Aspects of quasinormal modes of black holes with scalar hair

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What are black hole quasinormal modes?

The response of a black hole to a perturbation of an external field or the perturbation of the metric is manifested in the form of a damped wave emitted by the black hole, characterized by a complex frequency, called the quasinormal frequency. The real part of the frequency corresponds to the actual frequency of the wave motion while the imaginary part takes care of the damping factor.

Charged spherically symmetric BH with scalar hair

Starting from the action of general relativity coupled to a Maxwell field $F^{\mu\nu}$ and and conformally coupled to a scalar field ψ,

$$I = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - F_{\mu\nu}F^{\mu\nu} - 8\pi G\left(\nabla_\mu\psi\,\nabla^\mu\,\psi + \frac{R}{6}\psi^2\right)\right] \tag{1}$$

Astorino* arrived at the RN black hole of mass, M and charge, e endowed with a scalar hair, s

$$ds^2 = -f\left(r\right)dt^2 + f\left(r\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{2}$$

where,

$$f\left(r\right) = \left(1 - \frac{2M}{r} + \frac{e^2+s}{r^2}\right) \text{ and } \psi = \sqrt{\frac{6}{8\pi G}}\sqrt{\frac{s}{s+e^2}}. \tag{3}$$

$$\text{Horizon radii:} \quad r_\pm = M \pm \sqrt{M^2 - e^2 - s} \tag{4}$$

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*M. Astorino, Phys. Rev. D **88**, 104027 (2013).

Charged spherically symmetric BH with scalar hair(contd..)

The net stress-energy tensor looks like

$$T^{\mu}_{\nu} = \frac{e^2 + s}{r^4} diag\,(-1, -1, 1, 1)\,. \tag{5}$$

- The scalar hair is a primary hair since the scalar field ψ survives even in the absence of the electromagnetic field.
- The trace of the energy momentum tensor due to the scalar field alone is also zero. Thus the existence of this hair is completely consistent with the theorem given by Banerjee and Sen[†].

[†] N. Banerjee, S. Sen, Pramana 85, 1123 (2015)

Quasinormal Modes ...

A scalar field Φ of mass μ and charge q propagating in the background (2) obeys the Klein-Gordon equation

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]\Phi = 0 \tag{6}$$

$A_\nu = -\delta^0_\nu e/r \rightarrow$ electromagnetic vector potential of the black hole.

$$\Phi_{lm}(t, r, \theta, \phi) = e^{-i\omega t} Y_{lm}(\theta) R(r) e^{im\phi}, \tag{7}$$

Radial Part:

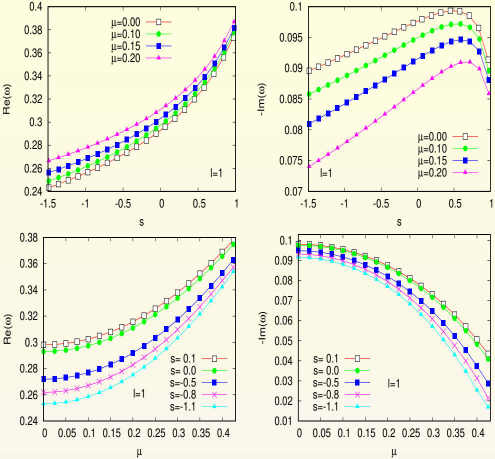
$$\frac{d}{dr}\left(\Delta \frac{dR}{dr}\right) + \frac{U}{\Delta} R = 0, \tag{8}$$

$$\Delta = r^2 f(r) \text{ and } U = \left(\omega r^2 - eqr\right)^2 - \Delta\left[\mu^2 r^2 + l(l+1)\right]$$

Boundary condition:

$$R \sim \begin{cases} \text{pure ingoing waves} & \text{as} \quad r \rightarrow r_+ \\ \text{pure outgoing waves} & \text{as} \quad r \rightarrow \infty. \end{cases} \tag{9}$$

