Plausible signature of noncommutativity in type Ia supernovae with super-Chandrasekhar white dwarfs?

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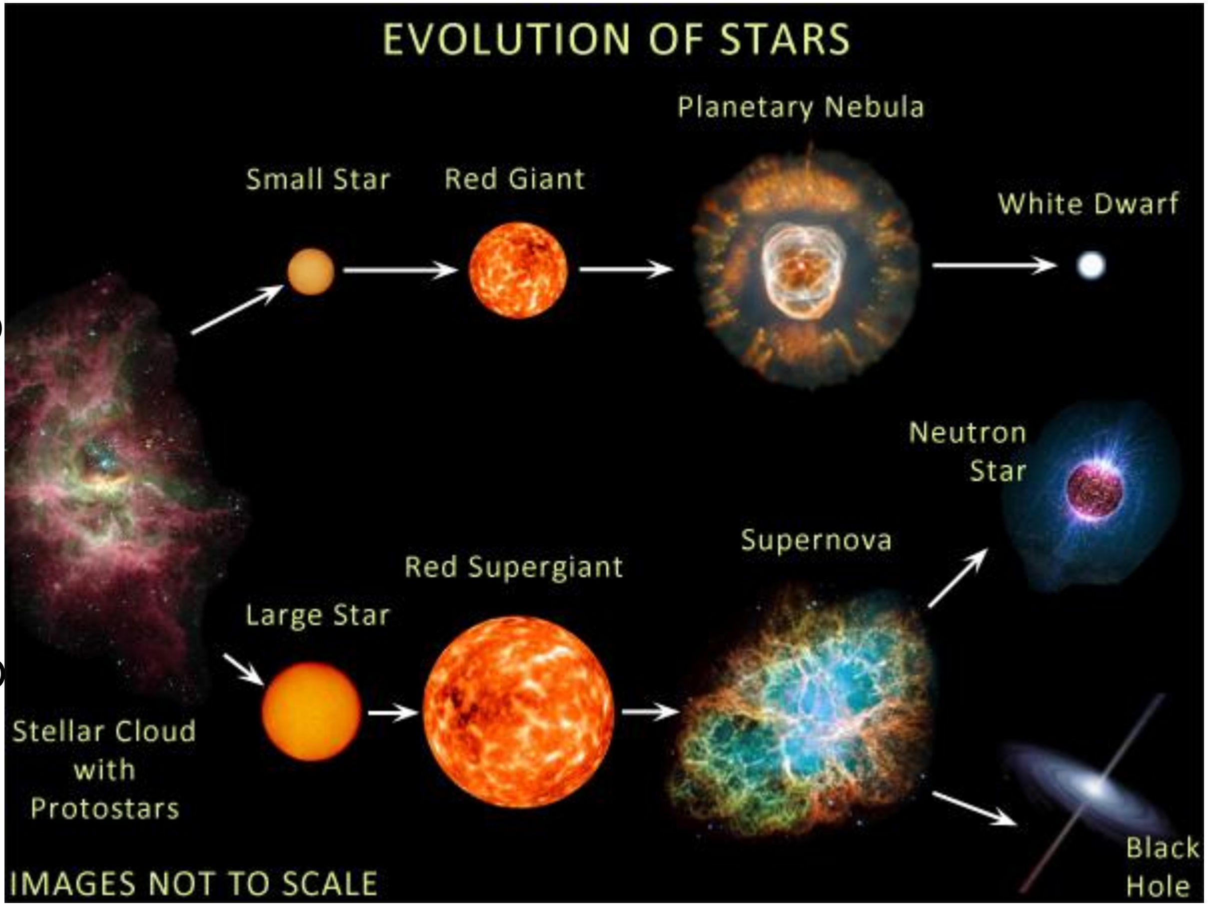
Evolution of Stars

$M \leq 8M_{\odot}$



Chandrasekhar

$M \geq 8M_{\odot}$

TOV, Kerr

- ❑ **In one of his celebrated papers, S. Chandrasekhar showed that maximum mass of a non-magnetized, non-rotating white dwarf ≈ 1.44 solar mass → Chandrasekhar limit**
- ❑ **Including effects of general relativity (GR), limit decreases to 1.4 solar mass**

How to arrive at the Chandrasekhar mass-limit?

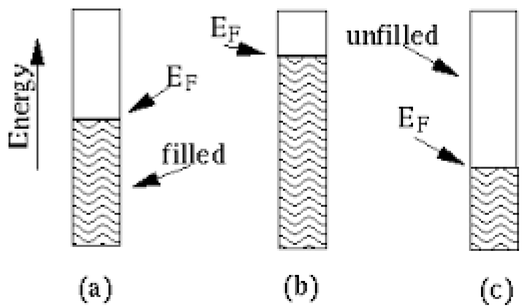


Figure 4. Band diagram of the conductor with (a) no potential applied, (b) negative potential applied (c) positive potential applied

- Gas pressure in white dwarfs is dominated by degenerate electrons

- Electrons become degenerate when all the states of the system below the Fermi level are filled → arisen at high density, during collapse/contraction of the star → Pauli's exclusion principle restricts number of fermions (here electrons) in energy states

- We have to obtain the equation of state: pressure-density relation, of an electron degenerate gas

$$P_\alpha = \frac{\pi m_e^4 c^5}{3h^3}\left[x(2x^2-3)\sqrt{x^2+1} + 3\sinh^{-1} x\right] \quad \textit{and} \quad \rho_\alpha = \frac{8\pi \mu_e m_H (m_e c)^3}{3h^3} x^3$$

where, $x = p_F/(m_e c)$

How to arrive at the Chandrasekhar mass-limit?

