The effect of entanglement and correlations on tripartite two-level dipole coupled atoms

Quantum Optics and Quantum Information Processing

Prasanta K. Panigrahi

Professor
Department of Physical Sciences
IISER Kolkata

January 29, 2018

Plan of The Talk

1. Intensity and radiation statistics of three two-level atoms
    - The Model
    - The intensity characteristics of light emitted by three atoms in a Line-configuration
    - Photon Statistics: Line Configuration
    - The intensity characteristics of light emitted by three atoms in a Loop-configuration
    - Photon Statistics: Loop Configuration
2. New frontiers on quantum correlations: Timelike curves can increase entanglement with LOCC

The Model

The Hamiltonian for the system of three identical two-level atoms coupled through dipole-dipole interaction is given by

$$H = \omega \sum_{i=1}^{3} S_i^z + \sum_{i \neq j=1}^{3} \Omega_{ij}(S_i^+ S_j^- + H.C), \qquad (1)$$

where Ω_{ij}, the dipole dipole interaction strength, is a function of the inter-atomic separation 'd'. In the above, ω is the atomic transition frequency, $S_i^+ = |1\rangle_i\langle 0|$ and $S_i^- = |0\rangle_i\langle 1|$ are the raising and lowering operators of the i^{th} atom.

The Model

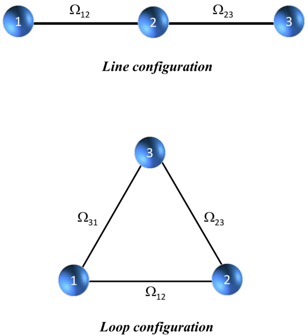


Figure: Schematic representation of Line(open loop) and Loop (closed loop) configurations for three qubits coupled via dipole-dipole interaction.

The intensity characteristics of light emitted by three atoms in a line configuration

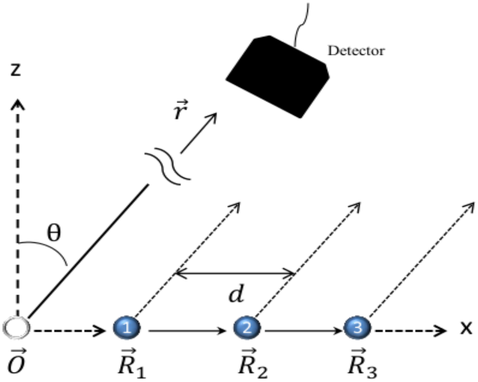


Figure: Schematic diagram of the system: where three identical two-level atoms are localized at positions $\bar{R}_1$ to $\bar{R}_3$. A detector at position $\bar{r}$ records a photon scattered by the atoms, in the far field regime.

The int. charac. ..... line configuration contd...

The electric field operator is given as

$$\hat{E}^{(+)} = -\frac{e^{ikr}}{r}\Sigma_i \vec{n}x(\vec{n}x\vec{p}_{ge})e^{-i\phi_j}\hat{S}_j^- \quad (2)$$

where $\vec{p}_{ge}$ is the dipole moment of the transition and ϕ_j is

$$\phi_j(\vec{r}) = \vec{n}.\vec{r} = jkdsin(\theta). \quad (3)$$

We also assume $\vec{p}_{ge}$ to be along the y direction and $\vec{n}$ in the x-z plane, so that $\vec{p}_{ge}.\vec{n} = 0$.

The int. charac. ..... line configuration contd...

These assumptions give rise to dimensionless expressions for the amplitudes and hence intensities, resulting in the following expression for the radiated intensity at $\vec{r}$

$$I(\vec{r}) = \sum_{i,j} \langle \hat{S}_i^+ \hat{S}_j^- \rangle e^{i(\phi_i - \phi_j)}$$

$$= \sum_i \langle \hat{S}_i^+ \hat{S}_i^- \rangle + \left(\sum_{i \neq j} \langle \hat{S}_i^+ \rangle \langle \hat{S}_j^- \rangle + \sum_{i \neq j} (\langle \hat{S}_i^+ \hat{S}_j^- \rangle - \langle \hat{S}_i^+ \rangle \langle \hat{S}_j^- \rangle) \right) e^{i(\phi_i - \phi_j)} \quad (4)$$

Thus, the characteristics of the intensity would depend on the incoherent terms $\langle \hat{S}_i^+ \hat{S}_i^- \rangle$, the non vanishing of the dipole moments $\langle \hat{S}_i^+ \rangle$ and the quantum correlations like $\langle \hat{S}_i^+ \hat{S}_j^- \rangle - \langle \hat{S}_i^+ \rangle \langle \hat{S}_j^- \rangle$.

