Non-linear effects on holographic superconductors

(An analytical study)

Debabrata Ghorai

Department of Theoretical Sciences,
S.N. Bose National Centre for Basic Sciences,
Kolkata, India.

Under joint supervision of
Dr. Sunandan Gangopadhyay & Prof. Biswajit Chakraborty

Holographic superconductors

Holographic Superconductors : A gravitational model to describe high T_c superconductors.

- **SC :** Infinite conductivity, Meissner effect. **Type:** Low & High T_c SC (>30K)
- Weakly coupled superconductors $\leftrightarrow$ The BCS theory of superconductivity.
- From the BCS theory $\frac{\Delta_0}{k_B T_c} = 1.76$ (Weak coupling limit $VN_0 << 1$)

- High T_c superconductors (Bednorz & Muller in 1986) and $\frac{\Delta_0}{k_B T_c} \approx 3.72$ (Expt.).
- This theory fails to explain the strongly coupled superconductors (cuprates).
- Gauge/gravity duality provides a new tool to understand high T_c superconductors.

- **Why Holographic:** To describe superconductivity of a material, one has to consider one higher (spatial) dimensional gravity theory in AdS spacetime.

Holographic superconductors

Holographic Superconductors : A gravitational model to describe high T_c superconductors.

- **SC :** Infinite conductivity, Meissner effect. **Type:** Low & High T_c SC (>30K)
- Weakly coupled superconductors $\leftrightarrow$ The BCS theory of superconductivity.
- From the BCS theory $\frac{\Delta_0}{k_B T_c} = 1.76$ (Weak coupling limit $VN_0 << 1$)

- High T_c superconductors (Bednorz & Muller in 1986) and $\frac{\Delta_0}{k_B T_c} \approx 3.72$ (Expt.).
- This theory fails to explain the strongly coupled superconductors (cuprates).
- Gauge/gravity duality provides a new tool to understand high T_c superconductors.

- **Why Holographic:** To describe superconductivity of a material, one has to consider one higher (spatial) dimensional gravity theory in AdS spacetime.

Why gravity theory : The AdS/CFT correspondence [Maldacena, 1998]

The AdS/CFT duality claims that 5-dim. weakly coupled gravity theory in AdS spacetime is dual with 4-dim. strongly coupled gauge theory in the boundary.

- **Two consequences :**
  (1) The appropriate variables are the weakly coupled variables for describing strongly coupled gauge theory.
  (2) Weakly coupled gravity theory makes analysis much easier.
- It is an analytic method for describing strongly coupled field theories.

- Construct simple gravity models - use this correspondence - one obtains properties which are similar to some of the basic properties of superconductors.

Caution: Models are too crude to make detailed comparison with any real world material (Not provide any mechanism of dual theory from bulk theory))

Gauge/gravity duality

Why gravity theory : The AdS/CFT correspondence [Maldacena, 1998]

The AdS/CFT duality claims that 5-dim. weakly coupled gravity theory in AdS spacetime is dual with 4-dim. strongly coupled gauge theory in the boundary.

- **Two consequences :**
  (1) The appropriate variables are the weakly coupled variables for describing strongly coupled gauge theory.
  (2) Weakly coupled gravity theory makes analysis much easier.
- It is an analytic method for describing strongly coupled field theories.

- Construct simple gravity models - use this correspondence - one obtains properties which are similar to some of the basic properties of superconductors.

Caution: Models are too crude to make detailed comparison with any real world material (Not provide any mechanism of dual theory from bulk theory)

Gauge/gravity duality

Why gravity theory : The AdS/CFT correspondence [Maldacena, 1998]

The AdS/CFT duality claims that 5-dim. weakly coupled gravity theory in AdS spacetime is dual with 4-dim. strongly coupled gauge theory in the boundary.

- **Two consequences :**
  (1) The appropriate variables are the weakly coupled variables for describing strongly coupled gauge theory.
  (2) Weakly coupled gravity theory makes analysis much easier.
- It is an analytic method for describing strongly coupled field theories.