From Chandrasekhar's original paper:

In hydrostatic equilibrium:

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho,$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

Boundary Conditions:

$$\rho(r=0) = \rho_c$$

$$\left(\frac{d\rho}{dr}\right)_{r=0} = 0$$

$$l_1 = \frac{1}{4\pi m\mu H}\left(\frac{3h^3}{2cG}\right) = 7{\cdot}720\mu^{-1} \times 10^8 \text{ cm.}$$

$P = K\rho^{\Gamma}$ → Extreme relativistic limit

with $\Gamma=4/3$

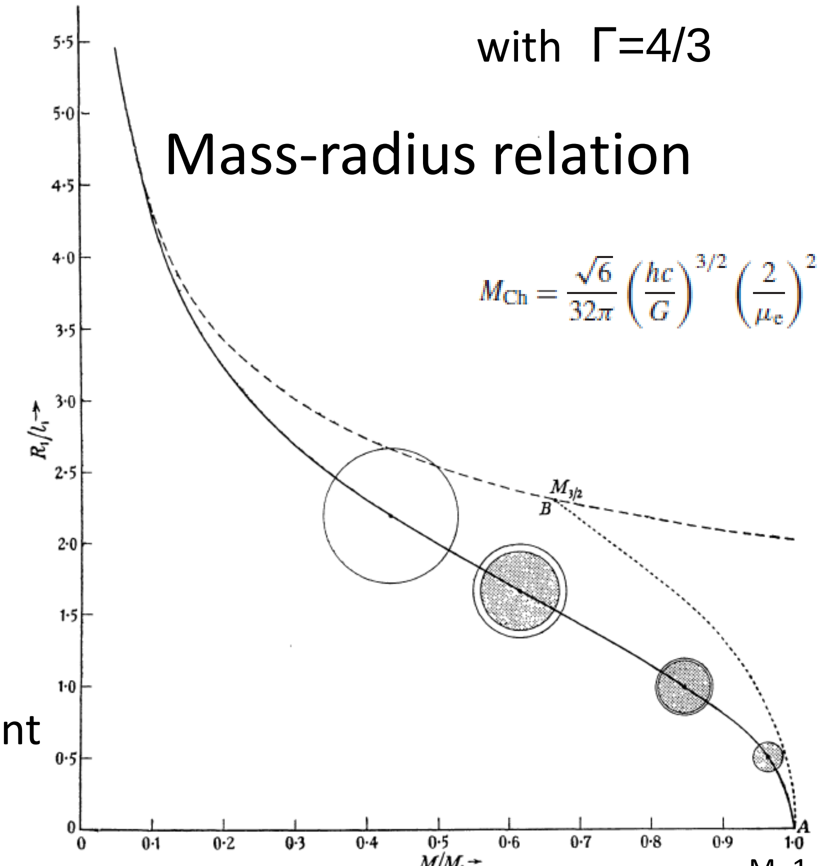
$R \propto \rho_c^{\frac{1-n}{2n}}$ $M \propto \rho_c^{\frac{3-n}{2n}}$ $n=1/(\Gamma-1)$

$R \propto M^{(1-n)/(3-n)}$

Clearly for n=3, mass becomes independent of density and radius becomes zero

Mass-radius relation

$$M_{\rm Ch} = \frac{\sqrt{6}}{32\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{2}{\mu_e}\right)^2 \frac{\xi_1^2|\theta'(\xi_1)|}{m_{\rm H}^2}$$



M=1.44M$_{\odot}$

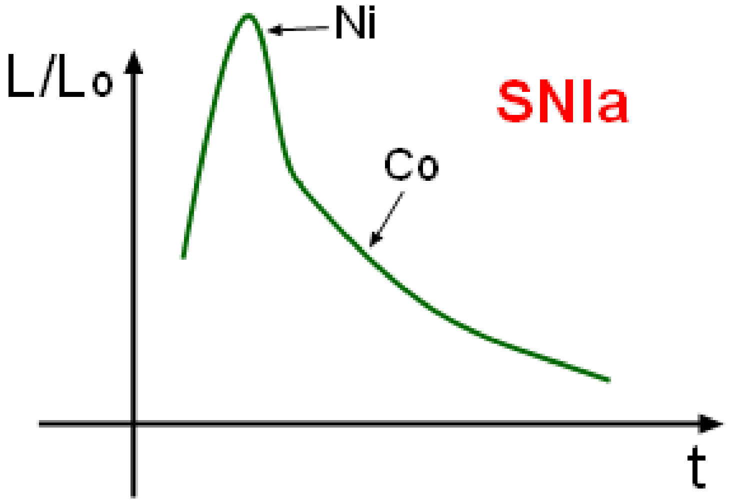
FIG. 2.—*The full line curve represents the exact (mass-radius)-relation for the highly collapsed configurations. This curve tends asymptotically to the - - - - curve as M → 0.*

Supernovae type Ia and its link to limiting mass of white dwarfs?