Results[†]: Uncharged scalar field



☞
- $s < 0$: $Re(\omega)$ and $|Im(\omega)|$ decrease with $|s|$.
- $s > 0$: Corresponds to a RN black hole[††] of unit mass and electric charge, $e = \sqrt{s}$

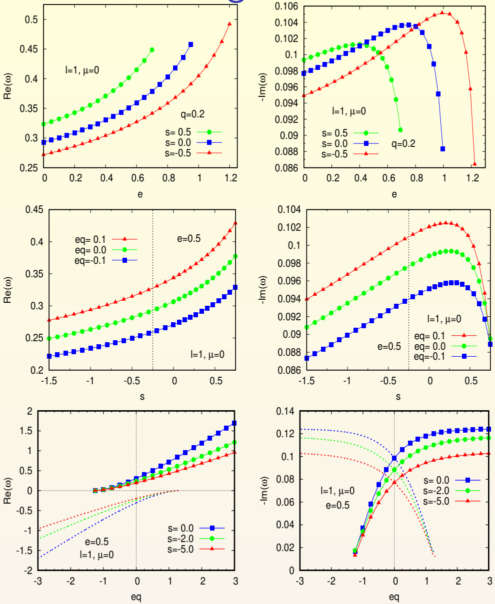
☞ At large μ the $|Im(\omega)|$ is vanishingly small.
$\rightarrow$ *Quasi-resonance modes*[§].

[†] A. Chowdhury, N. Banerjee, Eur. Phys. J. C **78**, 594 (2018)

[††] R.A. Konoplya, Phys. Rev. D **66**, 084007 (2002),R. Konoplya, Phys. Lett. B **550**, 117 (2002)

[§] A. Ohashi, M. aki Sakagami, Class. Quantum Grav. **21**, 3973 (2004).

Results[†]: Charged scalar field



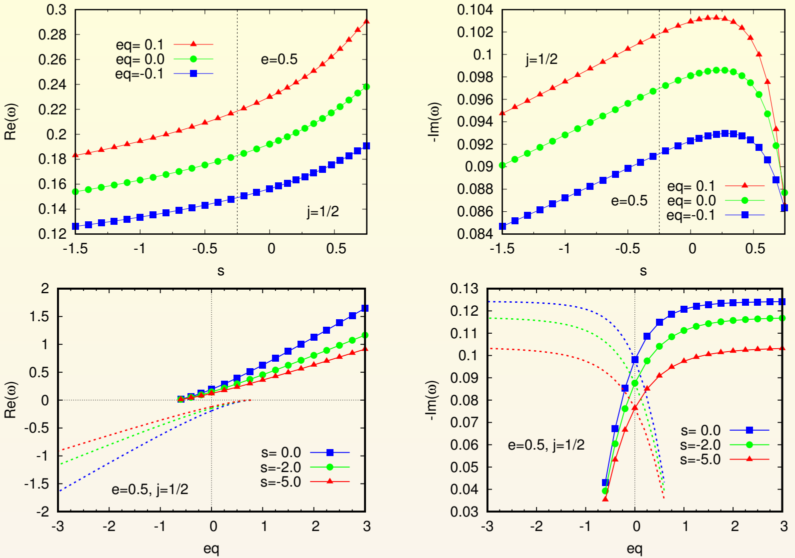
- Compared to the RN black hole, for fixed e and q, $Re(\omega)$ and $|Im(\omega)|$ are more for $s > 0$ and less for $s < 0$.
- For fixed e and q, $|Im(\omega)|$ increases as s changes from negative to positive and ultimately approaches the neutral one as the extremal value of s is approached.
- The symmetry of the QNMs with respect to the transformation $(eq \to -eq, \omega \to -\omega^*)$. The critical value of $|eq|$ at which the $Re(\omega)$ vanishes is unaltered.

[†] A. Chowdhury, N. Banerjee, Eur. Phys. J. C **78**, 594 (2018)

Differences with standard RN

- Uncharged Scalar field
  - $|Im(\omega)|$ decreases monotonically with $|s|$ for $s < 0$ unlike the occurrence of a distinct peak for the RN case.
  - For $s < 0$, quasi-resonance occurs at smaller field masses with lower frequencies.
- Charged Scalar field
  - Non-zero s has no analogue in the standard R.N. spacetime.

