Should photon be massless?

Zero mass, Asymptotic symmetries, Infrared divergence in QED

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PLAN

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Riemann Quotes:...

- Riemann in his famous habilitation lecture: *..Considerations become important in the extensions of these empirical determinations beyond the limits of observation to the infinitely great and infinitely small, since the latter may become more inaccurate beyond the limits but not the former....*
- *We must distinguish between unboundedness and infinite extent: the former belongs to extent relations and the later to the measure relations..*
- *The questions about infinitely great are for the interpretation of nature of useless questions, but this is not the case with infinitely small..*
- *On the hypotheses which lie at the bases of geometry: Bernhard Riemann, 1854*

- I gave a talk on ‘Is photon massless?’ at Dublin Inst of Advanced Studies in January 2018. To my surprise found Schrodinger talked about same question..in 1955.
- Must the Photon Mass be Zero? Author(s): L. Bass and E. Schrodinger Source: Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences,232,1188 (Oct. 11, 1955), pp. 1-6
- He was posing the question since massive photon will have an extra degree of freedom. While calculating in blackbody radiation energy density as a function of frequencies we multiply by a factor of 2 to account for the transverse degrees of freedom.
- Should we multiply by 3 to account for the additional longitudinal degree of freedom?
- If vector potential A_μ is coupled to conserved $j_\mu : \partial_\mu j^\mu = 0$ only a small change in the s-matrix contributions if m_γ is small.

Massive photon and conserved current

- Moral is we will still get the factor of 2 instead of 3
- But we can estimate the bound on the mass of the 'photon'
- Schrodinger himself estimated mass of the photon. The massive photon equation would be

$$(\nabla^2 - m_\gamma^2)\vec{A} = -\mu_0 \vec{J} \tag{1}$$

- The solution is

$$\vec{A} \approx \frac{\mu_0}{4\pi} \nabla \times \left(\vec{m} \frac{e^{-m_\gamma r}}{r} \right) \tag{2}$$

  where $\vec{m}$ is magnetic dipole moment.
- The magnetic field is:

$$B_z = \frac{m\ \mu_0}{4\pi} \left[\frac{3z^2 - r^2}{r^3} - \frac{2\ m_\gamma^2}{3\ r} e^{-m_\gamma\ r} \right] \tag{3}$$

Estimates of mass of the photon

- Considering the earth as a magnet we can estimate the photon mass using the above formula and comparing with data.
- The magnetic field has extra 'non-potential' contribution. which depends on the mass of the photon m_γ^2.
- It has a negative contribution and reduces the magnetic field.
- This can be compared with experimental estimates to obtain the mass limits..
- This gave Bass and Schrodinger $m_\gamma \ \leq \ 10^{-47} g$. By careful analysis of the geomagnetic field this was improved by Goldabher and Nieto to $\leq \ 10^{-48} g$.
- Schrodinger's estimates based on astronomical magnetic surveys are still very good. Based on dissipation of large scale magnetic fields in the galaxy there are now estimates $\leq \ 10^{-56} g$ which are in the list with question marks.

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Estimates of mass of the photon-contd

- Compton wavelength $\lambda = h/m\,c$ of these correspond to radius of solar sytem and galaxy or $\leq\ 10^{-16}eV$ and $10^{-24}eV$. Size of the universe provides a cutoff which can be maximum we can provide!!
- There are laboratory experiments too. Wavelength independence of velocity of light is one of the direct consequences of photon mass being zero.
- Using radio wave interferometer over a large frequency range veleocity difference has been measured. it was found to be $\frac{\Delta c}{c} \leq 10^{-5}$. This corresponds to $m_\gamma \ \leq\ 10^{-42}g$. Astronomical estimates are better.
- There are several other estimates with lot of uncertainties and the best so far can take is either Erwin Schrodinger estimate of $m_\gamma \leq 10^{-18}eV$ or galactic magnetic field estimate $m_\gamma \ \leq 10^{-24}eV$.