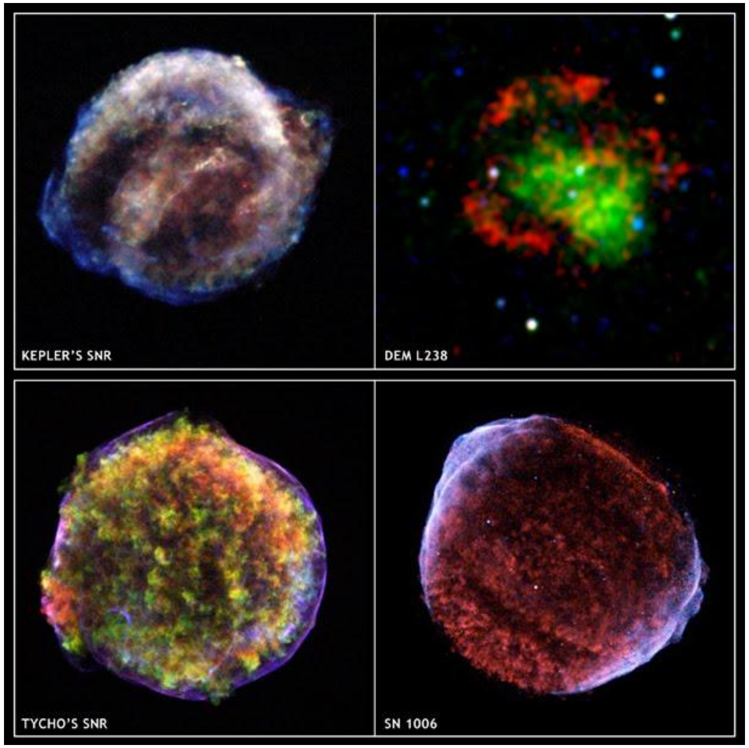
A **supernova** is an extremely luminous stellar explosion that involves the disruption of virtually an entire star.

Their optical spectra help in classifying them broadly in type I (no hydrogen lines in spectra) and II (show hydrogen lines in spectra).

Type Ia supernovae are believed to result from thermonuclear explosion of a carbon-oxygen white dwarf, when its mass approaches/exceeds the Chandrasekhar limit of 1.44 $M_\odot$ → **all look similar**



Chandra X-ray images of SN Ia

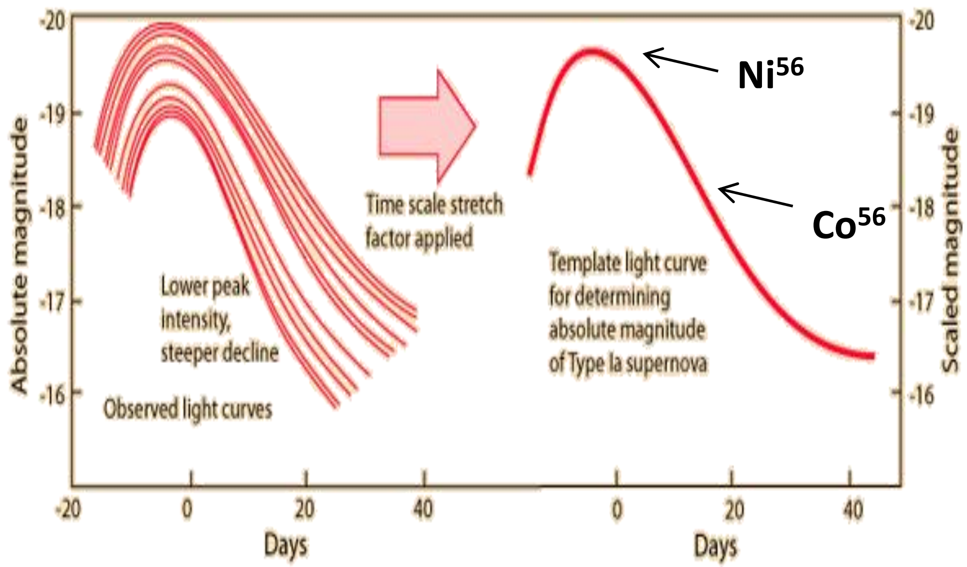


Type Ia supernovae are of great interest to astronomers because they have a characteristic light curve, which allows them to be used as **standard candles** and hence they help in investigating the expansion history of the Universe.

A long-standing puzzle in astronomy is the identification of supernova progenitors

Discovery of several peculiar over- and under-luminous type Ia supernovae provokes us to rethink the commonly accepted scenario of Chandrasekhar mass explosion of white dwarfs.

