Thermal Stability of Quantum Black Holes

Invited Talk at the Symposium on 'Current Developments in Quantum Field Theory and Gravity' at the S N Bose National Centre for Basic Sciences, Kolkata.

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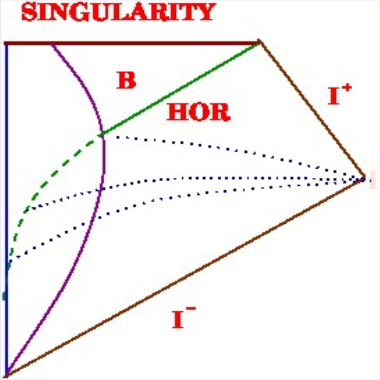
Spherical star collapsing to Black Hole (Penrose-Carter)



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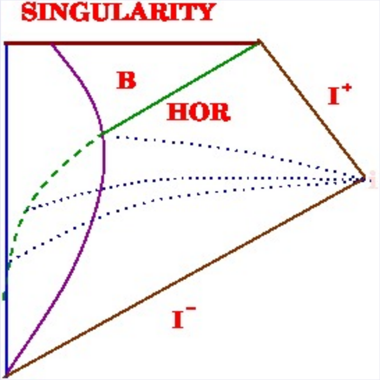

$$\mathbf{B} = \mathbf{M} - J^{-}(\mathbf{I}^{+}) \; ; \quad HOR = \partial \mathbf{B}$$

$$\begin{aligned} \delta A_{hor} &\geq 0 \\ \kappa_{hor} &= const \\ \delta M &= \kappa_{hor} \, \delta A_{hor} + \Phi \delta Q_{hor} + \cdots \end{aligned}$$

Black Holes Must Have Entropy !

Gen. Sec. Law of thermodynamics Bekenstein, 1973 :

$$\delta(S_{out} + S_{bh}) \geq 0 \Rightarrow S_{bh} = \frac{A_{hor}}{4A_P}$$

$A_P = l_P^2$, $l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33} cm \rightarrow$ Planck length signals onset of quantum gravity

Need to go beyond classical GR - compulsion, not aesthetics

$S_{bh} \sim l_P^{-2} \rightarrow$ **nonperturbative QG**

If black holes have entropy, they should have a temperature as well ! But what happens if $T_{bh} > T_{amb}$? Bekenstein (1973) : 'Paradox'

Hawking radiation : quantum fields near black hole horizon

Virtual particle-antiparticle pairs break-up near event horizon

One particle of each pair has finite probability to drift away to asymptopia, producing a thermal distribution (Hawking 1975)

$$F(\omega, T_H) \sim \frac{\omega^3 d\omega}{e^{\hbar\omega/k_B T_H} \pm 1}$$

$$T_H = \frac{\hbar}{2\pi} \kappa_{hor} \; , \; \kappa_{hor} \rightarrow \text{surface gravity at EH}$$

$T_H \sim M^{-1} \Rightarrow T_H << 1 \deg K$ **for stellar black holes**

Hawking radiation is swamped by CMB, hence unobservable

Semiclassical Thermal Instability

For a Schwarzschild black hole,

$$\begin{aligned} \delta M_{bh} &= \frac{\kappa_{hor}}{8\pi} \delta A_{hor} \\ T_{bh} &\equiv \frac{\hbar \kappa_{hor}}{2\pi} \sim \frac{1}{M_{bh}} \\ \Rightarrow \text{sp heat}\, C_{bh} &\equiv \frac{\partial M_{bh}}{\partial T_{bh}} < 0 \ ! \end{aligned}$$

$\Rightarrow$ **Thermal Instability : unabated Hawking rad !**

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- **Are all radiant black holes thermally unstable under Hawking rad ? No ! (Hawking-Page 1984)**
- **Apparently requires case-by-case analysis using classical black hole metrics**
- **Aim : Derive general thermal stability criteria irresp classical metric (PM 2007, Majhi-PM 2012, Sinha-PM 2016)**

Issues with Event Horizons : Trapping Horizons (Hayward 1997, Ashtekar et. al. 2000)

- Teleological
- Globally Stationary
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Trapping horizon :