The int. charac. ..... line configuration contd...

The one-atom excited states give rise to one class of W states while the two-atom excited states give rise to a second class of W states. One of the possible W state for the two-atom excitation is given by,

$$|W_{2,1}\rangle = \frac{1}{2}\left[|110\rangle + |011\rangle + \sqrt{2}|101\rangle\right], \quad \lambda_1 = \sqrt{2}g + \frac{\omega}{2} \tag{5}$$

where λ_1 is the eigenvalue. The corresponding intensity pattern can be exactly calculated,

$$I_{|W_{2,1}\rangle} = 2 + \frac{1}{2}\left[\cos(\phi_1 - \phi_3) + \sqrt{2}\{\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3)\}\right] \tag{6}$$

$$[I_{|W_{2,1}\rangle}]^{Max} = 3.914,$$

for $\theta = 0, \pi$.

The int. charac. ..... line configuration contd...

Another type of anti-symmetric W state for the two atom in the excited state is obtained as,

$$|\overline{W}_{2,1}\rangle = \frac{1}{2}\left[|110\rangle + |011\rangle - \sqrt{2}|101\rangle\right], \quad \lambda_2 = \frac{\omega}{2} - \sqrt{2}g, \qquad (7)$$

and the corresponding intensity is given by

$$I_{|\overline{W}_{2,1}\rangle} = 2 + \frac{1}{2}\left[\cos(\phi_1 - \phi_3) - \sqrt{2}\{\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3)\}\right] \qquad (8)$$

The intensity minima occurs at

$$[I_{|\overline{W}_{2,1}\rangle}]^{Min} = 1.086,$$

when $\theta = 0, \pi$.

The int. charac. ..... line configuration contd...

A second type of anti - symmetric state, of the GHZ type, with two atoms in excited state,

$$|\tilde{W}_{2,1}\rangle = \frac{1}{\sqrt{2}}\left[|011\rangle - |110\rangle\right], \quad \lambda_3 = \frac{\omega}{2} \tag{9}$$

has the intensity form

$$|I_{|\tilde{W}_{2,1}\rangle} = 2 - \cos(\phi_1 - \phi_3). \tag{10}$$

The int. charac. ..... line configuration contd...

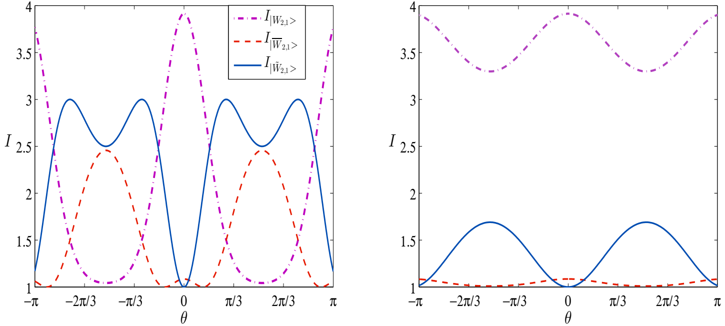


Figure: Line configuration: Intensity of the initial state $|W_{2,1}\rangle$(dot-dashed line), anti-symmetric state $|\overline{W}_{2,1}\rangle$(dashed line) and GHZ state $|\tilde{W}_{2,1}\rangle$(solid line) as a function of the observation angle θ for (a) weak dipole-dipole interaction strength ($\Omega_{12} = \Omega_{23} = 0.29\gamma$, for $d = \frac{\lambda}{3}$) and (b) strong dipole-dipole interaction strength ($\Omega_{12} = \Omega_{23} = 2.6\gamma$, for $d = \frac{\lambda}{10}$).

The int. charac. ..... line configuration contd...

