as possible to the original χ in some sense.

Howard, M., Twamley, J., Wittmann, C., Gaebel, T., Jelezko, F., & Wrachtrup, J. (2006). Quantum process tomography and Linblad estimation of a solid-state qubit. *New Journal of Physics*, ***8***(3), 33–33.

NCP and the quantum DPI



$$\nu = \max(0, I(R:S') - I(R:S))$$

$$B_{\text{neg}} = \sum_j |\lambda_j| - 2$$

Conclusion

- The reference system in the form of either a preparation device or the rest of the universe is a ubiquitous and unavoidable element in the analysis of any quantum process tomography experiment.
- It is not surprising that quantum process tomography experiments often lead to NCP dynamics
- The delocalized nature of quantum information gives the reference system also an important role in the open dynamics.

Thank You