- Nonstationary, marginally outer-trapped hypersurface
- TL or SL inner boundary with topology $\mathbf{R} \otimes \mathbf{S}^2$
- Can associate mass : $M_{hor} = M_{ADM} - M_{ext}$
- Equilibrates (thermally) to **Isolated Horizon** : non-stationary null inner boundary of const area

Isolated and Trapping Horizon Ashtekar et. al., 1997-2000

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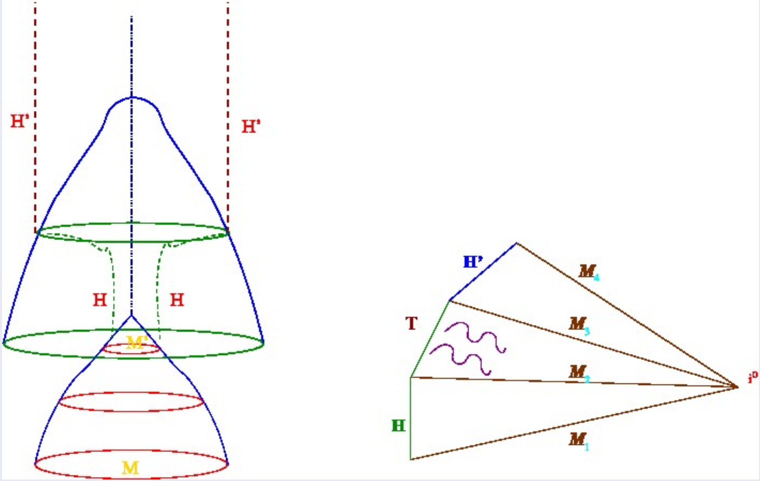
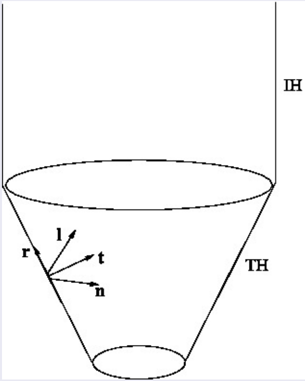


Trapping and Isolated Horizon (Ashtekar, Krishnan, 2005)



TH foliated by splk 2-surface : null normals **l** , **n** have
$\Theta_{\mathbf{l}} = 0$, $\Theta_{\mathbf{n}} < 0$ (marginally trapped)
Splk TH : accreting energy and growing : $L_{\mathbf{n}}\Theta_{\mathbf{l}} < 0$

Recap : Quantizing Isolated Horizons using LQG

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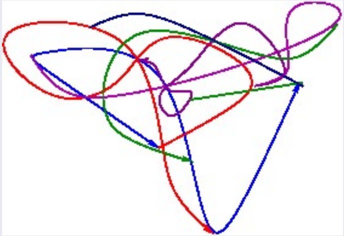

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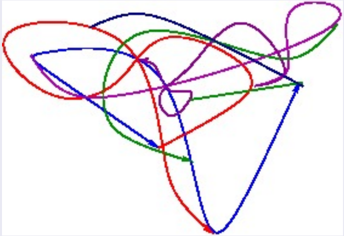

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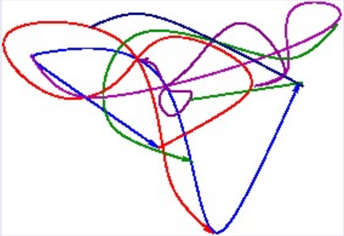

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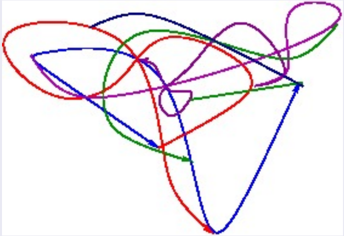

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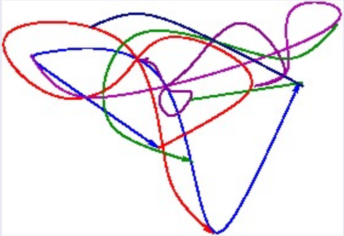

