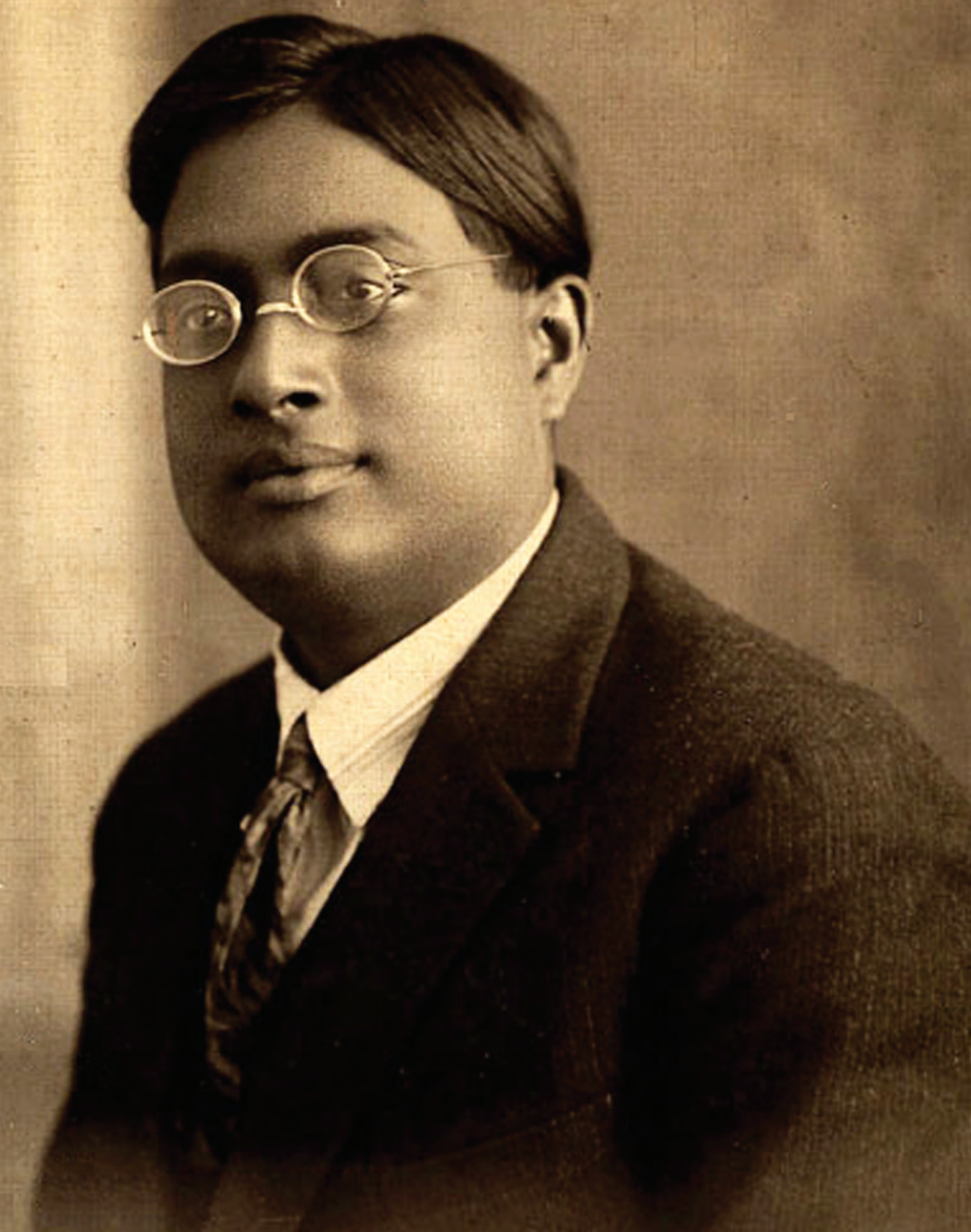
Reference: http://www.osa-opn.org/home/articles/volume_24/april_2013/features/satyendra_nath_bose_and_bose-einstein_statistics/#.VAJNdsVdXSs



Satyendra Nath Bose

and Bose-Einstein Statistics

Barry R. Masters

Satyendra Nath Bose is an often overlooked hero in the field of particle physics and quantum statistics.

Ever since researchers found evidence for the existence of the Higgs boson in 2012, the particle has made frequent appearances in scientific and mainstream media. Many people recognize Peter Higgs—one half of the particle's namesake, but fewer realize that the "boson" half of the moniker refers to Indian physicist Satyendra Nath Bose.