- Construct simple gravity models - use this correspondence - one obtains properties which are similar to some of the basic properties of superconductors.

Caution: Models are too crude to make detailed comparison with any real world material (Not provide any mechanism of dual theory from bulk theory)

Gauge/gravity duality

Why gravity theory : The AdS/CFT correspondence [Maldacena, 1998]

The AdS/CFT duality claims that 5-dim. weakly coupled gravity theory in AdS spacetime is dual with 4-dim. strongly coupled gauge theory in the boundary.

- **Two consequences :**
  (1) The appropriate variables are the weakly coupled variables for describing strongly coupled gauge theory.
  (2) Weakly coupled gravity theory makes analysis much easier.
- It is an analytic method for describing strongly coupled field theories.

- Construct simple gravity models - use this correspondence - one obtains properties which are similar to some of the basic properties of superconductors.

Caution: Models are too crude to make detailed comparison with any real world material (Not provide any mechanism of dual theory from bulk theory).

Gravitational dual for superconductors

Superconductor (Gauge)	Gravity
Temperature	Black hole's temperature
Condensate	Charged scalar field

- Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.
- **Scalar hair :** A non-zero condensate corresponds to a static non-zero field outside a black hole.
- Matter fields outside a black hole wants to fall into the horizon (or radiate out to infinity in the asymptotically flat case).
- Asymptotically *AdS* spacetime acts like a confining box.

- Action for formation of scalar hair (Gubser, 2008)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}(R-2\Lambda) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\nabla_\mu\psi - iqA_\mu\psi|^2 - m^2|\psi|^2\right]$$

Using this model, Hartnoll, Herzog,and Horowitz computed numerically the ratio $\frac{\Delta_0}{k_B T_c} \approx 4$ and reproduced some properties of superconductors in 2008.

Gravitational dual for superconductors

Superconductor (Gauge)	Gravity
Temperature	Black hole's temperature
Condensate	Charged scalar field

- Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.
- **Scalar hair :** A non-zero condensate corresponds to a static non-zero field outside a black hole.
- Matter fields outside a black hole wants to fall into the horizon (or radiate out to infinity in the asymptotically flat case).
- Asymptotically *AdS* spacetime acts like a confining box.

- Action for formation of scalar hair (Gubser, 2008)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}(R-2\Lambda) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\nabla_\mu\psi - iqA_\mu\psi|^2 - m^2|\psi|^2\right]$$

Using this model, Hartnoll, Herzog,and Horowitz computed numerically the ratio $\frac{\Delta_0}{k_B T_c} \approx 4$ and reproduced some properties of superconductors in 2008.

Motivation

Investigated the properties of holographic superconductors in Born-Infeld electrodynamics with backreaction in Gauss-Bonnet gravity for d-dimension.

- We consider the action

$$S = \int d^d x \frac{\sqrt{-g}}{2\kappa^2}\left(R - 2\Lambda + \frac{\alpha}{2}(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}) + 2\kappa^2\mathcal{L}_m\right)$$

- $\mathcal{L}_m = \frac{1}{b}\left(1 - \sqrt{1 + \frac{b}{2}F^{\mu\nu}F_{\mu\nu}}\right) - (D_\mu\psi)^* D^\mu\psi - m^2\psi^*\psi$

$\Downarrow$ Born-Infeld electrodynamics (Non-linear) instead of Maxwell electrodynamics

$\Downarrow$ **Motivation** $\Downarrow$ Whether all parameters (b, κ, α) favour or disfavour for condensation.

$\Downarrow$ Gauss-Bonnet gravity

- Improve a new analytic tech. (thermodynamic geometry approach) and use it.
- Analytical investigate the conductivity in HSC with BI electrodynamics.
- The non-linear effects on holographic free energy and thermodynamic geometry.

Motivation

Investigated the properties of holographic superconductors in Born-Infeld electrodynamics with backreaction in Gauss-Bonnet gravity for d-dimension.