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Asymptotic symmetries in QED

- QED has massless photons, gauge theory, and charged particles. Local gauge invariance and global gauge invariance tied up nicely.
- They get seperated by the asymptotic properties of the gauge transformations. Global gauge transformations lead to current conservation and charge as superselection. Within QED there is no charge quantisation unless one evokes monopoles which we will discuss later.
- Local gauge invariance gives redundant degrees of freedom which can be eliminated only by gauge fixing.
- The degrees of freedom describe photons which are massless and is responsible for the long range interactions between charged particles. This interaction affects the freedom of charged particles even at spatial asymptotic infinity.This gives the description of 'in' and 'out' states with additional dressing.
- Also leads to certain additional global symmetries at asymptotic infinity.

Infrared divergences in QED

- Masslessness of the photons give propagators which go like $\frac{1}{k^2}$. This leads to a divergence due to long wavelength/low frequency photons in several processes.
- These photons are also tied up with asymptotic dressing of the charged particles.
- Interestingly the divergences are cancelled by using a coherent state of the charged particles along with 'soft photons'
- The above description was the way text books are written and calculations are performed for all the known processes.
- Recently this question is revisited in QED, QCD and gravity theories from some new perspective. We will focus on QED in 3 and 4 dimensions only.
- Strominger and his collaborators, Laddha and Campligia have made studies on the behaviour potentials at null infinity and produced a mapping which has a discontinuity from the past and future.

Mass of the photon

- Interestingly similar things were shown atleast a decade back by Andrzeg Herdegen on the asymptotic structure. Bal, Sachin & co have also relooked at the question with reference to Lorentz symmetry. Bucholz also had a relook at the algebraic formulation of superselections.
- Since the issues are tied up with the mass of the photon we can look at observationally what are the limits on this.
- A S Goldhaber and M M Nieto: Mass limits: Solar magneticfield $10^{-16}eV$.Cosmic magnetic field limit: $10^{-24}eV$.
- Roughly these correspond to Compton wavelength to be $\geq$ AU (astronomical unit) or radius of the galaxy.
- Using the ultimate size of the universe as bound we can get: $m_\gamma \ \leq \ 10^{-33}eV$
- Neutrino was expected to be massless and later established to be massive, but our experimental conclusions cannot be sensitive when we push the barrier at this level.

Proca and Stueckelberg theory

- If we introduce mass term of the photon to the conventional Maxwell action, it breaks local gauge invariance. But global invariance is still there, and current is conserved and charge is still superselected.
- But the massive Proca theory describing spin 1 has 3 degrees of freedom unlike Maxwell theory which has only 2. Hence there is discontinuity in the degrees of freedom when we take $m_\gamma \to 0$.
- But the Massive QED is renormalisable (ultraviolet). If we make the mass to be tiny but nonzero we will find the contribution of the longitudinal photon to several processes are extremely small as used by the text book of Banks! There is no infrared divergence either because of 'mass'!
- Also what happens in the $m \longrightarrow 0$ limit, for the infrared divergence and the question of lack of local gauge invariance which has been our guiding principle. Stueckelberg theory avoids discontinuity and gauge invariance question nicely.

Massive gauge theory

- We can preserve local gauge invariance and still give mass to the photon in three ways. (1) Stueckelberg theory (2) Higgs mechanism (3)topological massive $B \wedge F$ theory.
- The Lagrangian for Stueckelberg theory is:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}m^2\left(A_\mu - \frac{1}{m}\partial_\mu\phi\right)^2 + \bar{\psi}[\gamma^\mu(i\partial_\mu + eA_\mu) - M]\psi \tag{4}$$

- The gauge fixing: $-\frac{1}{2}(\partial_\mu A^\mu + m\,\phi)^2$. The gauge transformations are:

$$\psi \to e^{i\lambda(x)}\psi, \;\; A_\mu \to A_\mu - \partial_\mu\lambda(x), \;\; \phi \to \phi + m\lambda(x) \tag{5}$$

where ϕ is Stueckelberg scalar field.

