Signature of Noncommutative geometry on resonant detectors of Gravitational Wave

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- Likewise, quantization of the space $\Rightarrow$ Restriction on the simulteneous determination of different components of space coordinates $\Rightarrow$ the NC Heisenberg algebra

  $$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \;, \quad [\hat{x}_i, \hat{x}_j] = i\theta_{ij} \;, \quad [\hat{p}_i, \hat{p}_j] = i\bar{\theta}_{ij} \;.$$

  $\theta, \bar{\theta} =$ constant antisymmetric tensor

  $\Rightarrow$ noncommutative (NC) space.

NCQM/NCFT $\rightarrow$ commutative equivalent picture

- NCFT: Action with NC fields and star product;
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- This NC phase-space algebra $\to$ Standard Heisenberg algebra spanned by the operators $\hat{X}_i$ and $\hat{P}_j$ of the ordinary QM through the transformation equations
  $\hat{x}_i = \hat{X}_i - \frac{1}{2\hbar}\theta\epsilon_{ij}\hat{P}_j \ , \quad \hat{p}_i = \hat{P}_i + \frac{1}{2\hbar}\bar{\theta}\epsilon_{ij}\hat{X}_j$

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- Successful detection of GW in aLIGO, an interferometric detector. (but justifying a QM modeling is tricky.)
- Instead, we modelled the Resonant bar detectors of GW(still trying to achieve the needed sensitivity limit for successful detection) quantum mechanically in NC space .

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- Typical cylindrical Aluminium bar with $L \sim 3\,\mathrm{m}$, $r = 30\,\mathrm{cm}$, $M = 2 \times 10^3\,\mathrm{Kg}$.

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- SO we model a NCQM Harmonic oscillator interacting with GW.

Results

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- The same bar-detector resonating to two evenly placed frequencies, centering the intrinsic frequency of the bar can be a potential evidance of NC space.
- In such a scenario, since the intrinsic frequency ϖ is known, and resonance frequencies ϖ' would be observed, one can ontain Λ_θ.

Publications

1. AS, SG, PL**B 681** 96, 2009
2. AS, SG, SS, PR**D 83** 025004, 2011
3. SG, AS, MPL**A 35** 1250192, 2012
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5. SG, AS, SS, GRG **47** 6, 65 (2015)
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7. AS, SG, CQG **33** 205006 (2016)
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