Towards realistic Lorentzian wormholes

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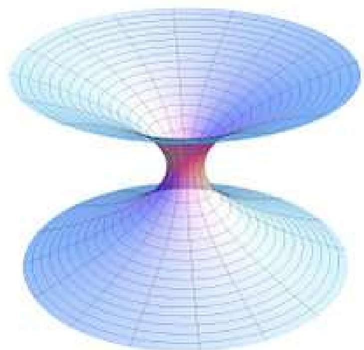
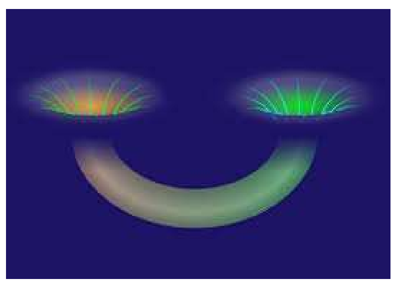
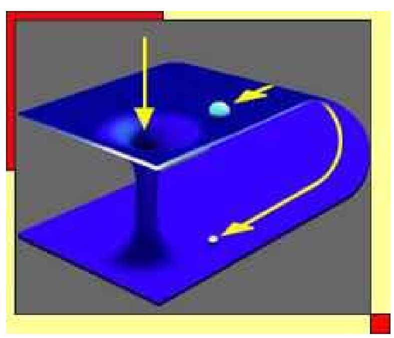
WORMHOLES

• **Spatial 2D section (t constant, $\theta = \frac{\pi}{2}$) embedded in 3D Euclidean space. Three types: first topologically different others.**







• **We will be concerned with the first type– two sheeted structure with a bridge.**

HISTORY

- **L. Flamm (1916)**
Shape of spatial section of Schwarzschild geometry. Flamm's paraboloid.

- **A. Einstein, N. Rosen (1935)**
Two-sheeted structure, particle model(?), new coordinates, degenerate metric.

- **C. W. Misner and J. A. Wheeler (1957)**
Physics as geometry, electric charge model, geons.

- **K. A. Bronnikov and H. G. Ellis (1973)**
Scalar field as a wormhole matter source.

- **M. S. Morris and K. S. Thorne (1988)**
Definition, energy condition violation in GR, time machine (MSM, KST and U. Yurtsever).

AIMS AND OVERVIEW

- **Definition (Morris-Thorne):**

A Lorentzian line element ($r \geq b_0$):

$$ds^2 = -e^{2\psi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\Omega^2.$$

$\psi(r)$ **and** $b(r)$ **satisfy (for a LW):**

$\rightarrow$ $e^{2\psi(r)}$ **has no zeros, is finite everywhere.**

$\rightarrow$ $b(r = b_0) = b_0$ **(Throat),** $\frac{b(r)}{r} \leq 1$

$\rightarrow$ $r \rightarrow \infty, \frac{b(r)}{r} \rightarrow 0$ **(asymp. flatness).**

Using proper radial distance l

$$ds^2 = -e^{2\chi(l)}dt^2 + dl^2 + r^2(l)d\Omega^2.$$

Example: Ellis-Bronnikov geoemtry:

$$ds^2 = -dt^2 + \frac{dr^2}{1-\frac{b_0^2}{r^2}} + r^2 d\Omega^2.$$

- **Problem with wormholes in GR**:

A Lorentzian wormhole geometry acts like a defocusing lens for null geodesic congruences flowing from one flat asymptotic region to another across the throat.

Hence, for wormhole existence, the convergence condition $R_{ij}u^i u^j \geq 0$ must be violated.

Since $G_{ij} = \kappa T_{ij}$ in GR, the required matter, violates all energy conditions.

- **Alternate theories?**

In any theory, the convergence condition is of course violated for wormholes.

But matter can still satisfy energy conditions.

Example: Scalar-Tensor Theories (STT).

- Scalar-tensor theories

Field equations: $G_{\mu\nu} = \frac{\bar{\kappa}^2}{l\Phi} T^M_{\mu\nu} + \frac{1}{\Phi} T^\Phi_{\mu\nu}$

Raychaudhuri equation:

$$\frac{d\Theta}{d\lambda} + \frac{1}{3}\Theta^2 + \Sigma^2 - \Omega^2 = -R_{\mu\nu}u^\mu u^\nu$$

Focusing theorem:

If $R_{\mu\nu}u^\mu u^\nu \geq 0$, then geodesics must focus ($\Theta \to -\infty$) within a finite affine parameter.