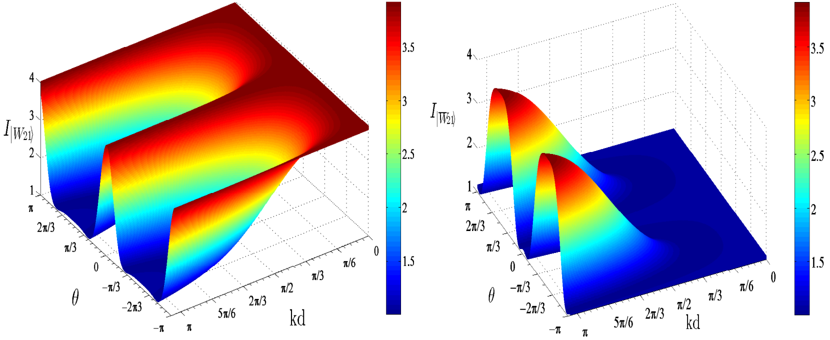


Figure: Surface Plot of Intensity $I_{|W_{2,1}\rangle}$ (left) and $I_{|\overline{W}_{2,1}\rangle}$ (right) as a function of observation angle θ and interatomic distance kd

Photon Statistics: Line Configuration

The second-order (intensity-intensity) correlation function of the radiation field, with zero time lag, is defined as

$$g^{(2)}(0) = \frac{\langle E^{-}E^{-}E^{+}E^{+}\rangle}{\langle E^{-}E^{+}\rangle\langle E^{-}E^{+}\rangle} \tag{11}$$

Violation of classical inequalities of this second-order correlation function points to non-classical character.

Photon Statistics: Line Configuration

The second-order correlation function of the radiation field, emitted by three atoms, which are initially in the $|W_{21}>$ state is obtained as

$$g^{(2)}(0) = \frac{4 + 2[\cos(\phi_3 - \phi_1) + \sqrt{2}(\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3))]}{\left[2 + \frac{1}{2}\{\cos(\phi_3 - \phi_1) + \sqrt{2}[\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3)]\}\right]^2} \tag{12}$$

The corresponding expression for the intensity correlation, when the two atoms are initially in $|\overline{W}_{2,1}\rangle$, denoted here by $\bar{g}^{(2)}(0)$, is given as

$$\bar{g}^{(2)}(0) = \frac{4 + 2[\cos(\phi_3 - \phi_1) - \sqrt{2}(\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3))]}{\left[2 + \frac{1}{2}\{\cos(\phi_3 - \phi_1) - \sqrt{2}(\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3))\}\right]^2} \tag{13}$$

Photon Statistics: Line Configuration

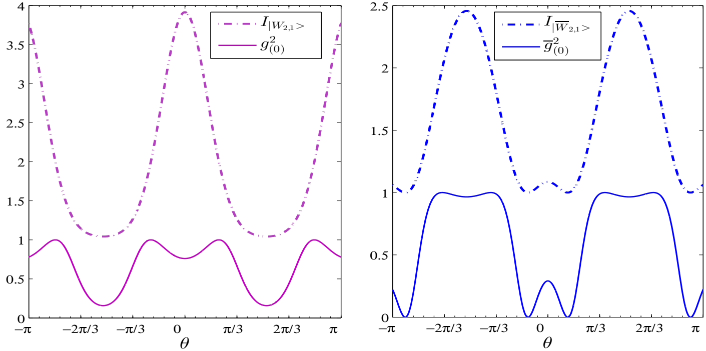


Figure: Line configuration: The intensity (dot-dashed line) and the second order correlation function of radiation field (solid line), emitted by two atoms which are initially in (a) the $|W_{2,1}\rangle$ state and (b)$|\overline{W}_{2,1}\rangle$ as a function of observation angle θ for $kd = \frac{2\pi}{10}$.