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- Graph : quantum state of space in **Spin network** basis
- **Geom observables : bounded, discrete spectra**

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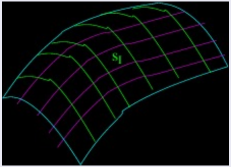

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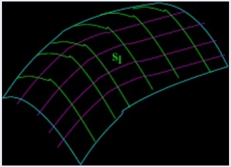

$$\hat{\mathcal{A}}_S \equiv \sum_{I=1}^{N} \int_{S_I} \det{}^{1/2}[{}^2g(\hat{E})]$$

$$a(j_1, \ldots, j_N) = 8\pi\gamma l_P^2 \sum_{p=1}^{N} \sqrt{j_p(j_p+1)}$$

$$\lim_{N\to\infty} a(j_1, \ldots j_N) \leq \mathcal{A}_{cl} + O(l_P^2) \text{ for } j_p \leq \frac{k}{2}$$

$$\text{Equispaced} \quad \forall j_p = 1/2$$

Quantum Black Hole (non-rotating)

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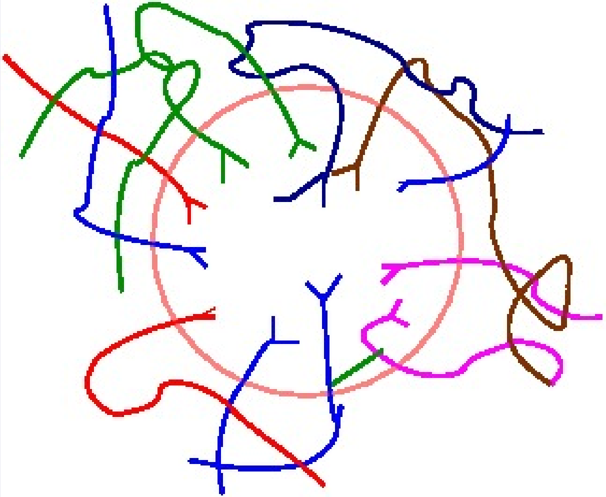

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- **IH described by $SU(2)$ Chern Simons theory, coupled to punctures created by bulk spin network edges carrying spin j_I , $I = 1, \ldots, N$, with the CS coupling $k \equiv A_{IH}/A_P \in \mathcal{Z}$.**

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- **IH described by $SU(2)$ Chern Simons theory, coupled to punctures created by bulk spin network edges carrying spin j_I , $I = 1, \ldots, N$, with the CS coupling $k \equiv A_{IH}/A_P \in \mathcal{Z}$.**
- IH area eigenvalues $a_{IH}(j_1, \cdots, j_N) \leq k \cdot A_P$

$$\hat{F}^i_{ab}|\Psi\rangle = -\frac{k}{2\pi} \sum_p a_{IH,ab}(p)\ \delta^{(2)}(x, x_p)\ J^i_{(p)}|\Psi\rangle$$

Recap : Black Hole Entropy (Kaul,PM, 1998, 2000; Majhi,PM 2014)

Count # of states of Chern-Simons quantum gauge theory with $\mathrm{total\ spin} = 0$

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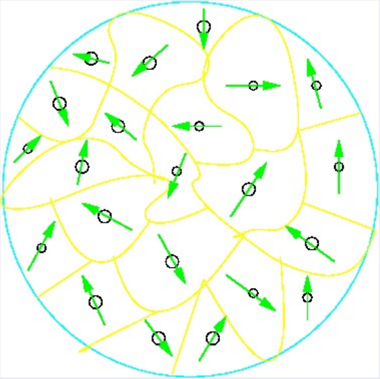
Dominant contribution from $j_l = 1/2$ if $A_{hor} >> A_P$ (macroscopic), but generalizes to arbitrary spin

$$S_{bh} = S_{BH} - \frac{3}{2} \log S_{BH} + O(S_{BH}^{-1}) \ , \ S_{BH} = k_B \ \frac{A_{hor}}{4A_P}$$

- **Systematic, finite quantum sptm fluctuation corrections to Bekenstein-Hawking entropy : signature of LQG**
- **Has been argued to have universal aspects : insensitive to charges and/or presence of cosmological constant**
- **Non-perturbative and background independent**