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Higgs mechanism

- Here introduce a complex scalar field Φ. This will have nonzero vacuum expectation value giving mass to the the photon.
- We can write in the symmetry broken phase $\Phi = R\, e^{i\phi}$.
- Phase of this field will be like Stueckelberg field and this mechanism in a specific limit of freezing the fluctuations of R goes to Stueckelberg theory.
- For topological massive theory we use two form $B = B_{\mu\nu} dx^{\mu} \wedge dx^{\nu}$ and $H = dB$.
- We can take as Lagrangian $\mathcal{L} = \dfrac{1}{2} H \wedge^{*} H + \dfrac{1}{2} B^2$
- Massive $B_{\mu\nu}$ will describe a spin -1 particle after elimination of constraints. But in the massless limit it describes a spin -0 particle!! Again a discontinuity of the degrees of freedom.
- In all these mechanisms extra degrees of freedom are introduced, but local gauge invariance gives the correct massive spin-1 theory.

Topologically massive gauge theory

- The Lagrangian is:

$$\mathcal{L} \;=\; -\frac{1}{2}F\wedge^{*}F + \frac{1}{2}H\wedge^{*}H + m\,B\wedge F + \bar{\psi}[\gamma^{\mu}(i\partial_{\mu} - eA_{\mu}) + M]\psi \quad (6)$$

- Again the combined gauge transformations leave the Lagrangian upto total divergence invariant.
- In $2+1\ D$ we also have Maxwell Chern Simon theory given by the Lagrangian:

$$\mathcal{L} \;=\; -\frac{1}{2}F\wedge^{*}F \;+\; m\,A\wedge\,F \;+\; \bar{\psi}[i\gamma\,D - M]\psi \quad (7)$$

- Stueckleberg theory in $2+1$ which is equivalent to Proca theory gives 2 degrees of freedom unlike 1 degree of freedom through Maxwell-Chern Simon theory.
- Both Maxwell and Maxwell CS theory describes a scalar field. but with different helicity. But we will focus on the 3+1 Stueckelberg QED.

Stueckelberg QED

- For perturbative calculation we need to fix gauge. The gauge fixing term is $-\frac{1}{2}(\partial_\mu A^\mu + m\phi)^2$.
- It is known to be renormalizable, and due to the mass infrared divergence is not there. What happens in the limit $m \to 0$ limit? *To facilitate that we will look at a transformed gauge field.*

$$\bar{A}_\mu = A_\mu - \frac{1}{m}\partial_\mu \phi \tag{8}$$

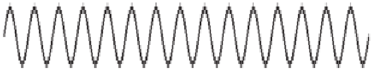
- *This changes the interaction as:*

$$e\bar{\psi}\gamma_\mu A^\mu \psi = e\bar{\psi}\gamma_\mu \bar{A}^\mu \psi + \frac{e}{m}\bar{\psi}\gamma^\mu \psi \, (\partial_\mu \phi) \tag{9}$$

- *In this form $\bar{A}^\mu$ is gauge invariant (transverse component). But the fermion interacts with Stueckelberg field or in effect the longitudinal component!*

Stueckelberg QED

- The gauge field propagator:



$$\frac{-i\, g_{\mu\nu}}{p^2 - m^2} \tag{10}$$

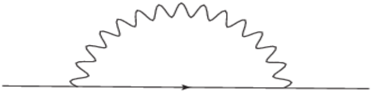
The scalar field propagator:

$$\frac{i}{p^2 - m^2} \tag{11}$$

- Gaugefield vertex: $i\, e\ \ \gamma^{\mu}$
- scalar field vertex: $-\ \dfrac{e}{m} p^{\mu} \mathcal{I}$

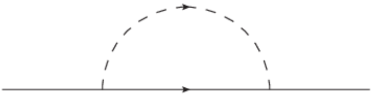
Stueckelberg QED

- The self energy diagram has two components.
- Gauge field part:



$$< \bar{\psi}\psi >_A \;=\; (-i\,e)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu\,(-ig_{\mu\nu})\,(\not{p}+M)\,\gamma^\nu}{[(p-k)^2-m^2)](p^2-M^2)} \qquad (12)$$

- scalar field part:

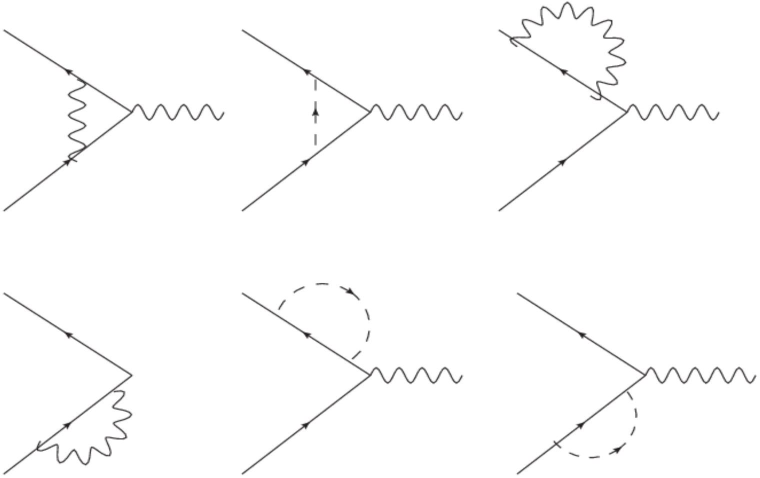


$$< \bar{\psi}\psi >_\phi \;=\; \frac{e^2}{m^2} \int \frac{d^4k}{(2\pi)^4} \frac{(\not{k}-\not{p})[-i(\not{p}\;+\;M)](\not{k}\;-\;\not{p})}{(p^2-M^2)[(p-k)^2-m^2]} \qquad (13)$$

- The infrared divergence cancels in the $m \to 0$ limit.

Stueckelberg QED

- For the vertex there are six diagrams to the lowest order. (cancels, caution: verification)



Soft photon theorem

- Soft photon/graviton theorem is a manifestation of asymptotic symmetries. They are rules for the modification of transition rates to deal with the infrared divergences which arise due to the exact masslessness of these particles. They manifest in the virtual transmission of very soft photons between external particle legs.
- The theorem is: If one considers a reaction $\alpha \to \beta$ involving photons, the rate has to be modified to account for the possible emissions of soft photons having total energy less than or equal to a minimum energy resolution ΔE. The modification takes the form:

$$\Gamma_{\beta\alpha} = \left(\frac{\Delta E}{\Lambda}\right)^{A} b(A)\Gamma^{0}_{\beta\alpha} \tag{14}$$

Γ^0 is the bare reaction rate. If we repeat the calculations for Stuckelberg QED the above equation gets modified to:

$$\Gamma_{\alpha\beta} = F(\Delta E)\Gamma^{0}_{\alpha\beta} \tag{15}$$

where $F(\Delta E)$ is a new function.

Lienard Wiechert Potential

- But we have to carefully take the limit of $m_\gamma \rightarrow 0$ to obtain Weinberg's result. But the infrared divergence will not be there due to charge conservation itself.
- In the presence of a photon mass, modifications to the classical formulae of Lienard and Wiechert can be obtained
- There are discontinuities in the potentials at infinity, and it would be interesting to see if they indeed arise in our theory as well if the massless limit is taken.
- We may now look at the formula in retarded coordinates $u = t - r$ and take the limit $r \rightarrow \infty$.

$$\lim_{r,t\rightarrow\infty,u=const} F_{rt} = \frac{e^2 Q}{4\pi r^2} \frac{\alpha + 1}{\gamma^2(1 - \hat{x} \cdot \overrightarrow{\beta})^2} e^{-\alpha} \tag{16}$$

where,

$$\alpha = \gamma(1 - \hat{x} \cdot \overrightarrow{\beta}) m_\gamma r. \tag{17}$$

- The limit of $m_\gamma \rightarrow 0$ is to be taken before $r \rightarrow \infty$.