- We consider the action

$$S = \int d^d x \frac{\sqrt{-g}}{2\kappa^2} \Big(R - 2\Lambda + \frac{\alpha}{2}(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}) + 2\kappa^2 \mathcal{L}_m \Big)$$

- $\mathcal{L}_m = \frac{1}{b}\left(1 - \sqrt{1 + \frac{b}{2}F^{\mu\nu}F_{\mu\nu}}\right) - (D_\mu\psi)^* D^\mu\psi - m^2\psi^*\psi$

$\Downarrow$ Born-Infeld electrodynamics (Non-linear) instead of Maxwell electrodynamics

$\Downarrow$ **Motivation** $\Downarrow$ Whether all parameters (b, κ, α) favour or disfavour for condensation.

$\Downarrow$ Gauss-Bonnet gravity

- Improve a new analytic tech. (thermodynamic geometry approach) and use it.
- Analytical investigate the conductivity in HSC with BI electrodynamics.
- The non-linear effects on holographic free energy and thermodynamic geometry.

Motivation

Investigated the properties of holographic superconductors in Born-Infeld electrodynamics with backreaction in Gauss-Bonnet gravity for d-dimension.

- We consider the action

$$S = \int d^d x \frac{\sqrt{-g}}{2\kappa^2} \left(R - 2\Lambda + \frac{\alpha}{2}(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}) + 2\kappa^2 \mathcal{L}_m \right)$$

- $\mathcal{L}_m = \frac{1}{b}\left(1 - \sqrt{1 + \frac{b}{2}F^{\mu\nu}F_{\mu\nu}}\right) - (D_\mu\psi)^* D^\mu\psi - m^2\psi^*\psi$

$\Downarrow$ Born-Infeld electrodynamics (Non-linear) instead of Maxwell electrodynamics

$\Downarrow$ **Motivation** $\Downarrow$ Whether all parameters (b, κ, α) favour or disfavour for condensation.

$\Downarrow$ Gauss-Bonnet gravity

- Improve a new analytic tech. (thermodynamic geometry approach) and use it.
- Analytical investigate the conductivity in HSC with BI electrodynamics.
- The non-linear effects on holographic free energy and thermodynamic geometry.

Basic Set up

- **Ansatz** : $A_\mu = (\phi(r), 0, 0, 0)$, $\psi = \psi(r)$
- **Metric** : $ds^2 = -f(r)e^{-\chi(r)}dt^2 + \frac{1}{f(r)}dr^2 + r^2(dx^2 + dy^2)$
- **The Hawking temperature** $\Leftrightarrow$ **Temperature of CFT on boundary**

$$T_H = \frac{f'(r_+)e^{-\chi(r_+)/2}}{4\pi}$$

- Asymptotic behaviour of matter fields

$$\phi(z) = \mu - \rho\frac{z}{r_+}, \quad \psi(z) = \psi_-\frac{z^{\Delta_-}}{r_+^{\Delta_-}} + \psi_+\frac{z^{\Delta_+}}{r_+^{\Delta_+}}$$

where $\Delta_\pm = \frac{3\pm\sqrt{9+4m^2L^2}}{2}$. μ and ρ are the charge density and the chemical potential of the boundary field theory respectively.

- We choose $\psi_- = 0$, so that ψ_+ is dual to the expectation value of the condensation operator J at the boundary. This is because we want the condensate to turn on without being sourced.
- So, near the boundary matter field $\psi(z) = \frac{\langle \mathcal{O}_+ \rangle z^{\Delta_+}}{r_+^{\Delta_+}}$

CENTRE FOR BASIC SCIENCES S. N. BOSE NATIONAL 1986 1894-1974 विज्ञानेन परिपश्यन्ति धीराः

Basic Set up

- **Ansatz** : $A_\mu = (\phi(r), 0, 0, 0)$, $\psi = \psi(r)$
- **Metric** : $ds^2 = -f(r)e^{-\chi(r)}dt^2 + \frac{1}{f(r)}dr^2 + r^2(dx^2 + dy^2)$
- **The Hawking temperature $\Leftrightarrow$ Temperature of CFT on boundary**

$$T_H = \frac{f'(r_+)e^{-\chi(r_+)/2}}{4\pi}$$

- Asymptotic behaviour of matter fields

$$\phi(z) = \mu - \rho\frac{z}{r_+}, \quad \psi(z) = \psi_-\frac{z^{\Delta_-}}{r_+^{\Delta_-}} + \psi_+\frac{z^{\Delta_+}}{r_+^{\Delta_+}}$$

  where $\Delta_\pm = \frac{3\pm\sqrt{9+4m^2L^2}}{2}$. μ and ρ are the charge density and the chemical potential of the boundary field theory respectively.

