EMERGENT UNIVERSE VIA WORMHOLE

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Gravity Equation in Cosmology

1915, GR : Einstein's Field Equation :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \qquad (1)$$

$$GRAVITY \Longleftrightarrow MATTER$$

Universe is homogeneous and isotropic. RW line element

$$ds^2 = -dt^2 + a(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin^2\theta d\phi^2) \right] \qquad (2)$$

The energy momentum tensor : $T^{\mu}_{\mu} = Diagonal(-\rho, p, p, p)$

Einstein's Field Equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{3}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G p \tag{4}$$

The conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{5}$$

where $H = \frac{\dot{a}}{a}$ represents Hubble parameter.

Emergent Universe

- A universe which is ever existing, large enough so that space-time may be treated as classical entities.
- No time like singularity
- The universe in the infinite past is in an almost static state but it eventually evolves into an inflationary stage

Ellis and Maartens, CQG 21 (2004) 223

- Considered a dynamical scalar field to obtain EU in a closed universe ($k = +1$). In the model a minimally coupled scalar field ϕ with a self interacting potential $V(\phi)$ was considered.
- In the case the initial size a_i of the universe is determined by the KE of the field.
- To understand we consider a model consisting of ordinary matter and minimally coupled homogeneous scalar field.

The Klein-Gordon equation for scalar field

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \quad (6)$$

$$\dot{\rho} + 3H(1+\omega)\rho = 0 \quad (7)$$

using EOS $p = \omega\rho$. Now the Raychaudhuri equation can be written as

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\frac{1}{2}(1+3\omega)\rho + \dot{\phi}^2 - V(\phi)\right] \quad (8)$$

First integral gives Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\left[\rho + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] - \frac{k}{a^2} \tag{9}$$

It leads to

$$\dot{H} = -4\pi G\left[\dot{\phi}^2 + (1+\omega)\rho\right] + \frac{k}{a^2} \tag{10}$$

Now, an accelerating universe ($\ddot{a} > 0$) demands

$$\dot{\phi}^2 + \frac{1}{2}(1+3\omega)\rho < V(\phi) \tag{11}$$

For a positive minimum

$$H_i = 0 \Rightarrow \frac{1}{2}\dot{\phi}_i^2 + V_i + \rho_i = \frac{3k}{8\pi G a_i^2} \tag{12}$$

where t_i may be infinte.

The Einstein static universe is characterized by $k = 1$ and $a = a_i = constant$. We therefore obtain

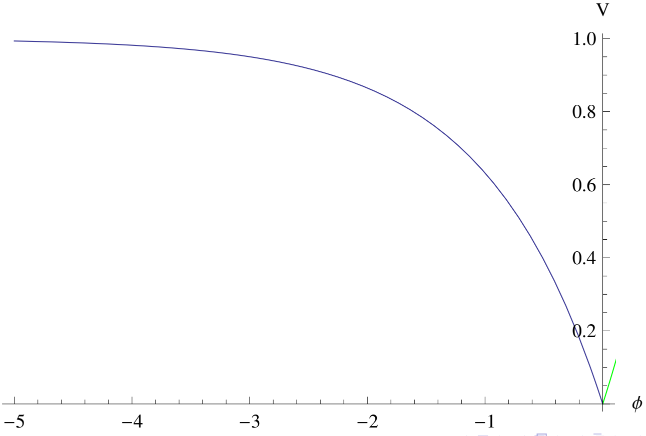
$$\frac{1}{2}(1-\omega)\rho_i + V_i = \frac{1}{4\pi G a_i^2} \tag{13}$$

$$(1+\omega)\rho_i + \dot{\phi}_i^2 = \frac{1}{4\pi G a_i^2} \tag{14}$$

- If the KE of the scalar field vanishes, there must be matter to obtain a static universe.
- If only scalar field with non-zero KE, then the field rolls at a constant speed along the flat potential.

- A simple potential for EU model with scalar field only is given by a potential having the characteristics $V(\phi) \to V_i$ as $\phi \to \infty$ and $t \longrightarrow -\infty$. But drops towards a minimum at a finite value ϕ_f.