TYPE Ia SUPERNOVAE



Courtesy:
Georgia State University

PECULIARITIES

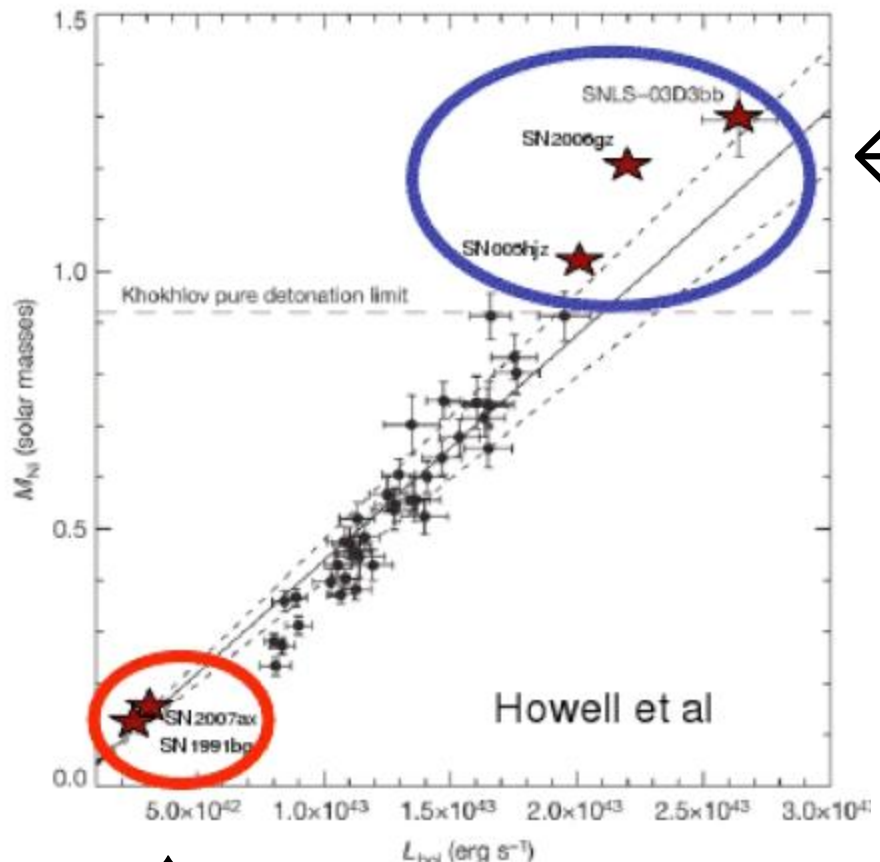
Over-luminous, very high Ni mass $\gtrsim 1.3 M_\odot$, violates luminosity-stretch relation, very low ejecta velocity

SN 2006gz, SN 2007if, SN 2009dc, SN 2003fg - seem to suggest super-Chandrasekhar-mass white dwarfs (2.1 - 2.8 $M_\odot$) as their most likely progenitors (Hicken et al. 2007, Howell et al. 2006, Scalzo et al. 2010).

Under-luminous, very low Ni mass $\sim 0.1 M_\odot$

SN 1991bg, SN 1997cn, SN 1998de, SN 1999by, SN 2005bl (Filippenko et al. 1992, Mazzali et al. 1997, Taubenberger et al. 2008) – suggest sub-Chandrasekhar explosion

Highly over-luminous, peculiar, type Ia supernovae along with standard type Ia supernovae



← Progenitors are argued to be significantly super-Chandrasekhar

$$L \rightarrow M_{wd}\, C^2 + m\, v^2$$

↓

low

↑

Progenitors are argued to be significantly sub-Chandrasekhar

Courtesy: talk of Mansi Kasliwal

Possible Origin and Our Avenue

- Since half a decade, we have been exploring progenitor of peculiar type Ia supernovae: Over-luminous and Under-luminous → Violation of Chandrasekhar mass-limit

- Brings violation of Chandrasekhar's limit: super-Chandrasekhar white dwarfs in limelight

- After our initiation in 2012, various groups come forward with many plausible mechanisms to violate Chandrasekhar mass-limit significantly: by e.g. magnetic fields, modified gravity, modifying uncertainly principle, doubly special relativity

- Not free from uncertainties

Main Idea

Introducing phase-space noncommutativity in the X-Y plane: $[\hat{p}_x, \hat{p}_y] = i\eta$ NCHA

Along with: $[\hat{x}, \hat{y}] = i\theta$ and $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$

$$[\hat{x}_i, \hat{z}] = [\hat{p}_i, \hat{p}_z] = 0; \quad i, j = 1, 2$$

In addition: HA

$$[x_i, p_j] = i\hbar\delta_{ij}; \quad [x\,,\,y] = 0 = [p_x, p_y]$$

With Bopp-shift transformation

$$\hat{x}_i = x_i - \frac{1}{2\hbar}\theta_{ij}p_j + \frac{1}{2\hbar}\lambda_{ij}x_j, \;\; \hat{p}_i = p_i + \frac{1}{2\hbar}\eta_{ij}x_j + \frac{1}{2\hbar}\lambda_{ij}p_j$$

$$\lambda_{ij} = \epsilon_{ij}\sqrt{-\theta\eta}$$

Hamiltonian/Energy

Dirac equation:

$$\hat{H}\psi = i\frac{\partial\psi}{\partial t} = E\psi$$

$$(E - m_e c^2)\phi = \vec{\sigma}.\vec{p}c\ \chi \text{ and } (E + m_e c^2)\chi = \vec{\sigma}.\vec{p}c\ \phi,$$