- We choose $\psi_- = 0$, so that ψ_+ is dual to the expectation value of the condensation operator J at the boundary. This is because we want the condensate to turn on without being sourced.
- So, near the boundary matter field $\psi(z) = \frac{\langle\mathcal{O}_+\rangle z^{\Delta_+}}{r_+^{\Delta_+}}$

Critical temperature

- Under the change of coordinates $z = \frac{r_+}{r}$, the field equations become
- $\left(1 - \frac{2\alpha z^2 f}{r_+^2}\right) f' - \frac{(d-3)f}{z} + \frac{(d-1)r_+^2}{L^2 z^3} - \frac{2\kappa^2 r_+^2}{(d-2)z^3} \times$
$\left[\frac{z^4}{r_+^2} f \psi'^2 + \frac{\phi^2 \psi^2 e^{\chi}}{f} + m^2 \psi^2 \frac{\phi^2 \psi^2 e^{\chi}}{f} + \frac{1}{b}\left(\left(1 - \frac{bz^4}{r_+^2}\phi'^2\right)^{-\frac{1}{2}} - 1\right)\right] = 0$
- $\left(1 - \frac{2\alpha z^2 f(z)}{r_+^2}\right) \chi'(z) - \frac{4\kappa^2 r_+^2}{(d-2)z^3}\left(\frac{z^4}{r_+^2}\psi'(z)^2 + \frac{\phi^2(z)\psi^2(z)e^{\chi(z)}}{f(z)^2}\right) = 0$
- $\phi''(z) + \left(\frac{\chi'(z)}{2} - \frac{d-4}{z}\right)\phi'(z) + \frac{d-2}{r_+^2} b e^{\chi(z)} \phi'(z)^3 z^3 -$
$\frac{2r_+^2 \phi(z)\psi^2(z)}{f(z)z^4}\left(1 - \frac{bz^4 e^{\chi(z)}}{r_+^2}\phi'(z)^2\right)^{\frac{3}{2}} = 0$
- $\psi''(z) + \left(\frac{f'}{f} - \frac{d-4}{z} - \frac{\chi'}{2}\right)\psi'(z) + \frac{r_+^2}{z^4}\left(\frac{\phi^2 e^{\chi(z)}}{f(z)^2} - \frac{m^2}{f}\right)\psi(z) = 0$

At T_c, $\psi = 0$. Solve $\chi(z)$ and $\phi(z)$ ▶ Obtain metric for Einstein and GB gravity ▶ Substitute all in $\psi(z)$ eq. near T_c and using Sturm-Liouville eigenvalue method, get relation between T_c and ρ ▶ Expand $\phi(z)$ in $\frac{\langle \mathcal{O}_\pm \rangle^2}{r_+^2}$.

$T_c = \frac{1}{4\pi}\left[(d-1) - \frac{(d-3)^2}{(d-2)}\kappa_i^2(\lambda^2|_{\kappa_{i-1}})\right]\left(\frac{\rho}{\lambda}\right)^{\frac{1}{d-2}}$: $\langle \mathcal{O}_+ \rangle = \beta T_c^{\Delta_+}\sqrt{1 - \frac{T}{T_c}}$.