Although not as well known as his contemporaries, S.N. Bose was arguably one of the most important scientists of the 20th century. His revolutionary way of viewing photon behavior and photon statistics—later known as Bose statistics—changed the field in a dramatic way. Albert Einstein played an integral role in publishing his work and also extended Bose's new statistics to atoms and predicted the Bose-Einstein condensate (BEC).

1047-6938/13/04/40/8-$15.00 ©OSA

In 1918, Bose and Meghnad Saha published a joint paper on the kinetic theory of gases in the prestigious English journal, *Philosophical Magazine*.

Several years ago, with the support of an OSA Fellow Travel Award (OPN, May 2004), I lectured in the various Indian Institutes of Technology and the two Bose Institutes in Calcutta, one of which was named after S.N. Bose (the second was named after Jagadish Chandra Bose). The trip inspired me to take a closer look at his work and his life. This article honors the seminal work of Bose, his statistics and the BEC.

Early life and education

Bose was born on 1 January 1894 in Calcutta, the capital of British India. At the age of five, he enrolled in the Normal School in North Calcutta and completed his final years of primary education at the Hindu School in Calcutta. When he was eight, Bose demonstrated extraordinary intellect and mathematical talent. He attributed to his upbringing his urbane, liberal and open-minded character; his deep curiosity and love of the arts; and his perfectionism.

Following his studies at the Hindu School, Bose studied at the Presidency College in Calcutta from 1909 to 1915. Although he suffered from weak vision, he loved to read; his favorite poets were Tagore and Tennyson. His motivation to study science was similar to that of many of his fellow students: The process of science could facilitate the advancement of development and prosperity in his country. "We wanted to put scientific knowledge to use through technology for the benefit of the people or to contribute to science by intensive study," Bose said.

Presidency College was the best place to study science in Calcutta, and the competition to get in was intense; about 2,000 applicants competed for 32 slots. Bose overcame the odds and earned himself a spot. He was impressed with his science teachers and especially with the lectures and laboratory demonstrations of his physics teacher Jagadish Chandra Bose, who developed novel instruments to produce and detect millimeter waves. On his final examinations, Bose achieved the highest ranking among the B.Sc. students and repeated this feat on his M.Sc. examinations.

In 1914, at the age of 20 and while still an M.Sc. student, Bose married Ushabati Ghosh, the 11-year-old daughter of a prominent physician in Calcutta. Of their nine children, two died in infancy; the surviving children consisted of five daughters and two sons.

Early professional life

In 1917, Bose became a lecturer in physics and applied mathematics at the University College of Science, Calcutta University. In 1918, Bose and Meghnad Saha published a joint paper on the kinetic theory of gases in the prestigious English journal, *Philosophical Magazine*. Subsequently they translated Einstein's papers into English so that Indian students who did not know German could read them. This work, *The Principle of Relativity*, which contained papers by Einstein and Hermann Minkowski, was published in 1919 by Calcutta University.

Bose worked at the University of Dhaka in East Bengal from its founding in 1921 to 1924. His initial post was as a reader (pre-professorial position) in the physics department. He taught courses on thermodynamics and electromagnetic theory to the M.Sc. students. For Bose, Dhaka was isolated from the European centers and thus many of the modern developments in relativity theory and quantum theory. But his colleague saved him from seclusion. Debendra Mohan Bose returned from his doctoral studies in Germany in 1919 and gave Bose a wonderful gift: Max Planck's book *Thermodynamik und Wärmestrahlung*.

Bose's 1924 publications

Bose read Planck's papers on the distribution of energy in blackbody radiation in order to teach this material to his class. He was disturbed by Planck's derivation of the law with its ad hoc assumptions. In 1924, Saha stayed with Bose in Dhaka and pointed out the 1923 papers of Wolfgang Pauli, Einstein and Paul Ehrenfest and their relation to Einstein's 1917 paper. This led him to develop Bose statistics—a new method to count states of indistinguishable particles—and applied it to his derivation of Planck's radiation law.