For $R = 0$ metrics (or null u^μ):

$$R_{\mu\nu}u^\mu u^\nu = \frac{\bar{\kappa}^2}{l\Phi} T^M_{\mu\nu}u^\mu u^\nu + \frac{1}{\Phi} T^\Phi_{\mu\nu}u^\mu u^\nu$$

Thus, even if $R_{\mu\nu}u^\mu u^\nu < 0$, $T^M_{\mu\nu}u^\mu u^\nu \geq 0$ is possible. Energy conditions hold, convergence condition violated.

Capozziello, Lobo, Mimoso (PLB 2014); SK, Lahiri, SenGupta (PLB2015)

Better, more realistic wormholes in STT? Many examples.

- Can such wormholes ever exist?

Imagine astrophysical processes (eg. mergers) creating such a wormhole–study its ringdown to a stable state.

- Find quasinormal modes

Scalar QNMs = Breathing mode in STT?

- Use the QNMs found:

To find the metric parameters, via comparison with GW signals.

To estimate percentage errors in the metric parameters in order to see how accurately they can be measured in future detectors.

Look for error minima which hint at existence possibilities, if the breathing mode is detected in GW obs.

• **Questions:**

What is the wormhole metric?

Which STT and why?

How do we distinguish between black holes, wormholes and naked singularities through GW obs.?

• **Collaborators:**

Recent: S. Aneesh, S. Bose.

Recent past: S. Lahiri, S. SenGupta, R. Shaikh.

Past: N. Dadhich, S. Mukherjee, M. Visser.

THE WORMHOLE

- **Schwarzschild spacetime:**

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega_2^2$$

- **Modified Schwarzschild:**

$$ds^2 = -\left(\kappa + \lambda\sqrt{1 - \frac{2M}{r}}\right)^2 dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega_2^2$$

- **Damour-Solodukhin (PRD2007):**

$$ds^2 = -\left\{\lambda^2 + (1 - \frac{2M}{r})\right\}dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega_2^2$$

- **GR, Braneworld contexts:**
Zero energy density, violates WEC.

Modified Schwarzschild first proposed:

(1) in GR by Dadhich, SK, Mukherjee, Visser (PRD 2002); (2) in Braneworld gravity by Casadio, Fabbri, Mazzacurrati (PRD, 2002)

MODIFIED SCHWARZSCHILD

$$ds^2 = -\left(\kappa + \lambda\sqrt{1-\frac{2M}{r}}\right)^2 dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega_2^2$$

- $\kappa, \lambda > 0$, $r \geq 2M$, **No horizon,** $R = 0$.

- **Throat at** $r = 2M$. **Spatial slice same as Schwarzschild.**

- **To get a full wormhole with two asymptotic regions, one would need to use the Visser surgery across timelike boundaries.**

- **Join two identical copies at** $r = 2M$ **across a thin-shell with surface stress-energy having zero energy density and positive pressure (similar surface stress energy noted in Lemos, Lobo, de Oliveira (PRD, 2003).)**

Question:

Is this wormhole a solution in a scalar-tensor theory with matter satisfying the WEC, but with the convergence condition violated, as required for any wormhole?

Answer:

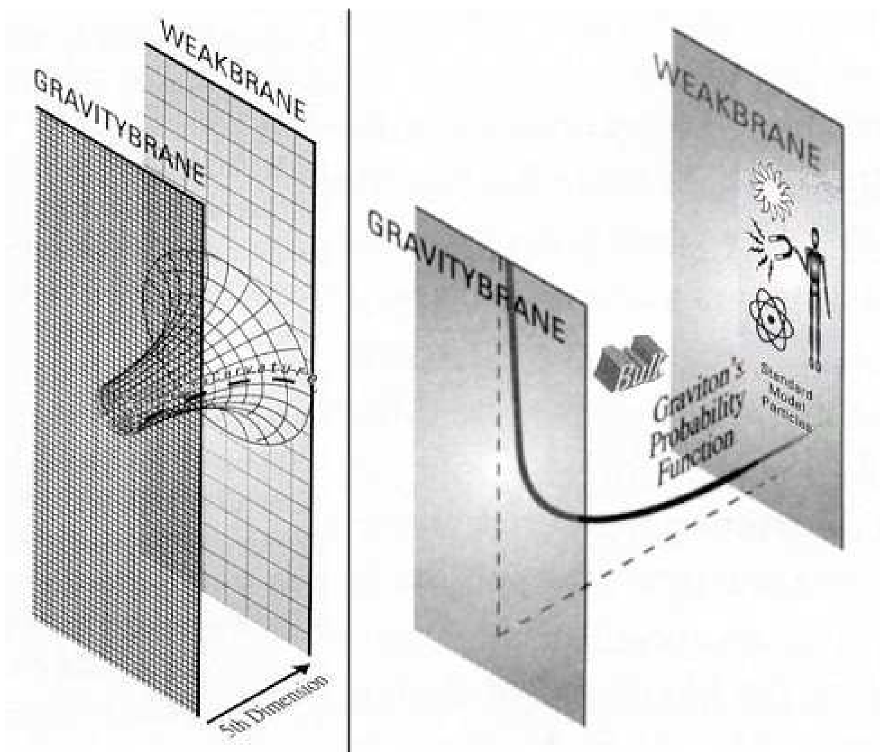
Yes, it is a solution in a particular Scalar Tensor Theory, with normal matter and an everywhere nonzero, finite scalar field.

SK, Lahiri, SenGupta (PLB2015), Shaikh, SK (PRD2016).

SCALAR TENSOR THEORY

• Consider a 5D universe with one warped extra dimension.

• Imagine, two 4D timelike hypersurfaces (curved) embedded in this five dimensional world.



• What is the gravity theory on one of these 4D hypersurfaces?

• What encodes information about the warped extra dimension?

• The gravity theory is a scalar-tensor theory with

$$\omega(\Phi) = -\frac{3\Phi}{2(1+\Phi)}.$$

• Φ is called the radion scalar which encodes information about the extra dimension.

• In particular, the radion is a measure of the distance between the two branes.

• There are similar effective scalar-tensor gravity theories on both the branes.

The details of the theories are avaiable in Kanno and Soda (PRD2002) ; Shiromizu and Koyama (PRD2003).

FIELD EQUATIONS FOR THE STT

- **The Einstein equations (Jordan frame):**

$$G_{\mu\nu} = \frac{\bar{\kappa}^2}{l\Phi} T^b_{\mu\nu} + \frac{\bar{\kappa}^2 (1+\Phi)}{l\Phi} T^a_{\mu\nu} + \frac{1}{\Phi} \left(\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla^\alpha \nabla_\alpha \Phi \right) - \frac{3}{2\Phi(1+\Phi)} \left(\nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \Phi \nabla_\alpha \Phi \right);$$

- **The scalar field equation:**

$$\Box \Phi = \frac{\bar{\kappa}}{l} \frac{T^b}{2\omega + 3} - \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} \nabla^\alpha \Phi \nabla_\alpha \Phi$$

$g_{\mu\nu}$ is the on-brane metric and covariant differentiation is defined w.r.t. $g_{\mu\nu}$.

$\bar{\kappa}^2$ is the $5D$ gravitational constant.

$T^a_{\mu\nu}$, $T^b_{\mu\nu}$ are the matter stress-energy on ‘a’ , ‘b’ branes, respectively.

We choose $T^a_{\mu\nu} = 0$.

• There is matter on the ‘b’ brane hypersurface. If we have an on-brane wormhole, does the on-brane matter satisfy the energy conditions?

• The on-brane matter does satisfy the WEC. The effective geometric stress energy helps avoid the WEC violation of on-brane matter.

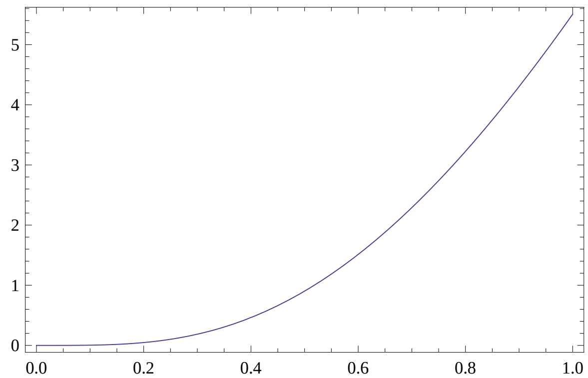
• The radion is finite and non-zero everywhere to avoid brane collisions or infinite separation.

• Thus we can have a Lorentzian wormhole on the brane with on-brane matter satisfying the energy conditions.