Photon Statistics: Line Configuration

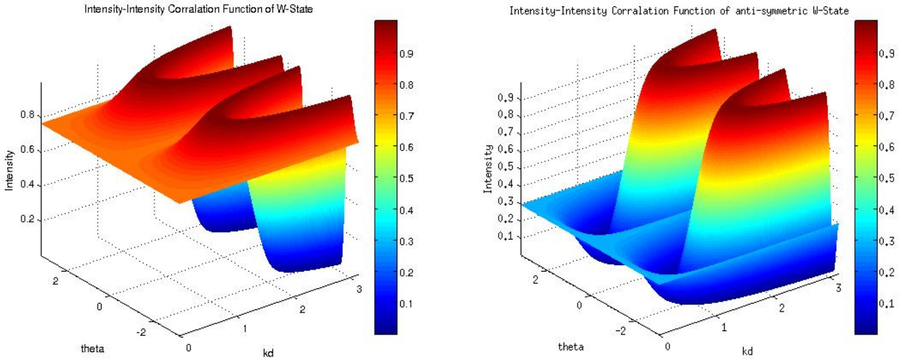


Figure: Surface Plot of Intensity-Intensity correlation $I_{|W_{2,1}\rangle}$ (left) and $I_{|\overline{W}_{2,1}\rangle}$ (right) as a function of observation angle θ and interatomic distance kd

The intensity characteristics of light emitted by three atoms in a loop configuration

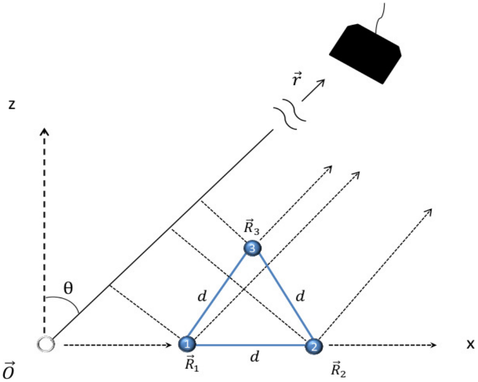


Figure: Schematic diagram of the system: where three identical and equidistant two-level atoms are localized at the vertices of an equilateral triangle. A detector at position $\bar{r}$ records the photon scattered by the atoms, in the far field regime.

The int. charac. ..... loop configuration contd...

In the loop configuration, the schematic of which is shown in Fig. **??**, the relative optical phase accumulated by a photon emitted at $\vec{R}_j$ and detected at $\vec{r}$ is given by

$$\phi_1(\vec{r}) \equiv \phi_1 = k\vec{n}.\vec{R}_1 = kd \sin\theta \tag{14}$$

$$\phi_2(\vec{r}) \equiv \phi_2 = k\vec{n}.\vec{R}_2 = 2kd \sin\theta \tag{15}$$

$$\phi_3(\vec{r}) \equiv \phi_3 = k\vec{n}.\vec{R}_3 = \frac{3kd \sin\theta + \sqrt{3}kd \cos\theta}{2} \tag{16}$$

For this configuration, the resulting symmetric W - state given by

$$|W_{2,1}\rangle = \frac{1}{\sqrt{3}} \left[|110\rangle + |011\rangle + |101\rangle\right], \quad \lambda_4 = 2g + \frac{\omega}{2} \tag{17}$$

and the corresponding intensity is found to be

$$I_{|W_{2,1}\rangle} = 2 + \frac{2}{3} \left[\cos(\phi_1 - \phi_3) + \cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3)\right]. \tag{18}$$

The int. charac. ..... loop configuration contd...

Similarly, one can construct two types of GHZ states when two atoms are present in the excited state.

$$|\overline{GHZ}_{2,1}\rangle = \frac{1}{\sqrt{2}}\left[|101\rangle - |110\rangle\right], \quad \lambda_5 = \frac{\omega}{2} - g, \tag{19}$$

with the intensity,

$$I_{|\overline{GHZ}_{2,1}\rangle} = 2 - \cos(\phi_2 - \phi_3), \tag{20}$$

For the second GHZ case,

$$|\widetilde{GHZ}_{2,1}\rangle = \frac{1}{\sqrt{2}}\left[|011\rangle - |110\rangle\right], \quad \lambda_6 = \frac{\omega}{2} - g, \tag{21}$$

the intensity of which is given by

$$I_{|\widetilde{GHZ}_{2,1}\rangle} = 2 - \cos(\phi_1 - \phi_3) \tag{22}$$

The int. charac. ..... loop configuration contd...