Critical temperature

- Under the change of coordinates $z = \frac{r_+}{r}$, the field equations become
- $\left(1 - \frac{2\alpha z^2 f}{r_+^2}\right) f' - \frac{(d-3)f}{z} + \frac{(d-1)r_+^2}{L^2 z^3} - \frac{2\kappa^2 r_+^2}{(d-2)z^3} \times$
  $\left[\frac{z^4}{r_+^2} f \psi'^2 + \frac{\phi^2\psi^2 e^{\chi}}{f} + m^2\psi^2 \frac{\phi^2\psi^2 e^{\chi}}{f} + \frac{1}{b}\left(\left(1 - \frac{bz^4}{r_+^2}\phi'^2\right)^{-\frac{1}{2}} - 1\right)\right] = 0$
- $\left(1 - \frac{2\alpha z^2 f(z)}{r_+^2}\right)\chi'(z) - \frac{4\kappa^2 r_+^2}{(d-2)z^3}\left(\frac{z^4}{r_+^2}\psi'(z)^2 + \frac{\phi^2(z)\psi^2(z)e^{\chi(z)}}{f(z)^2}\right) = 0$
- $\phi''(z) + \left(\frac{\chi'(z)}{2} - \frac{d-4}{z}\right)\phi'(z) + \frac{d-2}{r_+^2} b e^{\chi(z)}\phi'(z)^3 z^3 -$
  $\frac{2r_+^2\phi(z)\psi^2(z)}{f(z)z^4}\left(1 - \frac{bz^4 e^{\chi(z)}}{r_+^2}\phi'(z)^2\right)^{\frac{3}{2}} = 0$
- $\psi''(z) + \left(\frac{f'}{f} - \frac{d-4}{z} - \frac{\chi'}{2}\right)\psi'(z) + \frac{r_+^2}{z^4}\left(\frac{\phi^2 e^{\chi(z)}}{f(z)^2} - \frac{m^2}{f}\right)\psi(z) = 0$

At T_c, $\psi = 0$. Solve $\chi(z)$ and $\phi(z)$ ▶ Obtain metric for Einstein and GB gravity ▶ Substitute all in $\psi(z)$ eq. near T_c and using Sturm-Liouville eigenvalue method, get relation between T_c and ρ ▶ Expand $\phi(z)$ in $\frac{\langle\mathcal{O}_\pm\rangle^2}{r_+^2}$.

$$T_c = \frac{1}{4\pi}\left[(d-1) - \frac{(d-3)^2}{(d-2)}\kappa_i^2(\lambda^2|_{\kappa_{i-1}})\right]\left(\frac{\rho}{\lambda}\right)^{\frac{1}{d-2}} \quad : \quad \langle\mathcal{O}_+\rangle = \beta T_c^{\Delta_+}\sqrt{1 - \frac{T}{T_c}}.$$

Holographic free energy & thermodynamic geometry

- On-shell action
  $S_M = \int d^4x \left[\frac{\phi'^2(z)}{2} - \frac{F(z)\psi'^2(z)}{z^2} + \frac{\phi^2(z)\psi^2(z)}{z^2 F(z)} + \frac{2\psi^2(z)}{z^4} + \frac{b}{8} z^4 \phi'^4(z) + \mathcal{O}(b^2) \right].$
- The holographic free energy per volume :
  $\Omega = -T(S_{on} + S_c) = \beta T V_2 \left[-\frac{\mu\rho}{2} - \psi_+\psi_- + I \right] \quad \Rightarrow \quad \omega = \frac{\Omega}{V_2}$
- The thermodynamic metric is defined (Weinhold) : $g_{ij} = -\frac{1}{T} \frac{\partial^2 \omega(T,\rho)}{\partial x^i \partial x^j}$
- Metric component $\rightarrow g_{TT},\ g_{\rho\rho},\ g_{\rho T},\ g_{T\rho}$
- Riemannian scalar curvature (Ruppeiner) R

The critical point : $R \rightarrow \infty \Rightarrow det.g_{ij} = 0 \Rightarrow g_{TT} g_{\rho\rho} - g^2_{T\rho} = 0.$
The relation between the critical temperature and charge density.