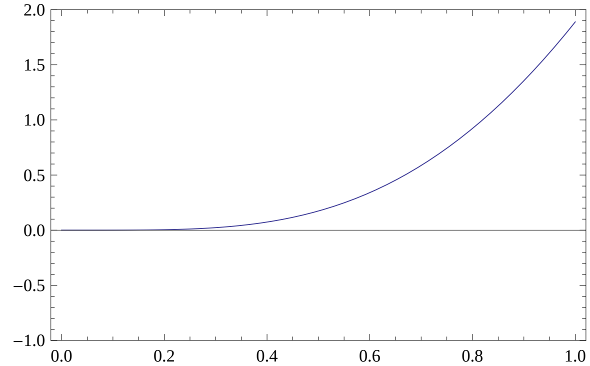
• The existence of such a wormhole is intimately connected to (a) a STT and (b) a warped extra dimension.

For details see, SK, S. Lahiri, S. SenGupta, PLB2015

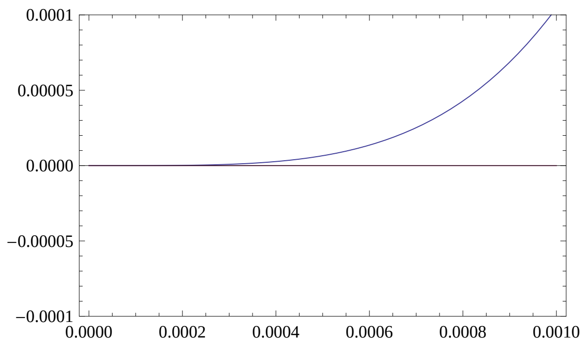
THE WEC INEQUALITIES



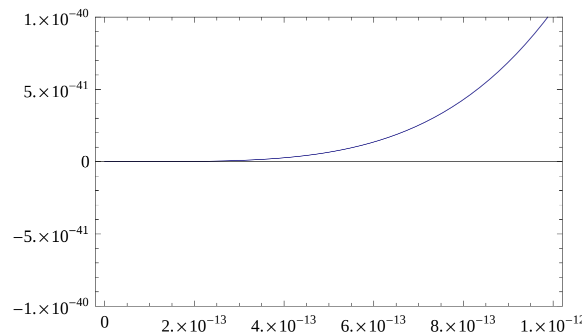
ρ **vs.** x



$\rho + \tau$ **vs.** x



$10^6(\rho + p)$ **vs.** x



$10^6(\rho + p)$ **vs. x, near** $x = 0$

$\frac{\kappa}{\lambda} = 0.5$, $x = \frac{M}{2r'}$, $r' \to$ **isotropic coordinate.**

Note from the graphs that the WEC ($\rho \geq 0$, $\rho + \tau \geq 0$, $\rho + p \geq 0$) holds.

DETECTING SUCH A WORMHOLE?

- Can the end-state of astrophysical collisions lead to wormholes? Not impossible!

- Study the ringdown by analysing perturbations of wormholes.

- In scalar-tensor theories, scalar perturbations may correspond to the breathing mode of gravitational waves.

- Obtain the scalar quasinormal modes by solving the massless scalar wave equation with outgoing boundary conditions at the asymptotic regions.

- Use the QNMs to obtain percentage errors on wormhole parameters, using inputs from observed GW signals like GW15091.

- Less error domains of the wormhole parameter $\frac{\kappa}{\lambda}$ are possible.

THE BREATHING MODE

• Near the detector, the background space-time is flat.

Consider perturbations of a flat Minkowski background and a constant background scalar field.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \Phi = \Phi_0(1+\epsilon)$$

where Φ_0 is a constant.

• The field equations become

$$\Box(-h_{\mu\nu} + \eta_{\mu\nu}\epsilon) = \frac{2\bar{\kappa}}{l\Phi_0} T_{\mu\nu}$$
$$\Box\epsilon = \frac{\bar{\kappa}}{l\Phi_1} T$$

where $\Phi_1 = \frac{3\Phi_0}{1+\Phi_0}$.

The choice of gauge is $\partial_\nu \bar{h}^{\mu\nu} = \partial^\mu \epsilon$ where $\bar{h}^{\mu\nu}$ is the trace-reversed metric perturbation.