Recap : It from Bit (Kaul-PM 2001, Das-Kaul-PM 2001)



$A_{plaq} \sim l_{Pl}^2 : A_H/A_{plaq} \equiv N_H >> 1$

$$\mathcal{N} = \frac{N_H!}{((N_H/2)!)^2} - \frac{N_H!}{(N_H/2+1)!(N_H/2-1)!}$$

Upon using Stirling approximation for $(N_H)!$, obtain $\mathcal{N} \simeq \exp N_H/N_H^{3/2}$

Aside : Thermal Holography PM 2001, 2007, 2009; Majhi, PM 2011; Sinha, PM 2016

'Quantum General Relativity' : indep qu fluct on bdy : $\mathcal{H} = \mathcal{H}_b \otimes \mathcal{H}_h$

$$|\Psi\rangle = \sum_{b,h} C_{bh} |\psi\rangle_b \otimes |\chi\rangle_h$$

$$\hat{H} = \hat{H}_b \otimes \mathbf{1} + \mathbf{1} \otimes \hat{H}_h$$

Hamiltonian constraint : bulk

$$\left(\hat{H}_b \otimes \mathbf{1} + \mathbf{1} \otimes \hat{H}_h\right) |\psi\rangle_b = 0$$

Aside : Bulk vs Boundary

Partition Function

$$
\begin{aligned}
Z(\beta) &= Tr_h Tr_b \ \exp -\beta \hat{H} \\
&= \sum_{b,h} |C_{b,h}|^2_h \langle \chi || _b \langle \psi | \exp -\beta \hat{H} | \psi \rangle_b | \chi \rangle_h \\
&= Tr_h \exp -\beta \hat{H}_h \cdot \sum_b |C_{b,h}|^2 || |\psi\rangle_b ||^2 \\
&= Tr'_h \exp \beta \hat{H}_h \equiv Z_h(\beta)
\end{aligned}
$$

Bulk states decouple! Boundary states determine bh thermodynamics $\rightarrow$ Thermal holography !

Weaker than Holographic Hypothesis 't Hooft 1992; Susskind 1993; Bousso 2002

... Given any closed surface, we can represent all that happens (gravitationally) inside it by degrees of freedom on this surface itself. This ... suggests that quantum gravity should be described by a **topological** *quantum field theory in which all (gravitational) degrees of freedom are projected onto the boundary.*

Aside : Canonical ensemble of Isolated Horizons

- Assume $Z_h(\beta)$ is determined by dynamics of A_h , A_{NS} (kinem)
- Assume macroscopic areas $A_h \simeq n \cdot A_P$, $n >>> 1$
- Rescale area $A_h \rightarrow A_h/A_P$
- Assume time-scale such that on every $\Sigma_t \rightarrow$ quasi-equil with IH of fixed $\bar{A}_h$
- Keep Gaussian fluct. (Das, PM, Bhaduri 2001; Chatterjee, PM 2004; PM 2007)

$$Z_h(\beta) \simeq \int dA_h \exp\left[S_h(A_h) - \beta M_h\right]$$

for large area eigenvalues $n >> 1$.

Evaluate Z_h by saddle-pt expansion around $\bar{A}_{IH}$ $\Rightarrow$: Gaussian approx

Aside : Thermal Stability Criterion for isolated black holes

$$\beta(\bar{A}_h) \; = \; \frac{S_{h,A_h}(\bar{A}_h)}{M_{h,A_h}(\bar{A}_h)}$$

Saddle pt condition (at $\bar{A}_h$, $\bar{A}_m$)

$$\beta\left[M_{h,A_hA_h}\right]|_{\bar{A}_h} > \left[S_{h,A_hA_h}\right]|_{\bar{A}_h}$$

This can be further expressed as (with slight change notation)

$$\left(\log\left[\beta(A_h)_{A_h}\right]\right)_{A_h}(\bar{A}_h) < 0$$

Stability of TH $\Rightarrow$ the local temperature must increase with area

Thermal Stability Criterion for horizon (IH) : if satisied, IH stable against Hawking radiation, otherwise : Hawking evaporation ?