The values of $\xi_{(\rho)} = \frac{T_c}{\sqrt{\rho}}$ for $d = 4,\ b = 0, \quad m^2 = -2 \Rightarrow \Delta_+ = 2$

M.M. (at $z = 0.33$)	$R \rightarrow \infty$	SL Method	Numerical
0.1190	0.1181	0.1180	0.1180

Holographic free energy & thermodynamic geometry

- On-shell action
$$S_M = \int d^4x \left[\frac{\phi'^2(z)}{2} - \frac{F(z)\psi'^2(z)}{z^2} + \frac{\phi^2(z)\psi^2(z)}{z^2 F(z)} + \frac{2\psi^2(z)}{z^4} + \frac{b}{8} z^4 \phi'^4(z) + \mathcal{O}(b^2)\right].$$
- The holographic free energy per volume :
$$\Omega = -T(S_{on} + S_c) = \beta T V_2 \left[-\frac{\mu\rho}{2} - \psi_+\psi_- + I\ \right] \quad \Rightarrow \quad \omega = \frac{\Omega}{V_2}$$
- The thermodynamic metric is defined (Weinhold) : $g_{ij} = -\frac{1}{T}\frac{\partial^2 \omega(T,\rho)}{\partial x^i \partial x^j}$
- Metric component $\rightarrow g_{TT},\ g_{\rho\rho},\ g_{\rho T},\ g_{T\rho}$
- Riemannian scalar curvature (Ruppeiner) R

The critical point : $R \ \rightarrow \infty \Rightarrow det.g_{ij} = 0 \Rightarrow g_{TT}g_{\rho\rho} - g^2_{T\rho} = 0.$
The relation between the critical temperature and charge density.

The values of $\xi_{(\rho)} = \frac{T_c}{\sqrt{\rho}}$ for $d = 4,\ b = 0,\quad m^2 = -2 \Rightarrow \Delta_+ = 2$

M.M. (at $z = 0.33$)	$R \ \rightarrow \infty$	SL Method	Numerical
0.1190	0.1181	0.1180	0.1180

Computation of conductivity

- The fluctuations in the Maxwell field in the bulk gives rise to the conductivity. For simplicity, we take $A_\mu = (0, 0, \varphi(r,t), 0)$ and $\varphi(r,t) = A(r)e^{-i\omega t}$ for the conductivity along the x-direction only.
- For $\Delta = \Delta_- = 1$ the EOM in z coordinate :
$$f(z)\frac{z^2}{r_+^2}\frac{d^2A}{dz^2} + \left[\frac{z^2}{r_+^2}f'(z) + \frac{zf(z)}{r_+^2}\right]\frac{dA}{dz} + \left[\frac{\omega^2}{z^2 f(z)} - \frac{2\psi^2(z)}{z^2}\right]A = 0$$
- From the definition of conductivity : $\sigma(\omega) = \frac{\langle J_x\rangle}{E_x} = -\frac{ir_+}{\omega}\frac{A'(0)}{A(0)}$
- The DC conductivity (low temp.) : $\text{Re.}[\sigma(\omega=0)] \sim e^{-\frac{E_g}{T}}$;
$E_g = \frac{3\langle\mathcal{O}_-\rangle}{2\pi}\left\{1 - \frac{\kappa^2\lambda^2}{6}\left(1 - \frac{b\lambda^2}{4}\right)\right\}$; $\quad E_g = 2\Delta_0$
- The conductivity (full spectra) :
$$\sigma(\omega) = \frac{i\langle\mathcal{O}_-\rangle}{\omega}\left\{1 + \frac{\kappa^2\lambda^2}{6}\left(1 - \frac{b\lambda^2}{4}\right)\right\}\sqrt{1 - \frac{\omega^2}{\langle\mathcal{O}\rangle^2}}$$

Imaginary part of conductivity has a pole $\frac{1}{\omega}$, using Kramers-Kronig relation one get a $\text{Re}[\sigma(\omega)] \propto \delta(\omega)$. So, DC conductivity is infinite at $\omega = 0$.
$\text{Im}[\sigma(\omega)] = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{Re[\sigma(\omega')]d\omega'}{\omega'-\omega}$

