Multi-partite entanglement can speed up quantum key distribution in networks

M. Epping, H. Kampermann, C. Macchiavello, and *D. Bruß*

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Outline

- Entangled-based quantum key distribution (QKD)

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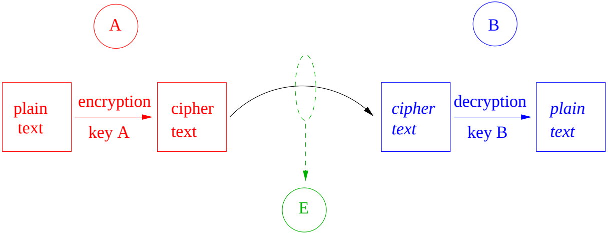
Quantum key distribution (QKD)



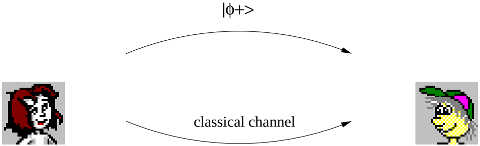
Vernam cipher ≡ "one-time pad" (1917):
Encoding with secret random key (only known to Alice and Bob, not to Eve). Proven to be secure.

How to establish secret random key?
↪ quantum cryptography ≡ quantum key distribution (QKD)

Entanglement-based QKD (between two parties)

A. Ekert, Phys. Rev. Lett. **67**, *661 (1991)*

Aim: secret random key for Alice and Bob



1) A sends half of a Bell state to Bob: $|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$
   A and B measure, use 2 bases randomly: $\uparrow\!\!\!\rightarrow$ or $\searrow\!\nearrow$
2) A and B exchange class. info about basis,
   keep matching cases: $1 \quad r \quad 0 \quad 0 \quad 1 \quad r \quad 0 \quad r$

$\hookrightarrow$ Alice and Bob have established secret random key!

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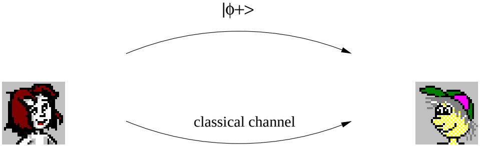

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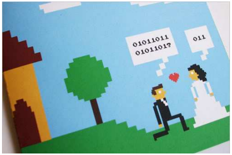
Security: monogamy of entanglement

Monogamy of entanglement

V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, *052306 (2000)*



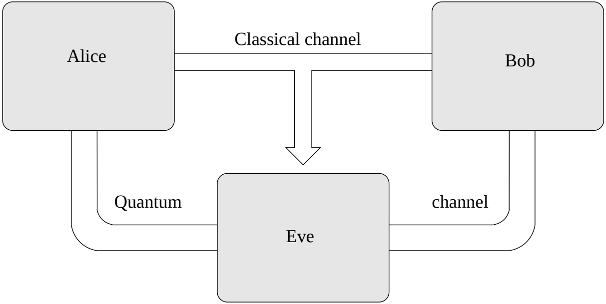
Impossible!

Possible

$$E(B|A) + E(B|C) \leq E(B|AC)$$

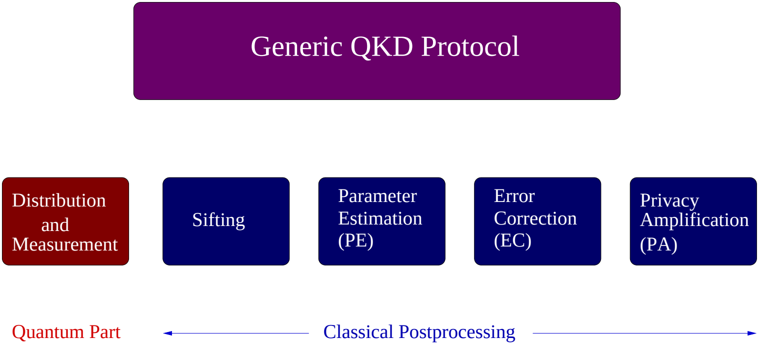
QKD in reality: noisy entangled state, $\rho = p|\phi^+\rangle\langle\phi^+| + (1-p)\frac{1}{4}\mathbb{1}$,
assume Eve to have purifying state (is partially correlated with A/B)
$\hookrightarrow$ security analysis

Quantum Key Distribution (QKD)



- Scenario: Alice und Bob have quantum channel (controlled by Eve) and classical channel (authenticated)
- Secure communication ⇔ Creation of a secret random key pair between Alice and Bob
- No restrictions on Eve