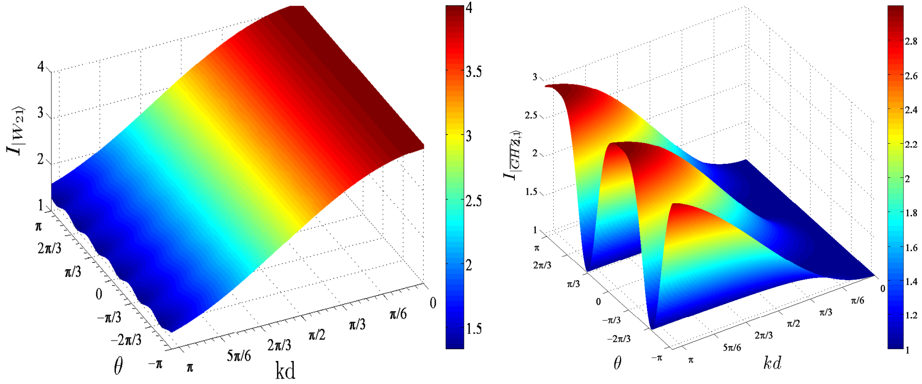


Figure: Surface Plot of $I_{|W_{2,1}\rangle}$ (left) and $I_{|\overline{GHZ}_{2,1}\rangle}$ (right) as a function of observation angle θ and interatomic distance kd.

Photon Statistics: Loop Configuration

The second-order equal - time correlation function when the system is initially in the $|W_{21}>$ state is given by,

$$g^{(2)}(0) = \frac{4 + \frac{8}{3}\left[\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3) + \cos(\phi_3 - \phi_1)\right]}{\left[2 + \frac{2}{3}\{\cos(\phi_1 - \phi_2) + \cos(\phi_2 - \phi_3) + \cos(\phi_3 - \phi_1)\}\right]^2}. \quad (23)$$

Photon Statistics: Loop Configuration

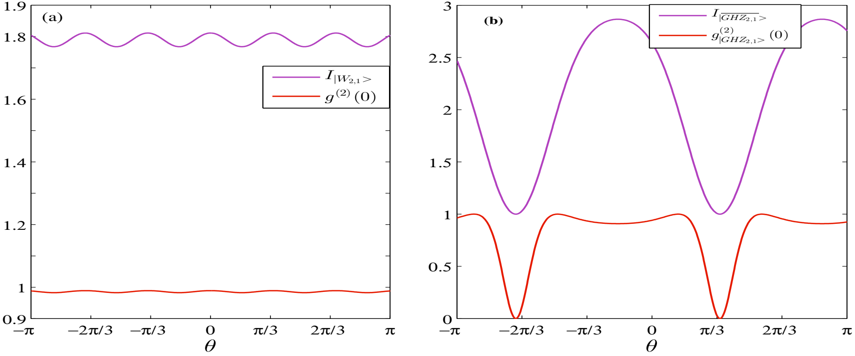


Figure: Loop configuration: The intensity (magenta line) and the second order correlation function (red line) of radiation field for (a) the symmetric $|W_{21}>$ state and (b) $|\overline{GHZ}_{2,1}\rangle$ state, as a function of observation angle θ for $kd = \frac{5\pi}{6}$.

Photon Statistics: Loop Configuration

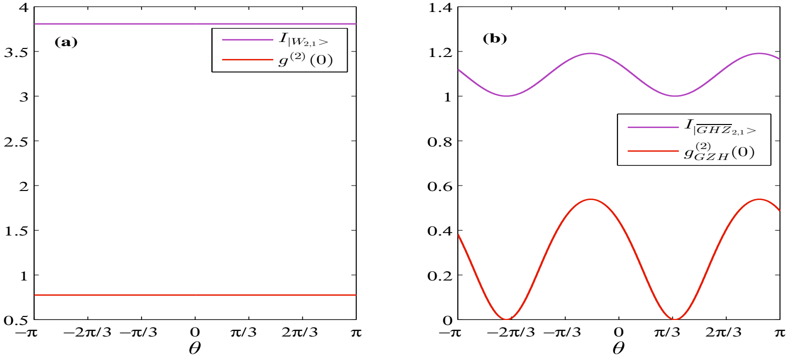


Figure: Loop configuration: The intensity (magenta line) and the second order correlation function (red line) of radiation field for (a) the symmetric $|W_{21}>$ state and (b) $|\overline{GHZ}_{2,1}\rangle$ state, as a function of observation angle θ for $kd = \frac{2\pi}{10}$.