• In vacuum, we have $\Box h_{\mu\nu} = 0 = \Box\epsilon$ and in the TT gauge, we get

$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ - \epsilon_0 & h_\times & 0 \\ 0 & h_\times & -h_+ - \epsilon_0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{i\omega(t-z)}$$

for a plane wave propagating in the z-direction.

The scalar field is $\Phi = \Phi_0(1+\epsilon)$ where $\epsilon = \epsilon_0 e^{i\omega(t-z)}$.

• Due to the scalar field, there is an additional polarization in the gravitational wave, which is known as the breathing mode.

• With a massive scalar, we also have a longitudinal mode.

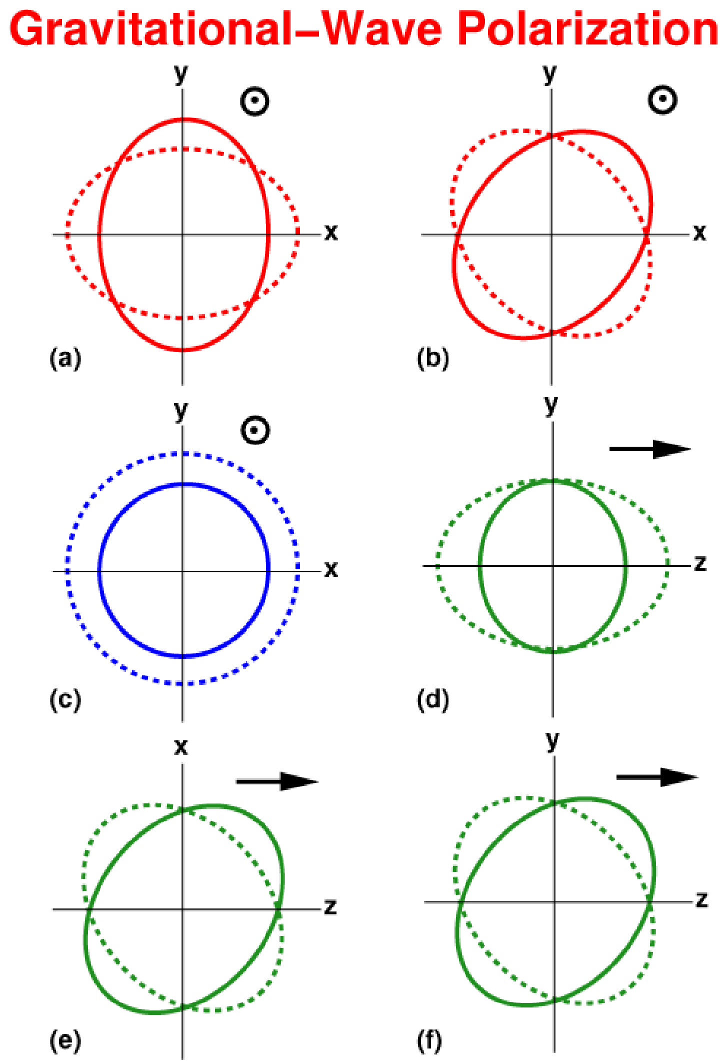


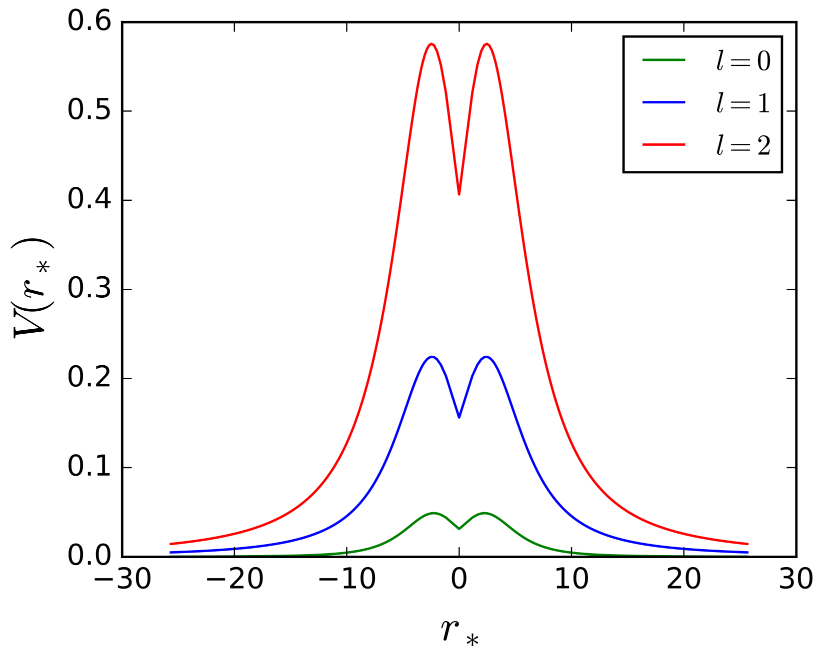
Figure taken from Will, Living Reviews

• In a curved background, with $\phi \to \phi + \delta\phi$, scalar perturbation equation in our STT, via a gauge choice, becomes:

$$\Box(\delta\Phi) = 0$$

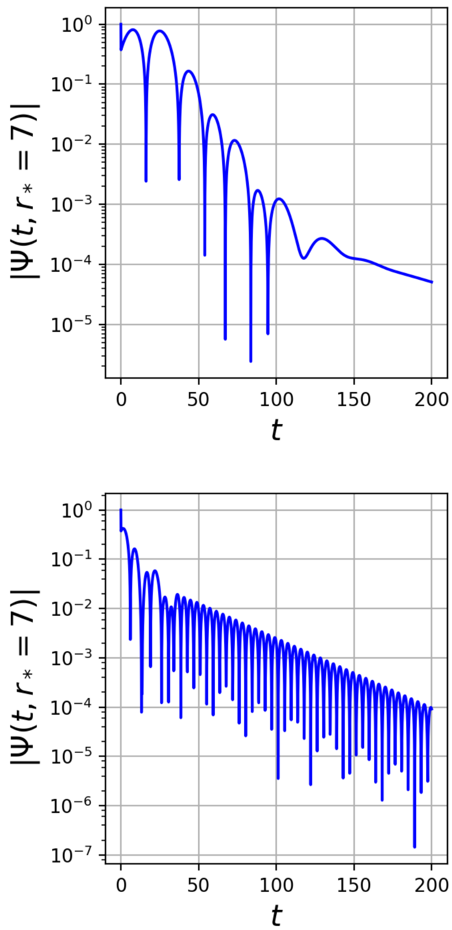
SCALAR PERTURBATIONS

- The effective potential for scalar perturbations:



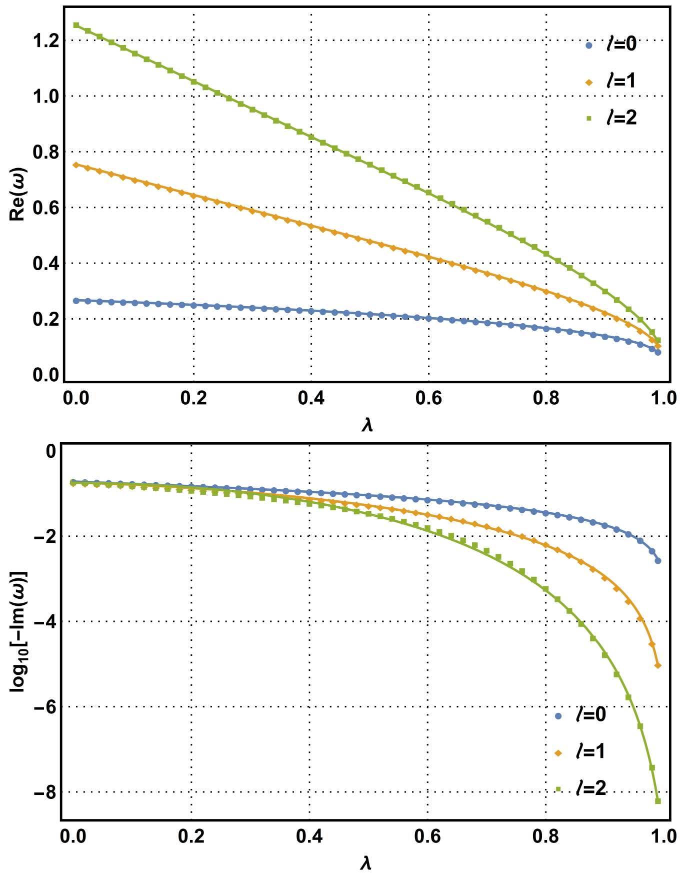
The horizontal axis is r^* (tortoise coordinate). Vertical axis $V(r^*)$. Note the double hump potential (characteristic of most black-hole mimickers). Double hump also responsible for echoes!