Hence

$$(E^2 - m_e^2 c^4)$$

$$= \left[A(p_x^2 + p_y^2) + B(x^2 + y^2) + C(xp_x + p_x x + yp_y + p_y y) + \frac{\eta}{\hbar}(yp_x - xp_y)\right.$$

$$\left. + \ p_z^2 - \sigma_z \eta\right] c^2,$$

when $A = 1 + \frac{\lambda^2}{4\hbar^2}\ ,\ \ B = \frac{\eta^2}{4\hbar^2}\ \text{ and }\ C = \frac{\eta\lambda}{4\hbar^2}$

Hamiltonian/Energy

In terms of appropriate ladder operators

$$a_j = \left(\frac{A}{2\hbar\sqrt{AB-C^2}}\right)^{\frac{1}{2}}\left(p_j + \frac{C}{A}x_j - i\frac{\sqrt{AB-C^2}}{A}x_j\right)$$

$$a_j^\dagger = \left(\frac{A}{2\hbar\sqrt{AB-C^2}}\right)^{\frac{1}{2}}\left(p_j + \frac{C}{A}x_j + i\frac{\sqrt{AB-C^2}}{A}x_j\right)$$

j=x,y

satisfying $[a_x, a_x^\dagger] = 1 = [a_y, a_y^\dagger]$ and $\hat{a}_1 = \frac{a_x + ia_y}{\sqrt{2}}, \quad \hat{a}_2 = \frac{a_x - ia_y}{\sqrt{2}}$

$$[a_1, a_1^\dagger] = 1 = [a_2, a_2^\dagger]$$

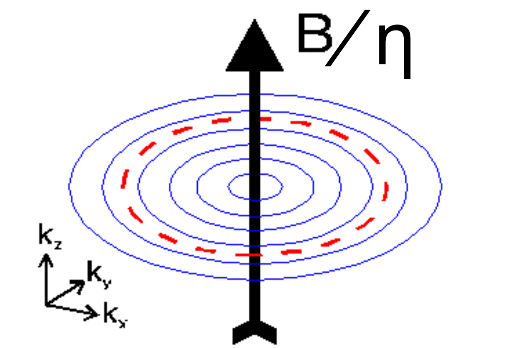
→ non-diagonal part of Hamiltonian

$$\hat{H}' = \eta(2a_1^\dagger a_1 + 1)$$

$$E^2 = p_z^2c^2 + m_e^2c^4 + 2m\eta c^2$$ with m=0,1,2,...

Approach: Equation of State

- **We consider a relativistic, degenerate electron gas at zero temperature in the presence of noncommutativity, neglecting any form of interactions: relativistic corresponds to $pc \geq mc^2$, zero temperature justifies as $E_F \gg k_B T$ ------- The energy states of a free electron are quantized into levels similar to the Landau orbitals in the presence of magnetic fields, which defines the motion of the electron in a plane perpendicular to the z-axis.**

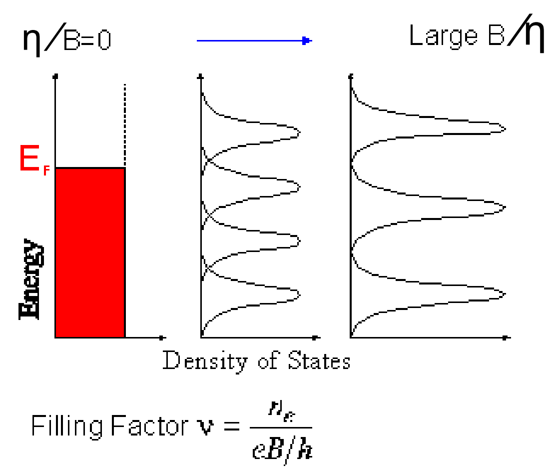


- **Energy eigenstates for the Dirac equation in noncommutativity is given by**

$$E^2(m) = p_z^2 c^2 + m_e^2 c^4 + 2m\eta c^2$$

- **Noncommutation effect modifies the density of states of the electrons as**

$$\frac{2}{h^3}\int d^3p \;\longrightarrow\; \int \frac{4\pi\eta}{h^3}\, dp_z$$



Proceeding is same as strong magnetic field effects: Landau quantization → B/B_c is replaced by $\eta/(m_e c)^2$