QKD: General description of a QKD protocol



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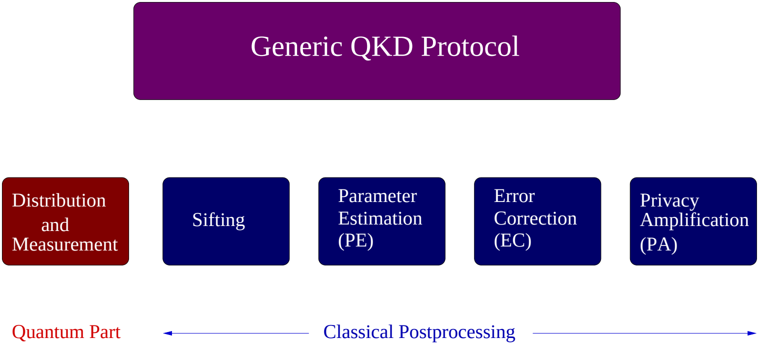


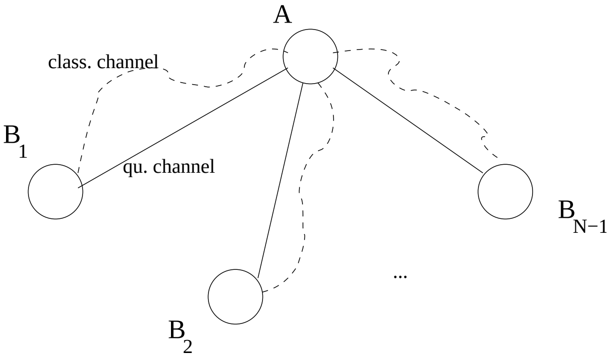
Equivalence of prepare+measure QKD with entanglement-based QKD

$\hookrightarrow$ In the following: use entanglement-based scheme

Generalisation of QKD to more than two parties

M. Epping, H. Kampermann, C. Macchiavello, and DB, New J. Phys. **19**, *093012 (2017)*

Aim: establish joint secret random key between N parties,
i.e. "conference key"



Establishing a conference key: Two possibilities

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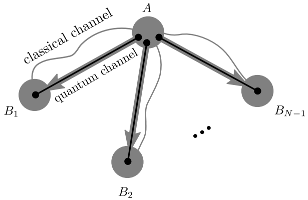
Using bipartite entanglement (2QKD):



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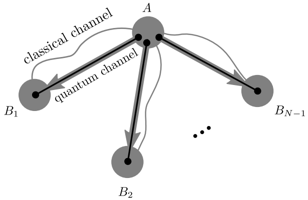
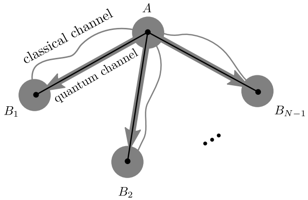


... or using multipartite entanglement (NQKD):



Multipartite entanglement

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Multipartite entanglement of composite (pure) states of N parties:

$$|\psi\rangle = |a\rangle_{1,\ldots,k} \otimes |b\rangle_{k+1,\ldots,N} \quad \hookrightarrow \quad \text{separable across bipartite split}$$
$$|\psi\rangle \neq |a\rangle_{1,\ldots,k} \otimes |b\rangle_{k+1,\ldots,N} \quad \hookrightarrow \quad \text{multipartite entangled}$$

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Example (separable): $|\psi\rangle = |0\rangle|0\rangle...|0\rangle$

Example (entangled): GHZ states of N qubits

$$|\psi_j^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|j\rangle \pm |1\rangle|\bar{j}\rangle)$$

where j takes values $0, ..., 2^{N-1} - 1$ in binary notation;
$\bar{j}$ is negation of j, e.g. if $j = 010$ then $\bar{j} = 101$

Multipartite entanglement for QKD

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Theorem (Perfect resource state for multipartite QKD)

For N qubits, with $N \geq 3$, the state
$|\phi_{corr}\rangle = a_{0,...,0}|0,...,0\rangle + a_{1,...,1}|1,...,1\rangle$ with $|\, a_{0,...,0}\,|^2 + |\, a_{1,...,1}\,|^2 = 1$
leads to perfect classical correlations between any number of parties, if and only if each of them measures in the z-basis.