- **Time domain profiles:**



Time domain profile and quasinormal ringdown. Profiles calculated for $\kappa = 0.5 = \lambda$, $M = 1$ **and** $l = 0, 2$ **at** $r_* = 7$. **Initial conditions are** $\psi_{lm}(u, 0) = \exp\left[-\frac{(u-10)^2}{100}\right]$ **and** $\psi_{lm}(0, v) = 1$. **The integration grid is** $u, v \in (0, 200)$ **with** $h = 0.1$. $l = 0$ **(top),** $l = 2$ **(lower).**

• The scalar QNMs (fundamentals):



Obtained numerically (Prony fit, direct integration). Continuous curves are best-fits.

• Wormhole QNMs and ringdown first studied by Konoplya and Molina (PRD2005). Also by S.W. Kim (PTP2008). Recent papers by Konoplya and collaborators, Kunz and collaborators.

EXAMPLE OF A QNM ν (in Hz), τ (in s)

For $l = 0$,

$$\nu = \frac{8628.13}{M}\left(1 - 0.29\frac{\lambda}{\lambda+\kappa}\right)\left(1 - \left(\frac{\lambda}{\lambda+\kappa}\right)^{2.36}\right)^{0.24}$$

$$\tau = \frac{M}{38908.58}\left(1 - 0.99\left(\frac{\lambda}{\lambda+\kappa}\right)^{0.94}\right)^{-1.02}$$

For $M = 68M_{solar}$ and $\frac{\kappa}{\lambda} = 0.1$, $\nu = 64$ Hz.

- **The breathing mode signal:**

$$h(t) = A\sin(2\pi\nu t)e^{-t/\tau}$$

where the strain amplitude A contains the breathing-mode antenna pattern.

- **Thus, knowing ν and τ from the signal, one can find the $\frac{\kappa}{\lambda}$ and M, in principle.**

But it is far far more tough! One must extract signal from detector noise. Estimate errors.

PERCENTAGE ERRORS

- The percentage errors on $\frac{\kappa}{\lambda}$ can be calculated via the Fisher information matrix formalism.

- We estimate how accurately the wormhole parameters will be measurable using future interferometric detectors like aLIGO.

- To estimate the error in κ/λ, we compute that matrix for the damped-sinusoid signal in a single aLIGO detector at design sensitivity for that parameter alone.

- The matrix is determined by the derivative of the signal h with respect to κ/λ, which influences both the frequency and the damping time-constant of the signal.

• We take M to be known.

• For reference the maximum QNM strain amplitude is assumed as 10^{-21}, which is approximately the maximum amplitude of the GW150914 signal.

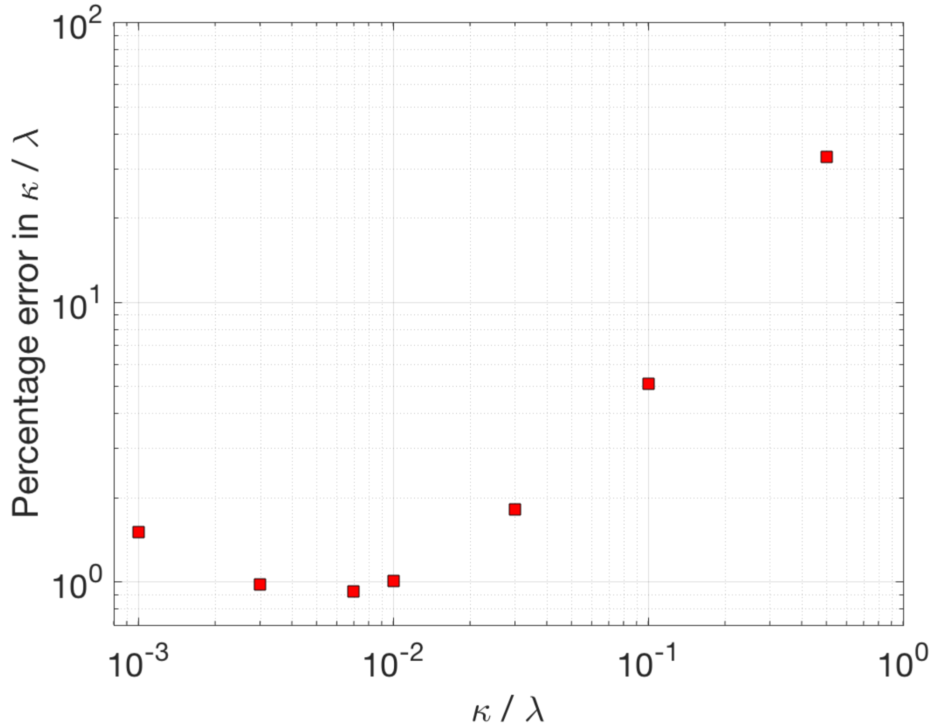
• Finally, we invert the information matrix to derive the estimated variance in the measured values of κ/λ.