Form of the Potential $V - V_f = (V_i - V_f)\left[Exp[\frac{\phi-\phi_i}{\alpha}] - 1\right]^2$



Scalar field Potential

The corresponding potential in EU is the reflection of the potential that obtained from higher derivative gravity. The different parts of the potential are :

- Slow-rolling regime or intermediate pre-slow roll phase
- Scale factor grows (slow-roll phase)
- inflation is followed by a re-heating phase
- standard hot Big Bang evolution

Ellis et al., CQG 21, 233 (2004)

Making use of R^2-modified gravity. The gravitational action

$$I = \int d^4x\sqrt{-g}[R + \alpha R^2] \tag{15}$$

Define a conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{16}$$

Here $\Omega^2 = 1 + 2\alpha R$, one obtains

$$\tilde{R} = \frac{1}{\Omega^2}\left[R - 6g^{\mu\nu}\nabla_\mu\nabla_\nu)(\ln\Omega) - 6g^{\mu\nu}\nabla_\mu(\ln\Omega)\nabla_\nu(\ln\Omega)\right] \tag{17}$$

Now set $\phi = \sqrt{3}\ln(1 + 2\alpha R)$ one obtains

$$I = \int d^4x\sqrt{-g}\left[\tilde{R} - \frac{1}{2}\tilde{g}_{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{4\alpha}\left(e^{-\frac{\phi}{\sqrt{3}}} - 1\right)^2\right] \tag{18}$$

EU Model in a spatially flat case

S Mukherjee, **BCP**, N K Dadhich, S D Maharaj, A Beesham, CQG 23, 6927 (2006)
Preamble : In looking for a model of emergent universe, the following features for the universe are assumed:

- The universe is isotropic and homogeneous at large scales.
- Spatially flat (WMAP results) :
- It is ever existing, No singularity
- The universe is always large enough so that classical description of space-time is adequate.
- The matter or in general, the source of gravity has to be described by quantum field theory.
- The universe may contain exotic matter (SEC violated)
- The universe is accelerating

EOS

$$p = A\rho - B\sqrt{\rho} \tag{19}$$

The Einstein equations for a flat universe in RW-metric ($G = \frac{1}{8\pi}$)

$$\rho = 3\frac{\dot{a}^2}{a^2} \tag{20}$$

$$p = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \tag{21}$$

Making use of the EOS we obtain

$$2\frac{\ddot{a}}{a} + (3A+1)\frac{\dot{a}^2}{a^2} - \sqrt{3}B\frac{\dot{a}}{a} = 0 \tag{22}$$

On integration

$$a(t) = \left(\frac{3\kappa(A+1)}{2}\left(\sigma + \frac{2}{\sqrt{3}B}Exp\left[\frac{\sqrt{3}Bt}{2}\right]\right)\right)^{\frac{2}{3(A+1)}} \tag{23}$$

- If $B < 0$ Singularity
- If $B > 0$ and $A > -1$ Non-singular (EU)

Using EOS, in $\frac{d\rho}{dt} + 3(\rho + p)\frac{\dot{a}}{a} = 0$ one obtains

$$\rho(a) = \frac{1}{(A+1)^2}\left(B + \frac{K}{a^{\frac{3(A+1)}{2}}}\right)^2 \tag{24}$$

where K is an integration constant.
This provides us with indications about the components of energy density in EU.

$$\rho(a) = \Sigma_{i=1}^{3}\rho_i \quad and \quad p(a) = \Sigma_{i=1}^{3} p_i \tag{25}$$

where we denote

$$\rho_1 = \frac{B^2}{(A+1)^2}, \quad \rho_2 = \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad \rho_3 = \frac{K^2}{(A+1)^2}\frac{1}{a^{3(A+1)}} \tag{26}$$

$$p_1 == -\frac{B^2}{(A+1)^2}, \quad p_2 = \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad p_3 = \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}. \tag{27}$$

Composition of EU model

Comparing with the barotropic EoS given by $p_i = \omega_i \rho_i$ one obtains

- $\omega_1 = -1$
- $\omega_2 = \frac{A-1}{2}$
- $\omega_3 = A$

Table-I

A	$\omega_2 = \frac{1}{2}(A-1)$	$\omega_3 = A$	Composition
$\frac{1}{3}$	- $\frac{1}{3}$	$\frac{1}{3}$	DE, Exotic Matter, Radiation
-$\frac{1}{3}$	-$\frac{2}{3}$	-$\frac{1}{3}$	DE, Exotic Matter, Cosmic String
1	0	1	DE, Exotic Matter, Stiff fluid
0	-$\frac{1}{2}$	0	DE, Exotic Matter, Dust

The three fluids model

The original emergent universe model is composed of three non-interacting fluids. Here we bring interaction among themselves at $t \geq t_0$,

$$\dot{\rho_1} + 3H(\rho_1 + p_1) = -Q', \tag{28}$$

$$\dot{\rho_2} + 3H(\rho_2 + p_2) = Q, \tag{29}$$

$$\dot{\rho_3} + 3H(\rho_3 + p_3) = Q' - Q, \tag{30}$$

where Q and Q' represent the interaction terms.