Results†: Massless charged Dirac field



† A. Chowdhury, N. Banerjee, Eur. Phys. J. C **78**, 594 (2018)

Thank You

Quasinormal Modes ...

Ansatz:

$$R = e^{i\Omega r}(r-r_-)^{\rho}\sum_{n=0}^{\infty} a_n u^{n+\delta} \tag{10}$$

where $\Omega = \sqrt{\omega^2-\mu^2}$, $u = \frac{r-r_+}{r-r_-}$, $\rho = \frac{i\left(i\Omega + M\left(\Omega^2+\omega^2\right) - eq\omega\right)}{\Omega}$ and $\delta = -\frac{ir_+^2\left(\omega - \frac{eq}{r_+}\right)}{r_+-r_-}$.

(10) $\rightarrow$ (8) $\Rightarrow$

$$\alpha_0 a_1 + \beta_0 a_0 = 0,$$
$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0,$$

where α_n, β_n and γ_n are functions of ω, q, μ and the black hole parameters.

The convergence of the series (10) at spatial infinity requires the recursion coefficients to satisfy an infinite continued fraction relation[‡]

$$0 = \beta_0 - \frac{\alpha_0\gamma_1}{\beta_1 -}\frac{\alpha_1\gamma_2}{\beta_2 -}\cdots\frac{\alpha_n\gamma_{n+1}}{\beta_{n+1}-}\cdots \tag{11}$$

[‡] G. Jaffé, Zeitschrift für Physik **87**(7), 535 (1934), E.W. Leaver, Proc. R. Soc. Lond. A. **402**(1823), 285 (1985), E.W. Leaver, Phys. Rev. D **41**, 2986 (1990), E.W. Leaver, Phys. Rev. D **43**, 1434 (1991).

Recurrence Coefficients

$$\alpha_n = -\frac{(n+1)^2\, r_- + (n+1) r_+ \left(-2ieq - n + 2ir_+\omega - 1\right)}{r_+ - r_-}, \tag{12}$$

$$\begin{aligned}\beta_n = \frac{1}{2\Omega(r_- - r_+)} \Big[& r_+ \Big\{ 2 \Big(-2e^2 q^2 (\Omega+\omega) + ie(2n+1)q\,(2\Omega+\omega) + \Big(l\,(l+1) + 2n^2 + 2n + 1\Big)\,\Omega\Big) \\ & + r_+ \Big(4\omega\,(\Omega+\omega)\,(3eq - 2in - i) + 3i\mu^2(2ieq + 2n + 1)\Big) + 2r_+^2 \Big(\mu^2\,(\Omega + 3\omega) - 4\omega^2\,(\Omega+\omega)\Big)\Big\} \\ & - 2r_- \Big\{ ir_+ \Big(-2(2n+1)\omega^2 + \mu^2(ieq + 4n + 2)\Big) + ie(2n+1)q\omega + \Big(l\,(l+1) + 2n^2 + 2n + 1\Big)\,\Omega \\ & + \mu^2 r_+^2\,(\Omega+\omega)\Big\} + i\mu^2(2n+1) r_-^2 \Big],\end{aligned} \tag{13}$$

$$\begin{aligned}\gamma_n = & \left[\frac{i\left\{eq\omega - \frac{1}{2}\left(\Omega^2 + \omega^2\right)(r_- + r_+)\right\}}{\Omega} + ieq + n - i\omega(r_- + r_+)\right] \\ & \left[n - \frac{i\left\{-2\,(r_+ - r_-)\left(eq\omega - \frac{1}{2}\,(r_- + r_+)\left(\Omega^2 + \omega^2\right)\right) + \Omega\,(r_- + r_+)\,(\omega\,(r_- + r_+) - 2eq) + \omega\Omega\,(r_- - r_-\right.}{2\Omega\,(r_+ - r_-)}\right.\end{aligned} \tag{14}$$