Bose wrote two papers in which he independently derived and further developed the previous work of Planck: "Planck's law and the light-quantum hypothesis," and "Thermal equilibrium in the radiation field in the presence of matter." He considered the papers as a single unit and submitted them to the *Philosophical Magazine*. He did not receive a reply from the prestigious English journal; therefore, on 4 June 1924, he sent the first paper to Einstein, along with a letter:

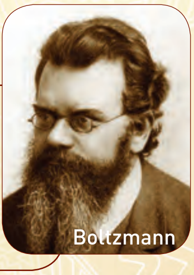
> *"If you think the paper worth publication, I shall be grateful if you arrange for its publication in* Zeitschrift für Physik*. Though a complete stranger to you, I do not feel any hesitation in making such a request. Because we are all your pupils though profiting only by your teachings through your writings."*

The two had corresponded before, when Bose had asked Einstein if he could translate his paper on generalized relatively into English. (Einstein acceded.) Bose's first paper stated that, in the thermal equilibrium of massless particles (photons), the number of particles are not conserved due to the absorption and emission of photons. These particles could be described in a new way—with Bose statistics, which used the counting of cells in one-particle phase space instead of the previous method of totaling the number of standing waves in a cavity in order to obtain the number of states.

Only one year before, Arthur Compton had published his seminal paper on the Compton effect. Bose interpreted Z, the

Statistical Mechanics

Prior to Bose's Statistics



1877

Ludwig Boltzmann develops a method of calculating probability by counting states and applied it to an ideal gas at equilibrium; he found the equilibrium state, which is the most probable state, is given by the Maxwell-Boltzmann distribution.

1900

Max Planck builds upon Boltzmann's prior work that related entropy and probability and develops the Boltzmann principle or equation: $S = k \ln W$, where S is entropy, k is Boltzmann's constant, and W is the probability of a distribution of elements into cells. Planck published his law of blackbody radiation based on Maxwell's classical electromagnetic theory and the ad hoc assumption that the energy ε of the oscillators or resonators was quantized. Some years later, Planck accepted that it was not possible to derive his law only from Maxwell's electromagnetic theory and Maxwell-Boltzmann statistics.

1905

Albert Einstein analyzes the energy fluctuations of radiation that obeyed Wien's law and concludes that the radiation consists of particles of discrete light-quanta with energy, $\varepsilon = h\nu$, where h is Planck's constant and ν is the frequency of the oscillator.

1910

Peter Debye publishes his derivation of Planck's law; instead of following Planck, who used the classical relation between energy density of the blackbody radiation and the average energy of a resonator, Debye calculates the probability of a given state of the radiation using the state without invoking resonators.

1911, 1914

From a historical perspective, the little-known work of **Ladislas Natanson** is significant. He shows that both Planck and Debye have made the tacit assumption of the indistinguishability of quanta in their derivations. Both **Paul Ehrenfest** and **Kamerlingh Onnes** reach the same conclusion.

Wikimedia commons

S.N. Bose viewing a photo of Albert Einstein (1953).

AIP Emilio Segre Visual Archives, Gift of Kameshwar Wali and Etienne Eisenmann

number of states in the frequency interval between ν and $\nu + d\nu$ as the number of cells that contain a given number of photons with frequency ν, and not the number of particles, as was previously done by Boltzmann.

On 14 June, he sent Einstein his second paper:

> *"I hope my first paper has reached your hands. The result to which I have arrived seems rather important (to me at any rate). You will see that I have dealt with the problem of thermal equilibrium between radiation and matter in a different way, and have arrived at a different law for the probability of elementary processes, which seems to have simplicity in its favour."*

Both papers put forward the idea that the fundamental assumptions of quantum theory are not compatible with the laws of classical electrodynamics. According to Bose, all previous derivations of the relation between energy density σ_ν and the average energy of a resonator, E, use classical assumptions and resulted in logical flaws. He stated that Einstein's 1917 derivation used "Wien's law based on classical theory and the Bohr correspondence principle, which assumes that the quantum theory agrees with the classical theory in certain limiting cases." Bose went on to say that "a new form of statistical mechanics is required to be compatible with quantum theory; these logical flaws can be solved by using statistical mechanics without any assumption of mechanism of elementary processes on which the energy exchange depends."

Bose followed Einstein's light-quantum hypothesis and considered blackbody radiation as light-quanta in a volume. He then continued in several steps, the most significant of which was how to calculate the number of states of a light-quantum in the frequency range of ν and $\nu + d\nu$. Bose extended Planck's "first quantization" of the material oscillators to the radiation field itself (photons). With the knowledge of the Compton effect, Bose assumed that the photon of frequency ν "has momentum of magnitude $h\nu/c$ in direction of its motion."