Computation of conductivity

- The fluctuations in the Maxwell field in the bulk gives rise to the conductivity. For simplicity, we take $A_\mu = (0, 0, \varphi(r,t), 0)$ and $\varphi(r,t) = A(r)e^{-i\omega t}$ for the conductivity along the x-direction only.
- For $\Delta = \Delta_- = 1$ the EOM in z coordinate :
$$f(z)\frac{z^2}{r_+^2}\frac{d^2A}{dz^2} + \left[\frac{z^2}{r_+^2}f'(z) + \frac{zf(z)}{r_+^2}\right]\frac{dA}{dz} + \left[\frac{\omega^2}{z^2 f(z)} - \frac{2\psi^2(z)}{z^2}\right]A = 0$$
- From the definition of conductivity : $\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{ir_+}{\omega}\frac{A'(0)}{A(0)}$
- The DC conductivity (low temp.) : $\mathrm{Re.}[\sigma(\omega = 0)] \sim e^{-\frac{E_g}{T}}$;
$E_g = \frac{3\langle \mathcal{O}_- \rangle}{2\pi}\left\{1 - \frac{\kappa^2\lambda^2}{6}\left(1 - \frac{b\lambda^2}{4}\right)\right\}$; $\quad E_g = 2\Delta_0$
- The conductivity (full spectra) :
$$\sigma(\omega) = \frac{i\langle \mathcal{O}_- \rangle}{\omega}\left\{1 + \frac{\kappa^2\lambda^2}{6}\left(1 - \frac{b\lambda^2}{4}\right)\right\}\sqrt{1 - \frac{\omega^2}{\langle \mathcal{O} \rangle^2}}$$

Imaginary part of conductivity has a pole $\frac{1}{\omega}$, using Kramers-Kronig relation one get a $\mathrm{Re}[\sigma(\omega)] \propto \delta(\omega)$. So, DC conductivity is infinite at $\omega = 0$.
$\mathrm{Im}[\sigma(\omega)] = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{Re[\sigma(\omega')]d\omega'}{\omega' - \omega}$

Conclusion

We derive the expressions for the relation between T_c and ρ, and the condensation operator in d-dimensions which yields the critical exponent to be $1/2$.

Higher value of the backreaction, Born-Infeld parameters and Gauss-Bonnet parameters result in a harder (disfavour) condensation to form in the all cases.

Analytically investigate the non-liner effects on the holographic free energy and thermodynamic geometry.

We have analytically computed the conductivity of holographic superconductors through two different approach in the framework of Born-Infeld electrodynamics. The band gap energy is found to be corrected by the backreaction and Born-Infeld parameters.

Conclusion

We derive the expressions for the relation between T_c and ρ, and the condensation operator in d-dimensions which yields the critical exponent to be $1/2$.

Higher value of the backreaction, Born-Infeld parameters and Gauss-Bonnet parameters result in a harder (disfavour) condensation to form in the all cases.

Analytically investigate the non-liner effects on the holographic free energy and thermodynamic geometry.

We have analytically computed the conductivity of holographic superconductors through two different approach in the framework of Born-Infeld electrodynamics. The band gap energy is found to be corrected by the backreaction and Born-Infeld parameters.

References and Acknowledgements

1. S.S. Gubser, Phys. Rev. D 78 (2008) 065034.
2. S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, Phys. Rev. Lett. 101, 031601 (2008).
3. G. Siopsis, J. Therrien, JHEP 05 (2010) 013.
4. F. Weinhold, J. Chem. Phys. 63 (6) (1975).
5. G. Ruppeiner, Rev. Mod. Phys. 67 (1995) 605.
6. **D. Ghorai**, S. Gangopadhyay, Eur. Phys. J. C, 76 (2016) 146.
7. **D. Ghorai**, S. Gangopadhyay, Phys.Lett. B 758 (2016) 106.
8. **D. Ghorai**, S. Gangopadhyay, EPL 118 (2017) 31001.
9. **D. Ghorai**, S. Gangopadhyay, Nucl. Phys. B 933 (2018) 1-13.

DG would like to thank DST-INSPIRE Fellowship, Govt. of India for financial support.

Thank you for your attention