Photon Statistics: Loop Configuration

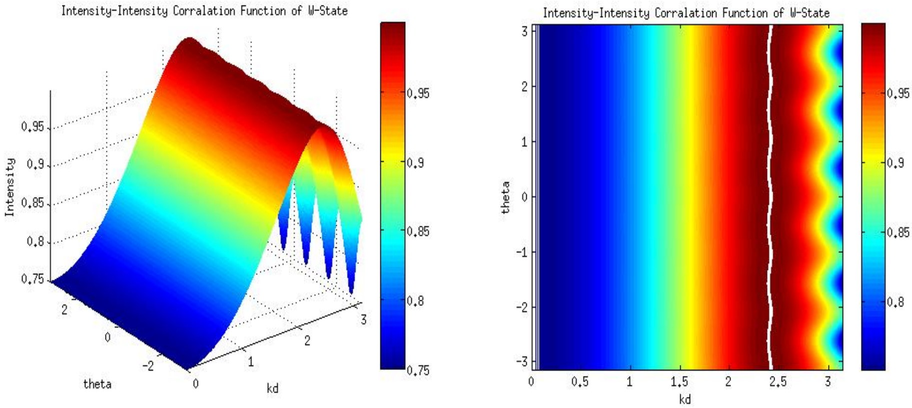


Figure: Surface (left) and Contour (right) Plot of $I_{|W_{2,1}\rangle}$ as a function of observation angle θ and interatomic distance kd.

Conclusion

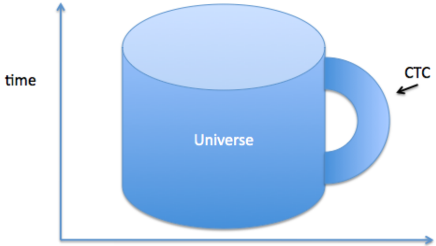
- Quantum photons on demand
- Intensity and statistics can be controlled in the far field regime

Work done with Shaik Ahmed, P. Anantha Lakshmi of University of Hyderabad.

On a new frontier in quantum correlations...

Timelike curves can increase entanglement with LOCC

Einsteins field equations allow the existence of closed timelike curves (CTC's), in certain exotic spacetime geometries
So what are CTCs?



TIME TRAVEL!
CTCs, if followed, allow a time traveler (human being or elementary particle) to interact with his/her former self.

But.. this got issues!

grandfather-like paradox: Our time traveller, accidentally or on purpose, kills her grandfather as young man before he as had any children. So, she does not exist; so, she cannot go to the past and kill her grandfather," (issue of the self-consistency of dynamics)

unproven theorem paradox: Our time traveller learns an elegant proof of a theorem in a conference and goes back in time and submits and presents the proof at the same conference, where she learnt the proof in the first place. (issue of indeterminacy)

Physicists to the rescue!

Deutsch suggested imposing a self-consistency condition, in context of Hilbert Space, postulating self-consistency conditions for the states that enter and exit the closed-timelike curve,

The Deutschian model of CTCs (D-CTCs) impose a boundary condition, in which the density operator of the CTC system that interacts with a chronology respecting (CR) system is the same, both before and after it enters the wormhole.

Formally,

$$\rho_{CTC} = \Phi(\rho_{CTC}) = Tr_{CR}\big(U(\rho_{CR} \otimes \rho_{CTC})U^{\dagger}\big)$$

where ρ_{CR} is the density matrix for chronology-respecting system, ρ_{CTC} is the initial density matrix of the qubit traveling along the closed timelike curve, and U is the interaction unitary.

Mathematically, this can be seen as nature finding a fixed point solution of the map, Φ, that depends on the chronology respecting system.

Other formulations are physically equivalent, transition probability CTCs (T-CTCs), postselected CTCs (P-CTCs)

CTC and Quantum Information

A different line of research has sought to understand the implications of CTCs, supposing they existed, for quantum mechanics, computation and information and now has a significant body of results in the quantum information literature. CTC-assisted models of computation can,

- Solve PSPACE problems in P
- Distinguish between non-orthogonal states
- Clone Quantum States (with arbitrary accuracy)
- ...
- Nature of entanglement?