EoS

$$P = \sum_{m=0}^{m_{max}} \frac{2\pi m_e^4 c^5 \eta_D}{h^3} g_m \left[\epsilon_F x_F(m) - (1 + 2m\eta_D) \log \frac{\epsilon_F + x_F(m)}{\sqrt{1 + 2m\eta_D}} \right]$$

$$n_e = \sum_{m=0}^{m_{max}} \frac{4\pi m_e^3 c^3 \eta_D}{h^3} g_m x_F(m) \qquad \rho = \mu m_n n_e,$$

At high enough density but with m=0 (ground level)

$$P = \frac{h^3}{8\pi \mu^2 m_n^2 m_e^2 c \eta_D} \rho^2 = K_{nc} \rho^2 = K_{nc} \rho^{1+1/n}$$

$$\rho = \rho_c = \frac{4\pi \mu m_n m_e^3 c^3}{h^3} \eta_D^{3/2} \sqrt{2m_1}$$

$$m < m_1 = (\epsilon_F^2 - 1)/2\eta_D$$ **with $m_1 < 1$**

$$\eta_D = \frac{\eta}{m_e^2 c^2}, \quad \epsilon_F = \frac{E_F}{m_e c^2}, \quad x_F(m) = \frac{p_F(m)}{m_e c}$$

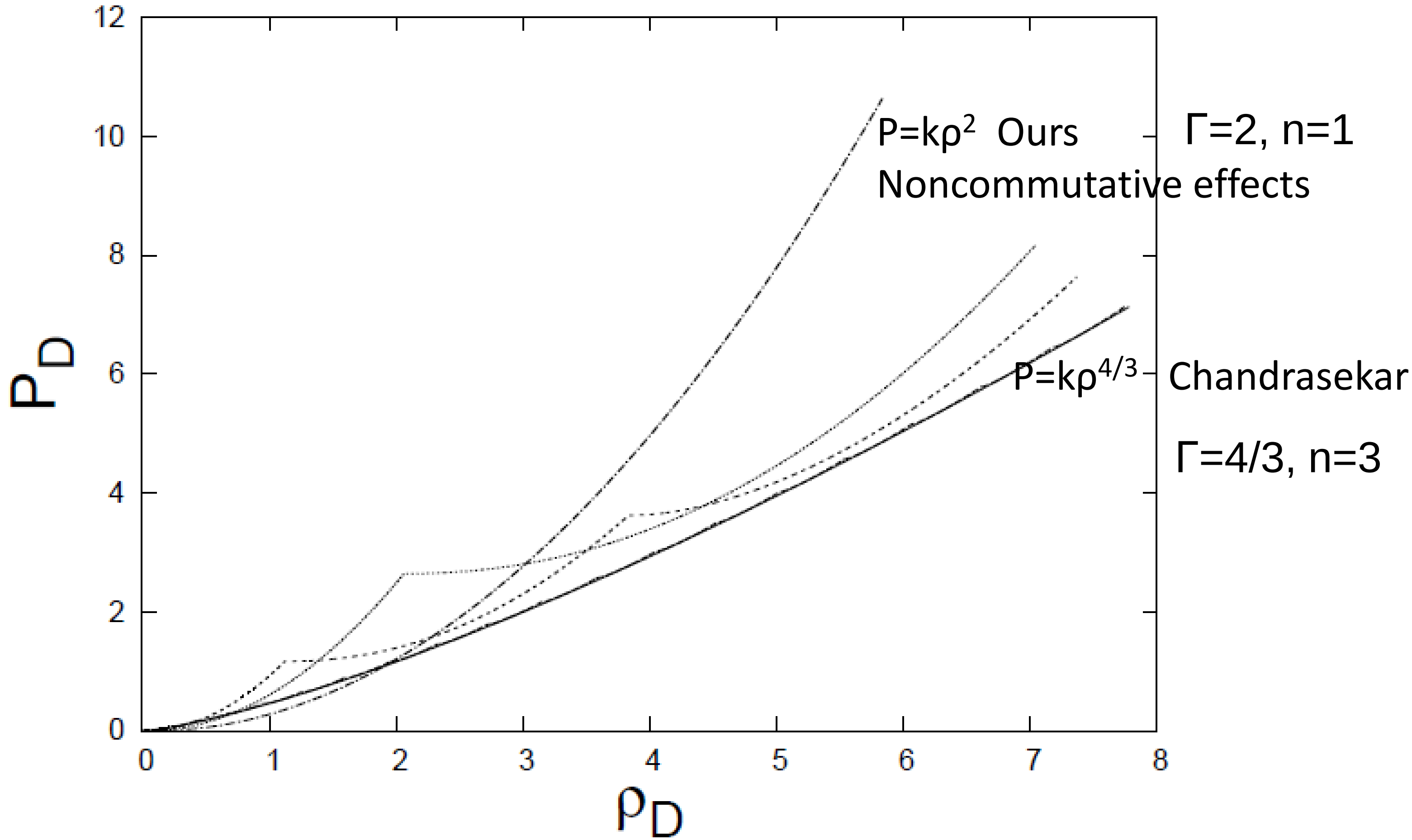
For $\rho_c = 2 \times 10^{10}/V$ gm/cc

$$\eta_D = \left(\frac{2 \times 10^{10} h^3}{4\pi \mu m_n m_e^3 c^3 \sqrt{2m_1} V} \right)^{2/3} \approx \frac{456}{(V\mu)^{2/3} m_1^{1/3}}$$