Here $\rho_1 \longrightarrow$ DE density, $\rho_2 \longrightarrow$ DM and $\rho_3 \longrightarrow$ normal matter.

- $Q < 0 \Rightarrow$ energy transfer from DM sector to the other two constituents,
- $Q' > 0 \Rightarrow$ energy transfer from DE sector to the other two fluids
- $Q' < Q \Rightarrow$ energy loss from the matter sector.
- $Q = Q' \Rightarrow$ DE interacts only with the DM.

The equivalent effective uncoupled model is described by the conservation equations:

$$\dot{\rho}_1 + 3H(1 + \omega_1^{eff})\rho_1 = 0 \tag{31}$$

$$\dot{\rho}_2 + 3H(1 + \omega_2^{eff})\rho_2 = 0 \tag{32}$$

$$\dot{\rho}_3 + 3H(1 + \omega_3^{eff})\rho_3 = 0 \tag{33}$$

where the effective equation of state parameters are

$$\omega_1^{eff} = \omega_1 + \frac{Q'}{3H\rho_1}, \tag{34}$$

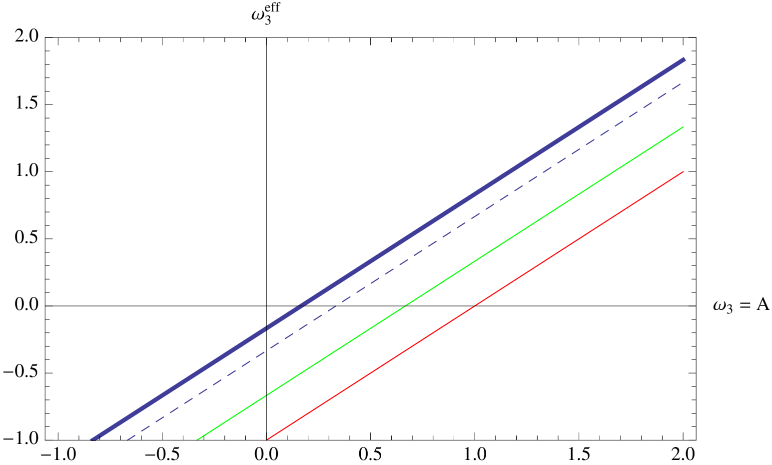
$$\omega_2^{eff} = \omega_2 - \frac{Q'}{3H\rho_2}, \tag{35}$$

$$\omega_3^{eff} = \omega_3 + \frac{Q - Q'}{3H\rho_3}. \tag{36}$$

Let us consider $Q - Q' = -\beta H \rho_3$, the effective state parameter for the normal fluid becomes

$$\omega_3^{eff} = \omega_3 - \frac{\beta}{3} \tag{37}$$

Plot the variation of effective equation of state parameter ω_3^{eff} with A which corresponds to ω_3 for different strengths of interaction determined by β. The strength of interaction is increased ω_3 which is actually determined by A for which $\omega^{eff} = 0$ (corresponds to matter domination). A universe with any A value is found to admit a matter domination phase which is not permitted in the absence of interaction.



ω_3^{eff}
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
ω_3 = A

- Gravitational action for massive Gravity

$$S = -\frac{1}{8\pi}\int\left(\frac{1}{2}R + m^2 L\right)\sqrt{-g}\,d^4x + S_m \tag{38}$$

- Massive Gravity Lagrangian

$$L = -\frac{1}{2}\left(K^2 - K^\mu_\nu K^\nu_\mu\right) + \frac{c_3}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}K^\mu_\alpha K^\nu_\beta K^\rho_\gamma + \frac{c_4}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta} K^\mu_\alpha K^\nu_\beta K^\rho_\gamma K^\sigma_\delta \tag{39}$$

where c_3, c_4 are constants and $K^\mu_\nu = \delta^\mu_\nu - \gamma^\mu_\nu$, $\gamma^\mu_\sigma \gamma^\sigma_\nu = g^{\mu\nu} f_{\sigma\nu}$, and $f_{\sigma\nu}$ is symmetric tensor.