What we already know about entanglement

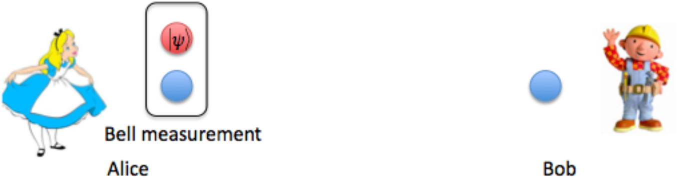
Cannot discriminate between Bell States using LOCC (Ghosh, Kar,Roy, Sen,Sen, PRL 2002)

Discriminate between any set bipartite entangled states?

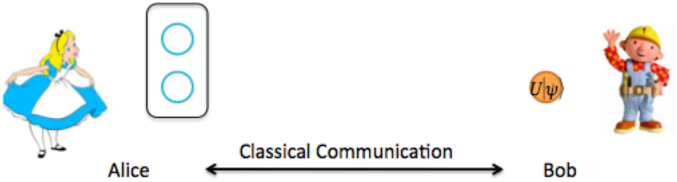
Cannot create entanglement using LOCC

Entanglement is monogamous

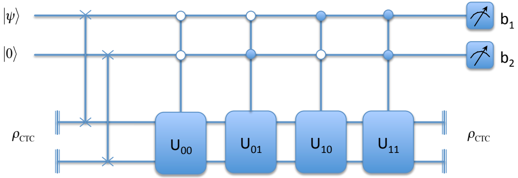
Another look at Bell State Discrimination



Another look at Bell State Discrimination



Bob recovers the state ψ, after CC from Alice and attempts to distinguish the states $\{\alpha 0 \pm \beta 1, \alpha 1 \pm \beta 0\}$



$$U_{00} = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \otimes \mathbb{I}, \qquad U_{01} = (X \otimes X) \circ (\begin{bmatrix} \beta & \alpha \\ \alpha & -\beta \end{bmatrix} \otimes \mathbb{I}),$$

$$U_{10} = (X \otimes \mathbb{I}) \circ (\begin{bmatrix} \beta & \alpha \\ -\alpha & \beta \end{bmatrix} \otimes \mathbb{I}), \qquad U_{11} = \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix} \otimes X$$

Measurements Outcomes, b_1, b_2	**State Identified by Bob**	**Conclusive Bell State**
0,0	$\alpha 0 + \beta 1$	Φ^+
0,1	$\alpha 0 - \beta 1$	Φ^-
1,0	$\alpha 1 + \beta 0$	Ψ^+
1,1	$\alpha 1 - \beta 0$	Ψ^-

The same idea can be extended to discriminate between any set bipartite entangled states,

$$\{\Sigma_{j=0}^{d}\lambda_j^{(i)}a_j^{(i)}b_j^{(i)}\}_{i=1}^{n}$$

using LOCC as well, which is again known to be impossible conventionally.

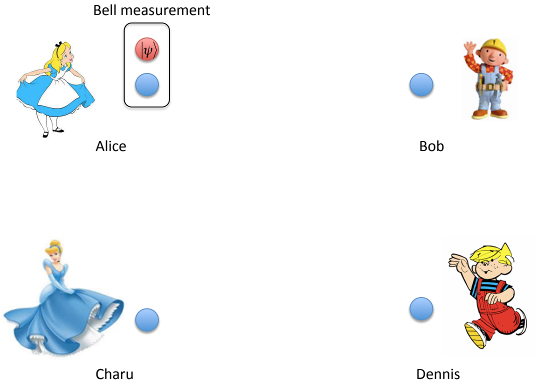
Detour: Smolin States

"A four-party quantum state which cannot be written in a separable form and from which no pure entanglement can be distilled by LOCC among the parties, and yet when any two of the parties come together in the same laboratory they can perform a measurement which enables the other two parties to create a pure maximally entangled state between them without coming together."