Aside : Thermal Stability Criterion

- Thermal Stability Criterion for radiant black hole (Chatterjee, PM 2005; PM 2007; Majhi, PM 2011; Sinha, PM 2016)
- Equil $\beta \Rightarrow$ th stab crit :

$$\frac{M_{hA_hA_h}}{M_{hA_h}} > \frac{S_{hA_hA_h}}{S_{hA_h}}.$$

- Generalizes to charged, rotating horizons (Majhi, PM 2011; Sinha, PM 2016)
- **No classical metric used in derivation**
- **Corrections to area law for S_{IH} makes stability criterion nontrivial**

$$S = S_{BH} - \frac{3}{2}\log S_{BH} + O(S_{BH}^{-1}) \ , \ S_{BH} \equiv \frac{1}{4}A_h$$

Aside : Fiducial Checks PM 2001

Schwarzschild black hole

$$M^2 = \frac{A}{16\pi}$$

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Schwarzschild black hole

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Examine explicitly thermal stability criterion :

$$\frac{M_{AA}}{M_A} = -\left(\frac{2}{A}\right) , \frac{S_{AA}}{S_A} = \frac{6l_P^2/A^2}{(1-6l_P^2/A)}$$
$$\Delta < 0$$

Violates stability bound for $A_h >> l_P^2 \rightarrow$ **thermally unstable**

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Violates stability bound for $A_h >> l_P^2 \rightarrow$ thermally unstable :

$$\text{AdS Schwarzschild : } M^2 = \frac{A}{16\pi} + \frac{A^2}{32\pi^2 l^2}$$
$$\Rightarrow \Delta > 0 \text{ for } A >> l^2 = (-\Lambda)^{-1/2}$$

AdS-Schwarzschild black hole is stable for $A >> (-\Lambda)^{-1/2}$

Generalization to charged, rotating bh Sinha, PM 2016

For charged, rotating black holes, assume

- $M = M(A, Q, J)$
- $A = al_P^2$, $Q = ql_P$, $J = jl_P^2$, $a, q, j \in \mathcal{Z}$, $a, q, j >> 1$

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Grand Canonical Partition Function

$$Z_G = \int dA \, dQ \, dJ \, \exp[S(A) - \beta(E(A, Q, J) - \Phi Q - \Omega J)]$$

$$\exp S(A) \equiv \frac{g(A(x), Q(y), J(z))}{\frac{dA}{dx}\frac{dQ}{dy}\frac{dJ}{dz}}$$

where, $S(A)$ is the microcanonical entropy.

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Saddle Point Approximation

- Expand around saddle point $\bar{A}, \bar{Q}, \bar{J}$ corresponding to Isolated horizon parameters
- Retain upto Gaussian fluctuations

Gaussian Approximation

$$
\begin{aligned}
Z_G &= \exp[S(\bar{A}) - \beta M(\bar{A}, \bar{Q}, \bar{J}) + \beta\Phi\bar{Q} + \beta\Omega\bar{J}] \\
&\times \int dadqdj \;\; \exp\{-\frac{\beta}{2}[(M_{AA} - \frac{S_{AA}}{\beta})a^2 + (M_{QQ})q^2 \\
&+ (2M_{AQ})aq + (M_{JJ})j^2 + (2M_{AJ})aj + (2M_{QJ})qj]\}
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$$
\beta = \frac{S_A}{M_A} \; , \;\; M_Q = \Phi, M_J = \Omega
$$

β is the inverse temperature and always assumed > 0

Thermal Stability Criteria

Convergence $\Rightarrow$ $\det H > 0$ where, H is the **Hessian** matrix

$$H = \begin{pmatrix} \beta M_{AA}(\bar{A},\bar{Q},\bar{J}) - S_{AA}(\bar{A}) & \beta M_{AQ}(\bar{A},\bar{Q},\bar{J}) & \beta M_{AJ}(\bar{A},\bar{Q},\bar{J}) \\ \beta M_{AQ}(\bar{A},\bar{Q},\bar{J}) & \beta M_{QQ}(\bar{A},\bar{Q},\bar{J}) & \beta M_{JQ}(\bar{A},\bar{Q},\bar{J}) \\ \beta M_{AJ}(\bar{A},\bar{Q},\bar{J}) & \beta M_{JQ}(\bar{A},\bar{Q},\bar{J}) & \beta M_{JJ}(\bar{A},\bar{Q},\bar{J}) \end{pmatrix}$$