Lightfront quantisation

- All finitely massive particles approximately behave like massless one in the infinite momentum frame.
- Hence light front quantisation is useful in understanding the questions. Light front QED has been studied by several people both from infrared and ultraviolet behaviour. Surprisingly there is no analysis of Stueckleberg QED from this perspective.
- The light front Hamiltonian is

$$P^{-} = H_0 + V_1 + V_2 + V_3 + V_\phi \tag{18}$$

- Here H_0 is free Hamiltonian including Stueckelberg field. V_1 is usual fermion gauge field vertex. V_2, V_3 are nonlocal terms appearing in light front formalism and do not worry us for infrared effects. V_ϕ is the interaction between fermion and Stueckelberg field.
- We have analysed the infrared question and as expected the infrared terms cancel at the amplitude level itself between Stueckelberg field and transverse photons!

Little group analysis-massless limit

- Since the issue of infrared question and asymptotic symmetries are related to massless particles, one can consider the limit of massive spin 1 representation of Poincare group becoming massless one.
- The little group of massive particle is given by $SO(3)$. That of massless one is $E(2)$.
- $SO(3)$ is described by $L_i = -i\epsilon_{(i)jk}$. But $E(2)$ is given by: L_3 and

$$P_1 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix} \tag{19}$$

- Def: $P_i = \frac{\epsilon_{ij}}{R} B^{-1} L_j B, \quad B(R) = Diag(1, 1, R)$ gives the Inonu Wigner contraction of $SO(3) \to E(2)$ in the limit $R \to \infty$.

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Little group analysis-massless limit

- Interestingly this is explained better by looking at the rotation and boost generators $L_i,\ K_i$ and considering $N_1 \ = \ K_1 \ - \ L_2, \ \ N_2 \ = \ K_2 \ + \ L_1$. Then L_3, N_1, N_2 give the $E(2)$.
- In light cone coordinates, boost generator

$$B(R) \ = \ e^{-ilog(R)K_3} \ = \ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & 0 & 0 & \frac{1}{R} \end{pmatrix}$$

  where $R \ = \ \sqrt{\frac{(1+\beta)}{(1-\beta)}}$ and β is the velocity.
- Using this we can easily write generators of $E(2)$.
- This is to be expected as these are generators in light front coordinates and all finite mass particles behave like massless particles in that limit. The local gauge transformations can be obtained as part of $E(2)$ tself.

Maxwell Chern Simons theory

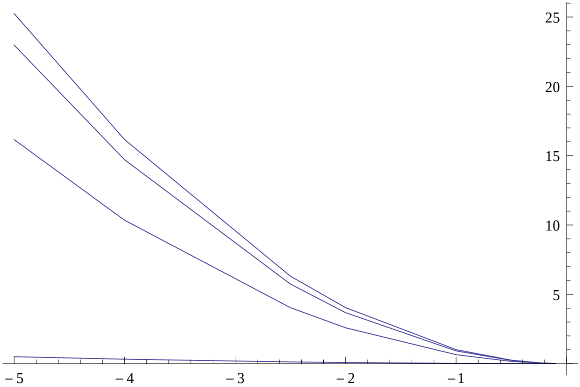
- We now approach the question from the issue of edge modes. For this we consider a simpler model namely 2+1 D Maxwell ED. Since we have a mass for the gauge field is 'm' we consider field modes in a disc of radius Compton wavelength $\frac{1}{m}$. Since the field is a scalar field (Deser, Jackiw, Templeton) we consider the same in the Disc with generic Robin boundary condition.
- We have explored the same (Bal, TRG, ...)in an earliar paper but the sign of the constant κ was positive to ensure positive definite 'Laplacian'. We can give up that and get edge states bounded to the edge.
- The edge eigenmodes of Laplacian on the Disc

$$-\nabla^2\psi = \lambda\psi, \quad \kappa\,\psi(R) \;+\; \partial_r\psi(R) \;=\; 0 \tag{20}$$

were computed earlier in a paper by us (TRG, Rakesh Tibrewala),and the edge modes $N \;\equiv\; \kappa R$.

Maxwell Chern Simons theory

- The disc radius is taken to be $R = \dfrac{1}{m}$. We want to scale $R \to \infty, m \to 0$.
- At the same time we take $\kappa \to 0$ so that N is fixed and large. We find as we scale the eigenvalues tend to zero as edge modes.
- The following figures exhibit the same.