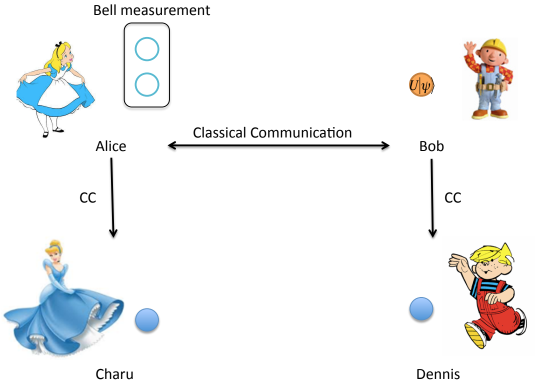
A four-party unlockable bound-entangled state,

$$\rho = \frac{1}{4}(\Phi^+\Phi^{+AB} \otimes \Phi^+\Phi^{+CD} + \Phi^-\Phi^{-AB} \otimes \Phi^-\Phi^{-CD} + \Psi^+\Psi^{+AB} \otimes \Psi^+\Psi^{+CD} + \Psi^-\Psi^{-AB} \otimes \Psi^-\Psi^{-CD})$$

Bell State Discrimination using CTC-assisted model of computation



Bell State Discrimination using CTC-assisted model of computation



So, Charu and Dennis now share a maximally entangled state with just LOCC, that they distilled from the Smolin State, without performing any joint operation, that is known to be impossible conventionally.

Open time-like curves

The same results hold true for Open time-like curves as well.
Open time-like curves (Pienaar et al, Phys. Rev. Lett. 110, 060501 (2013)) modelled the effects of such physical systems, where there was no interaction in the CTC, i.e. the Unitaries are Identity operators. Here, with entanglement between the qubits travelling along a timelike curve and an external chronology-respecting system, the self consistency conditions become,

$$\rho_{OTC\otimes CR} = Tr_{CR}(\rho_{OTC\otimes CR}) \otimes Tr_{OTC}(\rho_{OTC\otimes CR}) = \rho_{OTC} \otimes \rho_{CR}$$

where $\rho_{OTC\otimes CR}$ is a bipartite system, and one of the systems is sent through the OTC.

- Alice prepares the state $\psi = \alpha 0 + \beta 1$, s.t., $0 \leq \alpha \neq \beta \leq 1$ and $|2\alpha\beta - (\alpha^2 - \beta^2)| > 0$. Then they follow the same strategy as before and Alice teleports the state to Bob.
- Bob now has the state $\psi' \in \{\alpha 0 \pm \beta 1, \beta 0 \pm \alpha 1\}$ and needs to determine the exact state to conclude the Bell state.
- To do this, Bob uses the circuit depicted in Fig 2. The unitary $U_{b_1 b_2}$ is chosen based on Alice's Bell measurement outcomes $b_1 b_2$. The unitaries are defined as

$$U_{00} = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}, \qquad U_{01} = X \circ \begin{bmatrix} \beta & \alpha \\ \alpha & -\beta \end{bmatrix}$$
$$U_{10} = X \circ \begin{bmatrix} \beta & \alpha \\ -\alpha & \beta \end{bmatrix}, \qquad U_{11} = \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}$$

Table lists he corresponding Bell States Alice & Bob share. The first column corresponds to Alice's Bell measurements, and the first row lists the possible measurement outcomes for Bob who has an OTC assisted computer. Alice and Bob share the Bell state that is listed in the cell in row and column corresponding to their measurement outcomes. Here $\gamma = (\alpha^2 - \beta^2)^2$ and $\delta = (2\alpha\beta)^2$

	Bob Sees			
Alice's Bell Measurements b_1, b_2	All meas. 0	All meas. 1	γN meas. result in 0 and δN meas. result in 1	δN meas. result in 0 and γN meas. result in 1
0,0	Φ^+	Ψ^-	Φ^-	Ψ^+
0,1	Φ^+	Ψ^-	Φ^-	Ψ^+
1,0	Ψ^-	Φ^+	Ψ^+	Φ^-
1,1	Ψ^-	Φ^+	Ψ^+	Φ^-

Conclusion

Can discriminate between Bell States using LOCC
Can discriminate between any set bipartite entangled states
Can increase entanglement using LOCC

if there exists Deutschian CTCs or Open time-like curves

Moulick, S. R. and Panigrahi, P. K. Timelike curves can increase entanglement with LOCC. Sci. Rep. 6, 37958 (2016).

-: Thank You :-