Reference: http://www.osa-opn.org/home/articles/volume_24/april_2013/features/satyendra_nath_bose_and_bose-einstein_statistics/#.VAJNdsVdXSs

While Einstein had both of Bose's papers published in *Zeitschrift für Physik*, his appended comments on each were very different.

Bose divided the total phase space volume h^3 into cells of volume and multiplied the resulting formula by a factor of two. His paper explained that the factor of two was required "in order to take into account the fact of polarization."

While Einstein had both of Bose's papers published in *Zeitschrift für Physik*, his appended comments on each were very different. On Bose's first paper, Einstein was enthusiastic and viewed the work as a significant contribution. But Einstein's long appended comments on the second paper were very different:

> *"Your principle is not compatible with the following two conditions: (1) The absorption coefficient is independent of the radiation density. (2) The behavior of a resonator in a radiation field should follow from the statistical laws as a limiting case."*

Bose was disappointed with Einstein's comments on his second publication and on 27 January, 1925, he wrote his rebuttal to Einstein:

> *"I have written down my ideas in the form of a paper [his third] which I send under separate cover … I have tried to look at the radiation field from a new standpoint and have sought to separate the propagation of quantum of energy from the propagation of electromagnetic influence…"*

Bose later stated that, in his paper, he gave a quantum mechanical explanation for the factor of two, but that Einstein removed it and replaced it with the polarization argument. While the original Bose paper in English is missing from the archives, it is likely that Bose postulated the light-quanta had an intrinsic spin with only the values of $\pm h/2\pi$.

He felt that his postulated idea of photon spin was lost when Einstein presumably removed it from his paper and replaced it with the sentence about polarization.

Bose then stated: "The total number of cells must be regarded as the number of possible arrangements of a quantum in a given volume." He then showed that the factor $8\pi\nu^2/c^3$ in Planck's law is the total number of quantum states of the radiation. Next, he calculated the thermodynamic probability of a state. Furthermore, he stated, "… since each cell is to be counted as a single quantum state … consequently the cells can be partitioned into distinct classes characterized solely by their occupation numbers." Finally, he followed the standard techniques to derive Planck's law. Bose never cited the previous works of Natanson nor the works of Ehrenfest and Onnes.