Maxwell Chern Simons theory

- If we had taken up Stueckelberg theory, in 2+1 it will correspond to two Maxwell CS theory! with the sign of the mass term opposite.
- The Maxwell CS QED can be studied only in perturbation theory and the limits should be taken including the interaction with edge modes.
- This will be presented elsewhere.
- Further work: There is extension of Stueckelberg theory to Supersymmetric U(1) (P. Nath etal). Again the earlier studies have focussed only on ultraviolet renormalizability. The question of what happens to infrared question in the limit of massless gauge particles.
- Marolf considered earlier BMS symmetries in gravity by considering in a spacetime in a box along with twisting the boundary conditions. Then taking the limit of size of the box to ∞ in a suitable prescription one gets the asymptotic behaviour.

New conservation laws for zero rest mass fields

- In an interesting paper(1967) Newman and Penrose argue there are new conservation laws for zero mass fields in asymptotically flat space time.
- They also point out the role played by these conservation laws is difficult to measure.
- The physical significance of these constants, we may suppose that at any constant retarded time they may be thought of as composed of two parts, one defined by the multipole structure of the source, the other by as asymptotic incoming field
- One has the freedom, by manipulating the sources, to transfer, the contribution from the source terms to the wave terms and vice versa.
- More work is needed to understand the relation between sources and these conserved quantities.

Dark matter and BEC

- If ultralight bosons of mass $10^{-20} eV$ form dark matter the only chance is they forming BEC condensate.
- For that T_c should be compatible. The temperature at any epoch is $T = 2.7/a\ K$. In BEC average interparticle distance is $\frac{V}{N}^{\frac{1}{3}}$. N is the total number of bosons is comparable to thermal length $\frac{\hbar c}{kT}$. Hence $kT = \hbar\, c\, \frac{N}{V}^{\frac{1}{3}}$. Here $N = N_B + N_O$. where N_B is number of bosons in BEC and the N_O outside it.
- A good estimate of $\frac{N_B}{V} = \frac{\rho}{m} = \frac{.25\rho_{crit}}{ma^3}$
- Estimate for $\rho_{crit} = \frac{3H^2}{8\pi G}$ and is approximately $10^{-25}\ kg/m^3$

Dark matter and BEC

- Putting these factors and estimating the critical temperature which will sustain BEC condensate is $T_{crit} = \dfrac{1}{m^{1/3}a}$.
- The temperature at any epoch is $T(a) = \dfrac{2.7}{a}K$.
- The net result is if the mass of the bosons is less than $10^{-2}eV$ they will be able to sustain BEC condensate.
- The same arguments are used for ultralight axions or any other fuzzy dark matter candidate. But one should find such candidates. But for 'massive photon' it is already there.. and can serve as candidate?
- Noncommutative geometry may even help to reduce the critical density. Let me briefly explain this aspect..

Holography and Stueckelberg theory

- Dvali etal: propose holography can be formulated in terms of information capacity of Stueckelberg degrees of freedom.
- These degrees of freedom act as qubits to encode quantum information.
- The capacity is controlled by the inverse Stueckelberg energy gap to the size of the system.
- They relate the scaling of the gap of the boundary Stueckelberg edge modes Bogoliubov modes..
- ideas are not clear but needs further work...

Summary and Conclusions

- Revival of Infrared question through asymptotic symmetries is interesting. When we regulate theory through mass maintaining local gauge invariance gives the Stueckelberg scalar a new role. It regulates the divergence, and breaks the asymptotic symmetry. Can the charges due to the new symmetries be observed? Since they are tied up with masslessness of the photon in a limiting process probably they can be observed only to the extent we can measure the mass of the photon.
- What about QCD? Unfortunately there is no Stueckelberg theory for non abelian gauge theory...Speculations about $\frac{1}{N}$?.
- What about gravity? Probably massive gravity theories can shed some information.
- Last speculation? Can it help in dark matter? Stueckelberg field is not coupled to matter but can only gravitate...
- People involved in different parts: Ramadevi, Jai More, Ravindran, Rakesh Tibrewala, Nikhil Kalyanapuram.