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Proof: "$\Leftarrow$" clear;
"$\Rightarrow$": observable $\mathcal{M}_{ij}$ of two parties i and j:

$$\mathcal{M}_{ij} = (\vec{M}_i \cdot \vec{\sigma}) \otimes (\vec{M}_j \cdot \vec{\sigma}) = \sum_{\alpha,\beta \in \{x,y,z\}} M_i^\alpha M_j^\beta \sigma_i^\alpha \otimes \sigma_j^\beta,$$

$$\langle \phi_{corr}|\sigma_i^\alpha \otimes \sigma_j^\beta|\phi_{corr}\rangle = 0 \;\; \text{unless} \;\; \alpha = \beta = z,$$

also $\langle \phi_{corr}|\sigma_i^\alpha \otimes \sigma_j^\beta|\phi_{corr}\rangle = 2[p_i^\alpha(+)p_j^\beta(+) + p_i^\alpha(-)p_j^\beta(-)] - 1$,
thus $p_i^\alpha(+)p_j^\beta(+) + p_i^\alpha(-)p_j^\beta(-) \neq 1$, unless $\alpha = \beta = z$.

Multipartite QKD protocol

If one requires perfect correlations and uniformity of key,
the *only* possible resource state is $|GHZ\rangle = \frac{1}{\sqrt{2}}(|0,...,0\rangle + |1,...,1\rangle)$.

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4) *Classical post-processing:* As in the bipartite protocol, error correction and privacy amplification is performed.

Secret key rate for NQKD

Security analysis:

- Analogous to bipartite case, with modifications in worst-case error correction and depolarisation

  R. Renner, N. Gisin, and B. Kraus, Phys. Rev. A 72, 012332 (2005)

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- Figure of merit: secret fraction,
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$$r_\infty = \sup_{U \leftarrow K} \inf_{\sigma_{A\{B_i\}} \in \Gamma} [S(U|E) - \max_{i \in \{1,\ldots N-1\}} H(U|K_i)],$$

with $U \leftarrow K$: bitwise preprocessing channel on A's raw key bit K,
$S(U|E)$: conditional von-Neumann entropy of (class.) key variable and E,
$H(U|K_i)$: conditional Shannon entropy of U and B_i's guess of it,

Γ: set of all density matrices $\sigma_{A\{B_i\}}$ of A and B_i consistent with parameter estimation

Secret key rate: $R = r_\infty R_{\text{rep}}$ with repetition rate R_{rep}

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Introduce (extended) depolarisation procedure, $\hookrightarrow$ GHZ-diagonal state
$\hookrightarrow$ calculate secret fraction r_∞:

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$$
\begin{aligned}
r_\infty = \quad & \left(1 - \frac{Q_Z}{2} - Q_X\right) \log_2 \left(1 - \frac{Q_Z}{2} - Q_X\right) \\
& + \left(Q_X - \frac{Q_Z}{2}\right) \log_2 \left(Q_X - \frac{Q_Z}{2}\right) \\
& + (1 - Q_Z)(1 - \log_2(1 - Q_Z)) - h(\max_{1 \le i \le N-1} Q_{AB_i})
\end{aligned}
$$

with Q_Z: probability that at least one B_i obtains different result than A in z-measurement,
with Q_X: probability that at least one B_i obtains in x-measurement a result that is incompatible with noiseless state,
binary entropy: $h(p) = -p \log_2 p - (1-p) \log_2 (1-p)$,

Q_{AB_i}: probability that z-measurements of A and B_i disagree.

Example for explicit key rates

Noise model: mixture of GHZ-state and white noise (then $Q = Q_z$)

$$r_\infty(Q,N) = 1 + h(Q) - h\left(Q\frac{2^N-1}{2^N-2}\right) - h\left(Q\frac{2^{N-1}}{2^N-2}\right) + \left(\log_2(2^{N-1}-1) - \frac{2^N-1}{2^N-2}\log_2(2^N-1)\right)Q,$$

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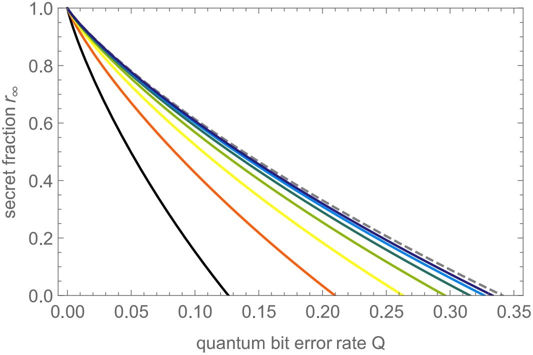

Key rates for $N = 2, 3, ..., 8$, from left to right.

Secret key rate as function of gate failure probability

Consider imperfect state preparation (depolarising noise): experimental creation of GHZ-state is more demanding with higher N!