Hence, for $\mu = 2$ and $V = 1$, $\eta_D > 287.3$

Effects of noncommutativity in Equation of State: Constant η

→Noncommuting length scale is similar to underlying Compton wavelength → At $\eta \sim (m_e c)^2$ effects becoming important



density in units of 2×10^9 gm/cc

pressure in units of 2.668×10^{27} erg/cc

New Mass-limit

Following Lane-Emden formalism:

Combining hydrostatic balance and mass equations

Assuming

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho, \quad \rho = \rho_c\theta^n,\ r = a\xi \quad a = \sqrt{K_{nc}/2\pi G},$$

$P = K\rho^{\Gamma}$

$$\frac{dM}{dr} = 4\pi r^2\rho \quad M = 4\pi\left(\frac{n+1}{4\pi G}\right)^{3/2}\left(\frac{h^3}{8\pi\mu^2 m_n^2 m_e^2 c}\right)^{3/2}\rho_c^{3(1-n)/2n}\frac{4\sqrt{2m_1}\pi\mu m_n m_e^3 c^3}{h^3}I_n$$

where $I_n = \int_0^{\xi_1}\theta^n\xi^2 d\xi$, $\rho = \rho_c\theta^n$, $r = a\xi$

radius of the star $R = a\xi_1$ and $a = \sqrt{K_{nc}/2\pi G}$, when at $\xi = \xi_1$, $\theta = 0$

For $\Gamma=2$, $n=1$, $I_n=\pi$, $\xi_1=\pi$

$$M = \left(\frac{hc}{2G}\right)^{\frac{3}{2}}\frac{m_1^{1/2}}{\mu^2 m_n^2} = 2.58M_\odot,$$

with $m_1 \approx 1$

$$R = \sqrt{\frac{\pi K_{nc}}{2G}}.$$

For $K_{nc} < 1.5\times 10^8$, $R < 594.2$ km

Comparison with Chandrasekhar

Combining hydrostatic balance and mass equations $\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho,$

$\frac{dM}{dr} = 4\pi r^2 \rho$ $\quad M \propto K^{3/2}\rho_c^{\frac{3-n}{2n}},$ $\quad R \propto K^{1/2}\rho_c^{\frac{1-n}{2n}}$ $\quad P = K\rho^{\Gamma}$

For high density regime $K = K_{nc} \rightarrow \eta_D \rightarrow \rho_c^{-2/3}$ where $\eta_D = \eta/(m_e c)^2$

Mass is independent of ρ_c and radius $R = \sqrt{\frac{\pi K_{nc}}{2G}}$

Ours

$\Gamma=2$ and hence $n=1$

$$M = \left(\frac{hc}{2G}\right)^{3/2}\frac{1}{(\mu_e m_H)^2} \approx \frac{10.312}{\mu_e^2}M_{\odot},$$

Chandrasekhar's

$\Gamma=4/3$ and hence $n=3$

$$M_{\rm Ch} = \frac{\sqrt{6}}{32\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{2}{\mu_e}\right)^2\frac{\xi_1^2|\theta'(\xi_1)|}{m_{\rm H}^2}$$

For $\mu_e=2$ (carbon-oxygen white dwarf)

$$M \approx 2.58 M_{\odot}.$$

$$1.44 M_{\odot}$$

For $m_1 \rightarrow 1$ (just filled ground level)

Another approach: Varying η
Equation of State

Surface and low density white dwarfs should follow commutating rule: η to decrease → ρ↓ η↓ keeping $m_1<1$

$$P = \frac{h^3}{8\pi\mu_e^2 m_n^2 m_e^2 c\eta_D}\rho^2 = K_{nc}\rho^2 = K_{nc}\rho^{1+1/n}$$

$$\rho = \rho_c = \frac{4\pi\mu_e m_n m_e^3 c^3}{h^3}\eta_D^{3/2}\sqrt{2m_1},$$

By eliminating η_D in adiabatic approximation

$$P = K_{ncm}\rho^{4/3}, \text{ with } K_{ncm} = \frac{hc}{2}\left(\frac{m_1}{2\pi\mu_e^4 m_n^4}\right)^{1/3} = 1.1\times 10^{15} m_1^{1/3}$$

Another approach: Mass-limit

Following Lane-Emden formalism:

Assuming

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho, \quad \rho = \rho_c\theta^n \quad r = a_1\xi, \quad a_1 = \sqrt{K_{nc}/2\pi G m_1^{1/3}}$$

P = K ρ^Γ with Γ=4/3

$$M = \int_0^R 4\pi r^2 \rho dr \qquad M = 4\pi a_1^3 \rho_c I_n, \text{ with } a_1 = \left[\frac{K_{ncm}\rho_c^{-2/3}}{\pi G m_1^{1/3}}\right]^{1/2}$$