- For Euclidean time

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G\rho}{3} - \frac{m^2}{3}\left(4c_3 + c_4 - 6 + 3C\frac{3 - 3c_3 - c_4}{a}\right) - \frac{m^2}{3}\left(3C^2\frac{c_4 + 2c_3 - 1}{a^2} - C^3\frac{c_3 + c_4}{a^3}\right) \tag{40}$$

Wormhole for EU

- Conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{41}$$

- The field equation can be rewritten as

$$\dot{a}^2 = V(a) \tag{42}$$

- $V(a) = -\frac{m^2}{3}\left[(4c_3 + c_4 - 6 + 3\Lambda)a^2 - \frac{c_3+c_4}{a} + X\right]$
-

$$\Lambda = \frac{8\pi G}{3m^2}\rho$$

$$X = 3(3 - 3c_3 - c_4)a + 3(c_4 + 2c_3 - 1)$$

$$p = A\rho - B\sqrt{\rho} \tag{43}$$

- we note existence of wormhole solution

$$\dot{a}^2 = 1 - \mu a^2 + \Sigma_{n=1}^{N}\frac{\nu_n}{a^{2n}}; \tag{44}$$

- The field equation can be rewritten as

$$\dot{a}^2 = \alpha - \beta a^2 - \frac{\gamma}{a^2} \tag{45}$$

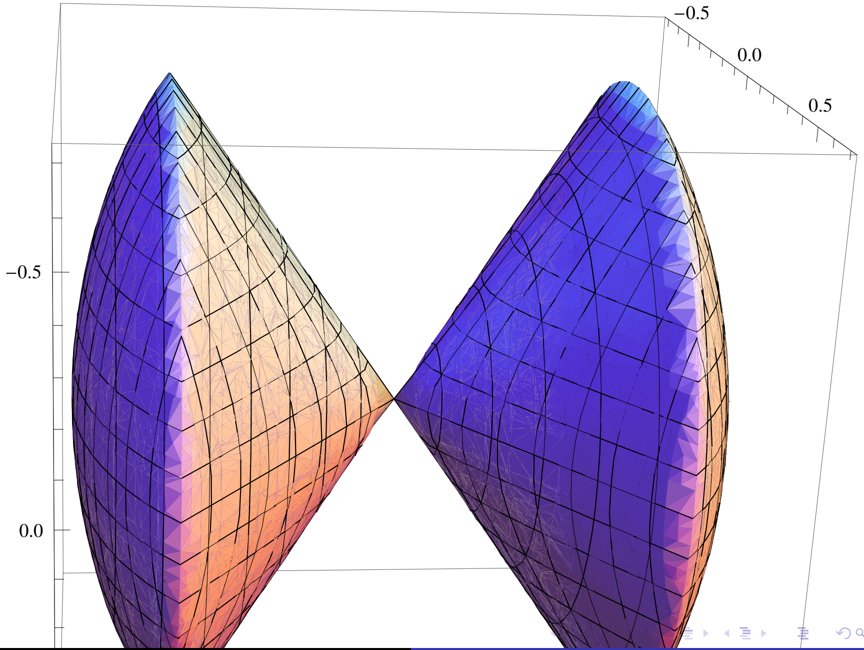
- where

$$\alpha = \frac{1}{2}\left(1 - \frac{4K}{B}\Lambda\right)$$

$$\beta = \frac{1}{2}(2\Lambda - 1) \quad \gamma = \left(\frac{K}{B}\right)^2 \Lambda$$

$$\frac{1}{2} < \Lambda < \frac{B}{4K}$$

- The above differential admits (i) $\dot{a}(\tau) < 0$ (ii)$\dot{a}(\tau) = 0$ (iii) $\dot{a}(\tau) < 0$
- It is found that in GTR with nonlinear EoS $\alpha < 0$ in flat universe. Consequently No WORMHOLE. However, in massive gravity $\alpha \geq 1$, WORMHOLE exists.


−0.5
0.0
0.5
−0.5
0.0

- The wormhole solution we obtain for EU model

$$a^2(\tau) = \frac{\frac{1}{4} - \frac{K}{B}\Lambda}{2\Lambda - 1} + a_o \; Cos\sqrt{2(2\Lambda - 1)} \;\; \tau \tag{46}$$

- where

$$a_o = \frac{\sqrt{\left(\frac{1}{4} - \frac{K}{b}\Lambda\right)^2 - 2\Lambda\left(\frac{K}{B}\right)^2}}{2\Lambda - 1}$$

$$\Lambda = \left(\frac{B}{A+1}\right)^2 \tag{47}$$

- Wormhole solution in GTR is not permitted in flat universe
- Wormhole solution in massive gravity theory is permitted in flat universe case.

Discussion

- Demonstrated the possibility of obtaining viable cosmological dynamics of the emergent universe. The initial static phase of the universe can be recovered.

THANK YOU