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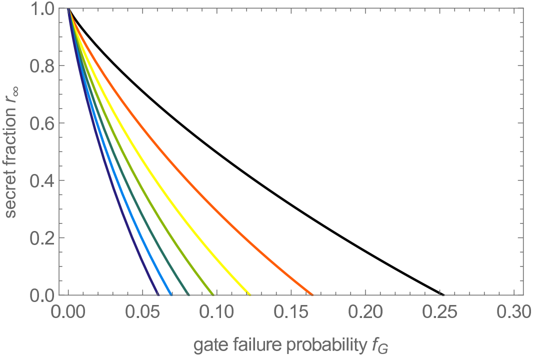
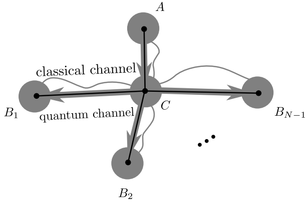


Key rates for $N = 2, 3, ..., 8$, from right to left.

Advantage of NQKD in quantum networks

Consider quantum networks with routers (can produce and entangle qubits), fixed channel capacity:



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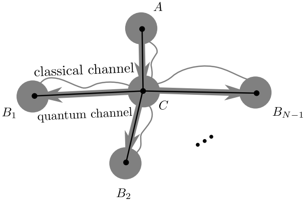
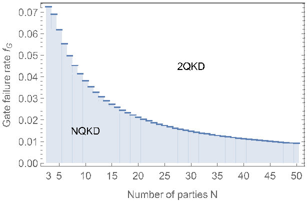


For small gate failure probability: NQKD is better than 2QKD!



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Processing of data at intermediate network nodes can improve throughput and increase robustness of quantum network with bottleneck.

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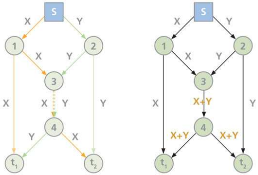

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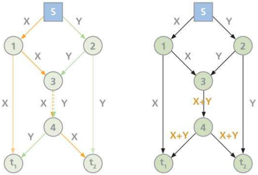

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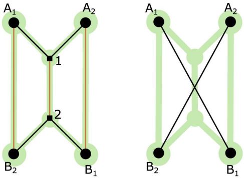

(a)Pre-measurement state. (b)Post-measurement state.

M. Epping, H. Kampermann, and DB, New J. Phys. **18**, *103052 (2016)*

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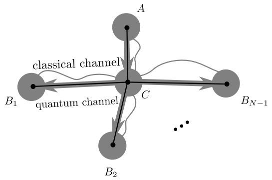
Distribution of GHZ-state in above network, with quantum operations at node C (router), and fixed channel capacities for all links:



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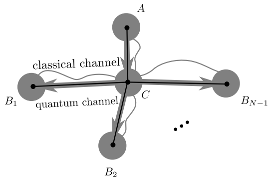

- A produces Bell state and sends only one qubit C to router: $|\text{—}\rangle_{CA} = \frac{1}{\sqrt{2}}(|0+\rangle + |1-\rangle)_{CA}$
- C produces $(N-1)$ qubits and entangles them with C via C_z gates: $|\psi_{\text{total}}\rangle = \frac{1}{\sqrt{2}}(|+\rangle_C |GHZ'\rangle_{AB_i} + |-\rangle_C X_{B_1} |GHZ'\rangle_{AB_i})$ where $|GHZ'\rangle$ is GHZ-state in X-basis.
- Router measures qubit C in X-basis and distributes qubits to B_i.
- Impossible to create $(N-1)$ Bell pairs by sending single qubit from A to router; need $(N-1)$ network uses.

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Summary and open questions

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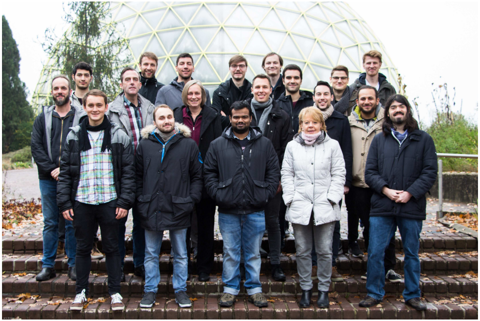
Quantum Information Theory in Düsseldorf

Institut für Theoretische Physik III, Heinrich-Heine-Universität Düsseldorf, Germany



from left to right: J. Bremer, J. M. Henning, D. Miller, H. Kampermann, T. Holz, G. Gianfelici, M. Zibull, DB, T. Backhausen, S. Datta, F. Bischof, T. Wagner, C. Liorni, C. Glowacki, F. Grasselli, C. Hoffmeister, B. Sanvee, L. Tendick, M. Battiato

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