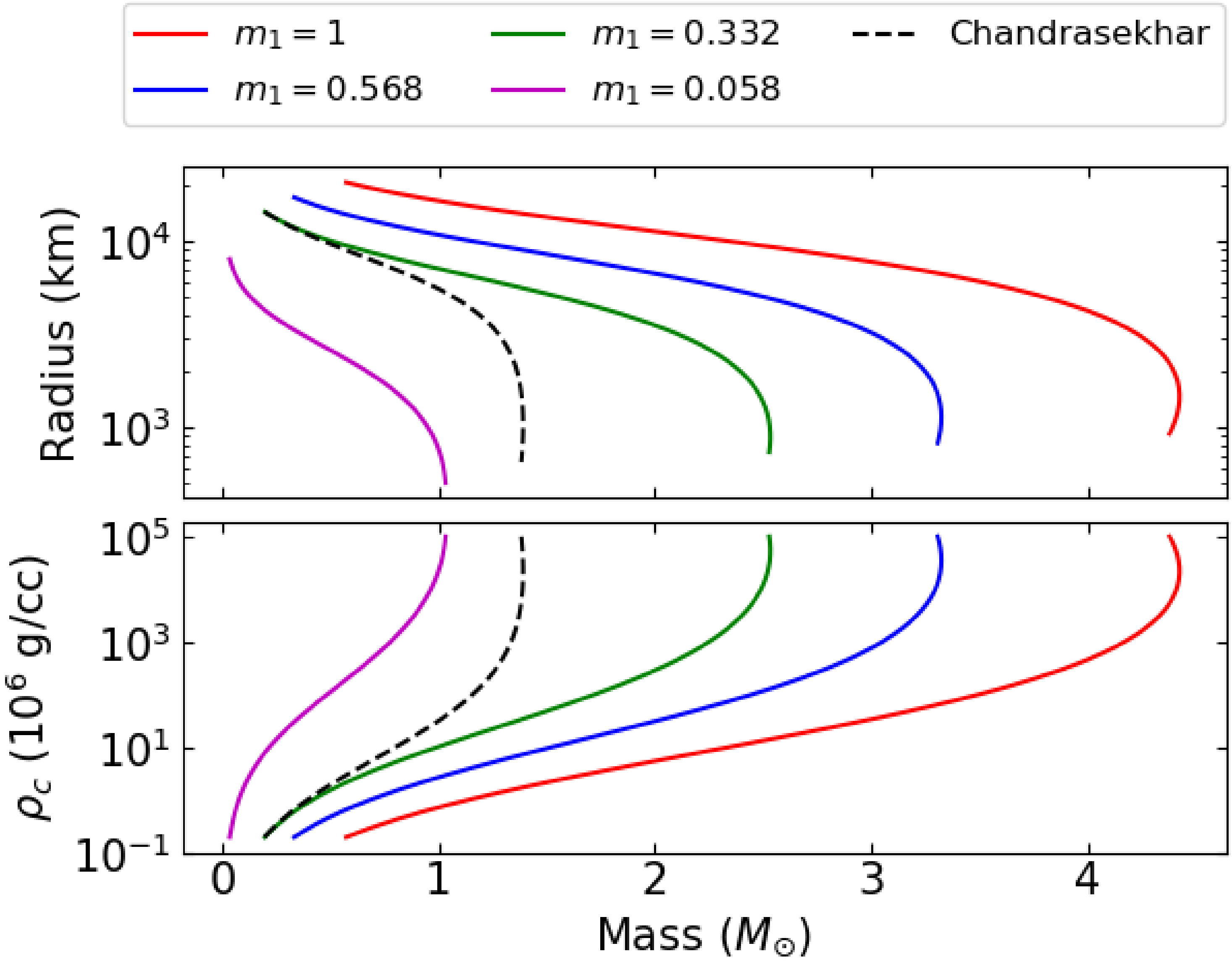
$$= 2.38 M_\odot I_n$$

$$I_n = \int_0^{\xi_1} \theta^n \xi^2 d\xi$$

$$\frac{d}{d\xi}\left[\xi^2\left(m_1^{1/3}\frac{d\theta}{d\xi} + \frac{\theta m_1^{-2/3}}{3(n+1)}\frac{dm_1}{d\xi}\right)\right] = -\xi^2\theta^n,$$

$$\text{hence } I_n = \left(\xi^2 m_1^{1/3} d\theta/d\xi\right)_{\xi_1}.$$

Mass-Radius Relation: General Relativity TOV solution



Summary

- Chandrasekhar-limit is "Sacrosanct", but the value of mass-limit is NOT
- New, generic, mass limit of white dwarfs seems to be around $2.6M_{\odot}$
- This violation may be due to non-commutative phase space at high density: Plausible observational signature of non-commutativity
- Next step should be to introduce z-directional non-commutativity and/or fuzzy sphere/disk → T. R. Govindarajan
- Once the limiting mass is approached, the white dwarfs explode exhibiting over-luminous, peculiar type Ia supernovae: inferred exploding mass $2.3 – 2.8\ M_{\odot}$
- This suggests a second standard candl: many far reaching significance