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This leads to the **Stability Criteria**

$$\begin{aligned} \Delta \equiv \beta M_{AA}(\bar{A},\bar{Q},\bar{J}) - S_{AA}(\bar{A}) &> 0 \\ \beta M_{QQ}(\bar{A},\bar{Q},\bar{J}) &> 0 \\ \beta M_{JJ}(\bar{A},\bar{Q},\bar{J}) &> 0 \\ M_{QQ}(\bar{A},\bar{Q},\bar{J}) M_{JJ}(\bar{A},\bar{Q},\bar{J}) &- (M_{JQ}(\bar{A},\bar{Q},\bar{J}))^2 > 0 \\ M_{JJ}(\bar{A},\bar{Q},\bar{J})(\beta M_{AA}(\bar{A},\bar{Q},\bar{J}) - S_{AA}(\bar{A})) &- \beta(M_{AJ}(\bar{A},\bar{Q},\bar{J}))^2 > 0 \\ M_{QQ}(\bar{A},\bar{Q},\bar{J})(\beta M_{AA}(\bar{A},\bar{Q},\bar{J}) - S_{AA}(\bar{A})) &- \beta(M_{AQ}(\bar{A},\bar{Q},\bar{J}))^2 > 0 \\ \det H &> 0 \end{aligned}$$

Predict thermal stability of all charged, rotating black holes !

Fiducial check : Quasi-stability (PM, Sinha 2017)

Classical bh metrics $\rightarrow M^2(A, Q, J)$ rather than $M(A, Q, J) \Rightarrow$

$$\Delta = \frac{M^2_{AA}}{M^2_A} - \frac{1}{2}\frac{M^2_A}{M^2} - \frac{S_{AA}}{S_A}$$

Generally, for most charged, rotating black holes

$$M^2 = \frac{1}{A}\left[a_0 + a_1\ A + a_2\ A^2\right]\ ,\ a_0, a_1, a_2 = f(Q, J) > 0$$

$$\Rightarrow \Delta \propto \left[\frac{3a_0^2}{a_2^2} + 4\frac{a_0 a_1}{a_2^2}A + \frac{6a_0}{a_2}A^2 - A^4\right]$$

Macroscopic black holes : For large $A >> A_P$, negative coeff of A^4 $\Rightarrow$ $\exists$ $[A_{min}, A_{Max}]$ such that $\Delta > 0$ for $A \in [A_{min}, A_{Max}]$

Fiducial check : Quasi-stability (PM, Sinha 2017)

Classical bh metrics $\rightarrow M^2(A,Q,J)$ rather than $M(A,Q,J) \Rightarrow$

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Quasi-stability : happens for AF black holes with charge and ang mom, absent for AF Schwarzschild black holes !

Application to Kerr-Newman black hole : Quasi-stability

$$ds^2 = -\frac{\Xi}{\Sigma}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\sin^2\theta}{\Sigma}((r^2+a^2)d\phi - a dt)^2 + \frac{\Sigma}{\Xi}dr^2 + \Sigma d\theta^2$$

where, $\Xi \equiv r^2 - 2Mr + a^2 + Q^2$, $\Sigma \equiv r^2 + a^2 cos^2\theta$, $a = \frac{J}{M}$

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Quasi-stability

$$\begin{aligned} \beta &> 0 \text{ for } A^2 > 16\pi^2(4J^2 + Q^4) \\ \Delta &\equiv (S_A/M_A)M_{AA} - S_{AA} \\ \Delta &> 0 \text{ for } 96\pi^2(4J^2 + Q^4) > A^2 > 16\pi^2(4J^2 + Q^4) \end{aligned}$$

In contrast to AF Schwarzschild bh, $\exists$ narrow region of parameter space in which all criteria but $\det H > 0$ **are OK ! ?**

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