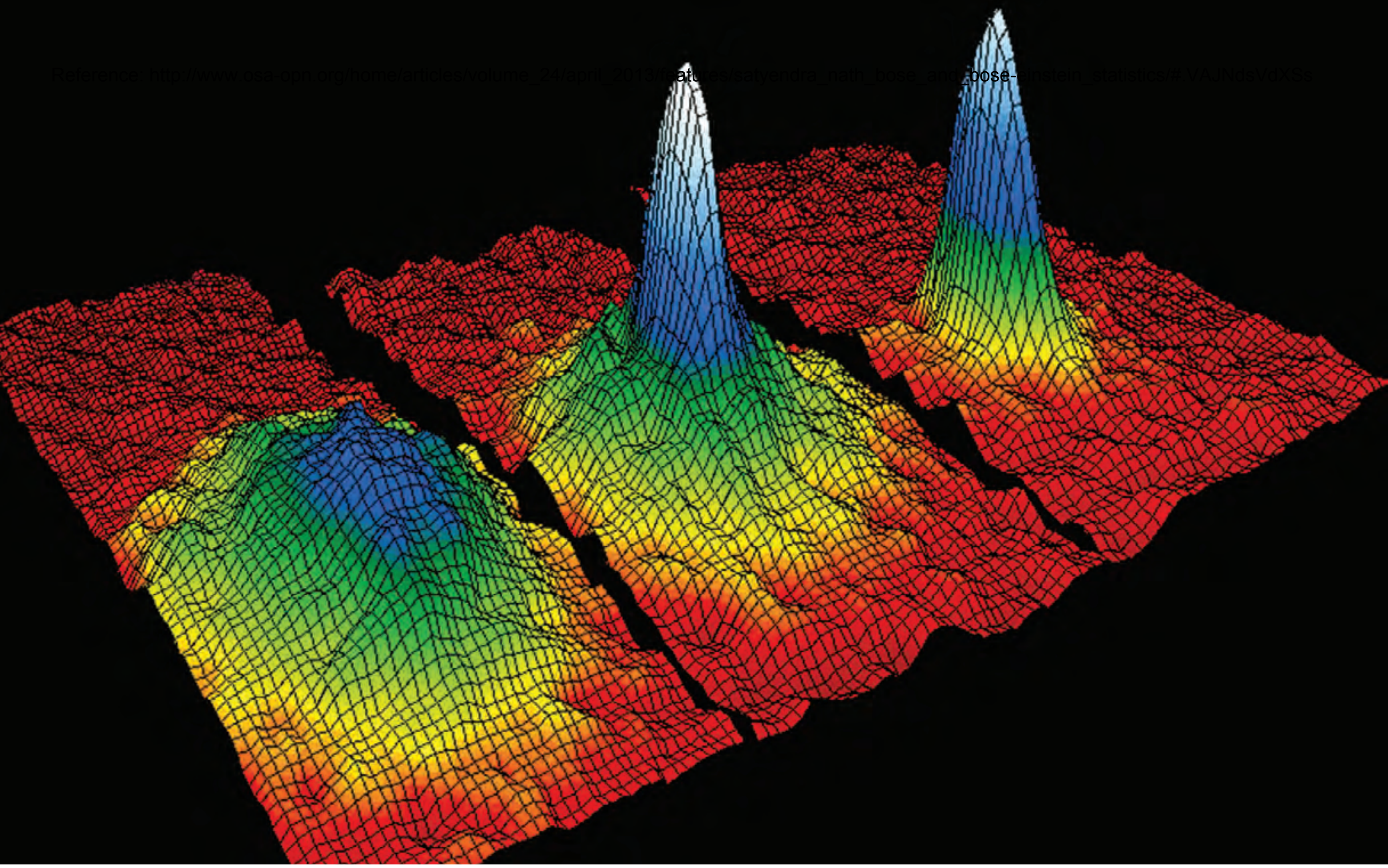

Fermi

Bosons and Fermions

In 1926, Wolfgang Pauli published a paper stating his exclusion principle, which said that no two identical particles with half-integer spin may occupy the same quantum state simultaneously. This was followed by Enrico Fermi's publication on the statistics of particles that obey the exclusion principle. In the same year, Paul Dirac connected the Bose and Fermi statistics of particles with the symmetry properties of their wave functions and named the particles "bosons" and "fermions."


Dirac

Bosons (e.g., photons) have integral spin and obey Bose-Einstein statistics. Moreover, two or more particles can occupy the same quantum state. Fermions (e.g., electrons), on the other hand, have half-integer spin and obey Fermi-Dirac statistics (discovered in 1926). Moreover, two or more Fermions cannot occupy the same quantum state due to the Pauli exclusion principle.



Velocity-distribution data of a gas of rubidium atoms, confirming the discovery of a new phase of matter, the Bose-Einstein condensate.

NIST/JILA/CU-Boulder

Bose believed that Einstein misunderstood the content of his paper. This appeared to dampen his desire to pursue his research. While in Germany, Bose did not publish any papers and even after he returned to India, he did not produce any original work in theoretical physics for the next 12 years.

Bose-Einstein condensate

One week after he received Bose's first paper, Einstein extended Bose's statistics and applied them to a monoatomic ideal gas whose numbers are conserved. He submitted three papers on this work to the Prussian Academy of Science in Berlin on 10 July 1924, 8 January 1925 and 29 January 1925. He did this without Bose's consent and without asking him to collaborate. Furthermore, in two of the papers, Einstein incorrectly cited Bose's statistics as having been devised by D. Bose, referring to another physicist, Debendra Mohan Bose.

The paper Einstein presented to the Prussian Academy on 8 January 1925 showed that, below a critical temperature, there was a collection of identical atoms, some of which would make a transition to the first quantum state—a state without kinetic energy. Einstein wrote: "A separation occurs; one part 'condenses,' the rest remains a 'saturated ideal gas.' "

The concept of a BEC changed from an idea to reality in 1938 when Fritz London made the proposal that the 1928 discovery of superfluidity in helium-4 was due to a BEC.

A modern description states that, at high temperatures, the particles in a Boson gas are distributed into energy levels given by Bose-Einstein statistics. Below a critical temperature, a macroscopic number of particles condense in the ground state and form a BEC. Thus, we can observe quantum behavior on a macroscopic scale and investigate many of the fundamental properties of matter. The first BEC of a gas was made in 1995 by Eric Cornell and Carl Wieman; they and Wolfgang Ketterle shared the 2001 Nobel Prize in Physics for their work.

Europe and late life

From 1924 to 1926, Bose lived in Europe. He spent the first year in Paris and then went to Berlin, where he was hoping to work with Einstein, since he knew that Einstein had published a subsequent paper that extended his work and predicted the Bose-Einstein condensation.