• Its square-root gives the lower bound on the statistical error in κ/λ.

Error estimates largely done by Sukanta Bose (IUCAA, WSU), Science Lead, LIGO-India.

ERROR ESTIMATES

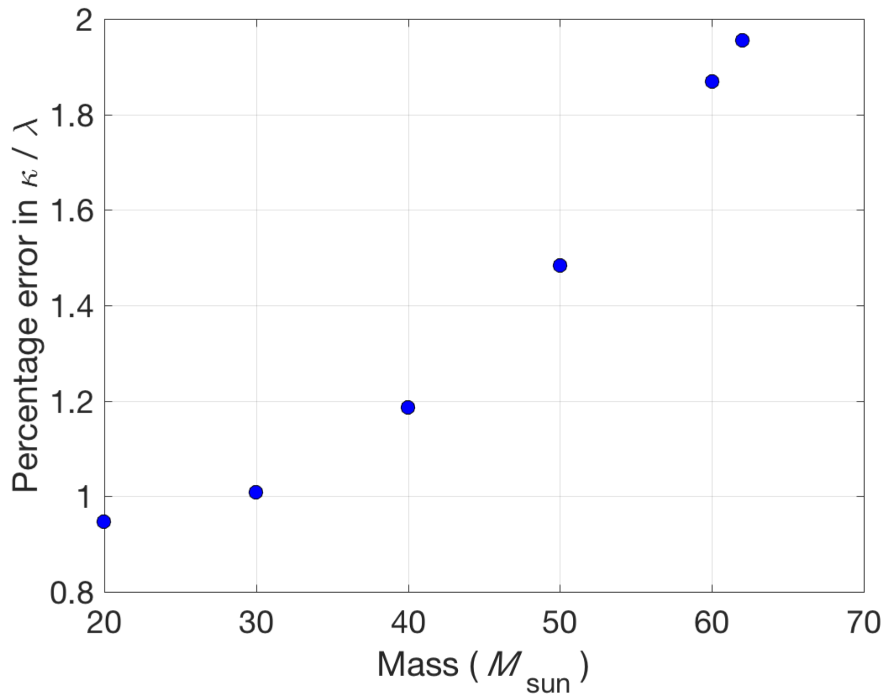
- $M = 30M_{solar}$.



- **Error intially reduces as $\frac{\kappa}{\lambda}$ increases. Mode freq. shifts to more sensitive parts of the detector band. For higher values (less than 1) the error increases due to decreasing time constant (heavy damping).**

- **Errors for $\frac{\kappa}{\lambda} > 1$ upto 10 also found using Einstein Telescope design sensitivities.**

• $\frac{\kappa}{\lambda} = 0.01$.



• **Error increases with increasing M because mode frequency decreases, placing the signal in the less sensitive part of the detector band.**

For more details and other plots see S. Aneesh, S. Bose and SK (PRD, June 2018). Also note interesting recent papers by R. Konoplya and collaborators on wormhole QNMs.

THE FUTURE

• We have outlined a way of knowing through GW observations, if wormholes could be/are there at all.

• Earlier attempts were largely using gravitational lensing. With new GW data, our proposal seems a better prospect. However, we badly need an astrophysical merger model. Also study gravitational perturbations .

• If we ever see the specific Lorentzian wormhole we proposed here, it will be indirect evidence for extra dimensions and modified gravity.

• Recent work on wormhole shadows (observable in the EVENT HORIZON telescope) and echoes in wormholes (observable in future GW detectors) will also be of use from an observational standpoint.

FINAL WORD

It is time to know whether Lorentzian wormholes which are the successors of the Einstein-Rosen bridge do exist in nature.

Geometrically, wormholes are ‘good’ spacetimes. No horizons, no singularities, asymptotically flat ....

Too good to be truly there ??

WAS EINSTEIN RIGHT ??