One week after he received Bose's first paper, Einstein extended Bose's statistics and applied them to a monoatomic ideal gas whose numbers are conserved.

When they finally met in Berlin, Einstein was working on a new subject: the unification of the electromagnetic and the gravitational fields. This change of research precluded them from working together; however, Einstein gave Bose a letter of introduction, which proved helpful for Bose to meet many German physicists.

With letters of recommendation from Albert Einstein, Paul Langevin and Herman Mark, Bose became a professor and the head of the department of physics when he returned to Dhaka, despite the fact that he did not have a doctoral degree. (Incidentally, D.M. Bose had previously turned down an offer for the same position.) Bose's mission was to introduce the latest developments in physics and chemistry to his university.

In 1945, Bose moved to Calcutta University, where he remained until 1956 when he became a professor emeritus in his retirement. He then taught physics at Santiniketan, the school founded by Rabindranath Tagore, a well-known Renaissance man, in 1921. Bose and Tagore were friends, and Tagore dedicated his book *Our Universe* to Bose. However, Bose faced severe opposition from the other teachers in the arts who feared that the teaching of science would move the schools' focus away from their subjects; therefore, after two years at Santiniketan, Bose again returned to Calcutta as a national professor.

Later in his career, Bose received many awards and honors for his contributions to science education and research in India. He was elected president of the Indian Physical Society in 1945 and held the position until 1948. He also served as president of the National Institute of Sciences of India from 1948 to 1950. In 1958, he was made a Fellow of the Royal Society. From 1954 to 1959, he was made a member of the upper chamber of the national parliament.

Bose died one month after his 80th birthday on 4 February 1974. As a tribute to his life's work, the government of India established the Satyendra Nath Bose National Center for Basic Sciences in Calcutta in 1986. Although they didn't always agree, Bose's work with Einstein changed the landscape of new physics in the 20th century. Nearly 40 years after his death, his additions to scientific knowledge continue to benefit the community and will do so for generations to come. OPN

ONLINE EXTRA

To view the complete set of letters between S.N. Bose and Albert Einstein, see the online version of this article at www.osa-opn.org.

Barry R. Masters (brmail2001@yahoo.com) is a Fellow of AAAS, OSA and SPIE. He is a visiting scientist with the department of biological engineering, Massachusetts Institute of Technology, Cambridge, Mass., U.S.A.

REFERENCES & RESOURCES

- S.N. Bose. "Plancks Gesetz und Lichtquantenhypothese," Zeitschrift für Physik **26**, 178 (1924).
- S.N. Bose. "Wärmegleichgewicht im Strahlungsfeld bei Anwesenheit von Materie," Zeitschrift für Physik **27**, 384 (1924).
- A. Einstein. "Quantentheorie des einatomigen idealen gases," Sitzungsberichte der Preussischen Akademie der Wissenschaften **XXII**, 261 (1924).
- A. Einstein. "Quantentheorie des einatomigen idealen gases. Zweite Abhandlung," Sitzungsberichte der Preussischen Akademie der Wissenschafte **I**, 3 (1925).
- S.N. Bose. *S.N. Bose, The Man and His Work, Life, Part I, Collected Scientific Papers*. S.N. Bose National Centre for Basic Sciences, Calcutta, India (1994).
- S.N. Bose. *S.N. Bose, The Man and His Work, Life, Part II, Lectures and Addresses, Miscellaneous Pieces*. S.N. Bose National Centre for Basic Sciences, Calcutta, India (1994).
- S. Dasgupta. *Jagadis Chandra Bose and the Indian Response to Western Science*, Oxford University Press, New Delhi, India (1999).
- J.R. Anglin and W. Ketterle. "Bose-Einstein condensation of atomic gases," Nature **416**, 211-18 (2002).
- E.A. Cornell and C.E. Wieman. "Nobel lecture: Bose-Einstein condensation in a dilute gas; the first 70 years and some recent experiments," Rev. Mod. Phys. **74**, 875-93 (2002).
- A. Einstein. Facsimile of Einstein's handwritten 1925 paper on BEC, Quantentheorie des einatomigen idealen Gases. Zweite Abhandlung. www.lorentz.leidenuniv.nl/history/Einstein_archive/Einstein_1925_manuscript/index_1.html
- W. Ketterle. When atoms behave as waves: Bose-Einstein condensation and the atom laser, Nobel Lecture, 8 December 2001. www.nobelprize.org/nobel_prizes/physics/laureates/2001/ketterle-lecture.pdf